

ESD.83 Historical Roots Assignment

METHODOLOGICAL LINKS BETWEEN OPERATIONS RESEARCH AND STOCHASTIC OPTIMIZATION

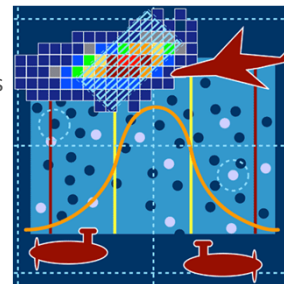
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Origins of Operations Research: World War II

- Need to bring scientific thinking to complex problems of warfare:
 - Determining optimal size of merchant convoy to minimize losses
 - Finding a target in efficient manner (search theory)
 - Planning bombing raids
- Birth of OR: 1936/37 British air force studied how radar technology could be used to control interception of enemy aircraft
- British and U.S. formed OR groups in air force and navy
 - U.S. Navy Antisubmarine Warfare Operations Research Group (ASWORG) organized in 1942 by **Philip Morse** (physicist)
 - U.S. Air Force Project SCOOP (Scientific Computation of Optimal Programs) in 1947 to “mechanize” planning procedures for training and supply activities (**George Dantzig** as mathematician)
- 1951 Philip Morse and George E. Kimball *Methods of Operations Research* introduced concepts of OR to fields outside of military and spurred method’s diffusion

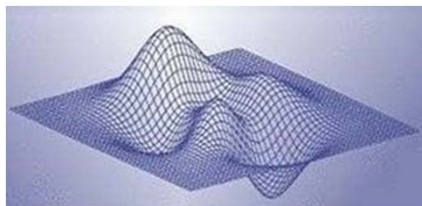


Operations research (OR)

- “a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control” (Morse & Kimball, 1951)
- “an interdisciplinary branch of applied mathematics and formal science that uses advanced analytical methods such as mathematical modeling, statistical analysis, and mathematical optimization to arrive at optimal or near-optimal solutions to complex decision-making problems” (Wikipedia)
- Broad field that applies well-developed methodologies to vast array of problems in various domains:
 - Transportation, resource allocation, industrial planning, agriculture, energy and environment, finance, manufacturing, supply chain, communications, marketing, scheduling, military operations
- “the most influential academic discipline you've never heard of” (Boston Globe)

Optimization in OR

- Optimization (or mathematical programming) : choosing the “best” element given some objective from a set of available alternatives
 - Often involves finding maximum or minimum of a function by systematically choosing values from within an allowed set



- OR typically involves finding best solutions that meet some objective (least cost, most profit, shortest distance, lowest fail rate, etc.)

→ Optimization is a natural tool in OR

Stochastic optimization

- Stochastic optimization combines OR methods of mathematical programming with advanced probability techniques to attack real world problems that involve uncertainty
 - Methodology really took off in 1990s

Deterministic Optimization

- Know all relevant information and no randomness
- Parameters defined by single values
- Maximize/minimize some function
- Produces same results from given set of initial conditions

Stochastic Optimization

- Uncertainty and randomness are present
- Parameters defined by probability distributions
- Maximize/minimize expectation of some function
- Produce different results from given set of initial conditions

Why Stochastic Optimization?

- 1) Large, computationally intractable deterministic problems
 - Deliberately introducing randomness into the solution search process can speed convergence, prevent getting stuck at local optima
- 2) Many problems are stochastic by nature → desire to better represent real systems which inherently contain uncertainty

Key players



Philip Morse

- Father of OR
- Founded ORSA
- Started MIT OR Center



George Dantzig

- Father of linear programming
- Simplex algorithm
- Early contributions to SO



John von Neumann

- Development of game theory
- Computer-based analysis
- Monte Carlo methods



Nicholas Metropolis

- Monte Carlo methods
- Metropolis-Hastings algorithm
- Greatly contributed to SO



John Little

- First PhD in OR (MIT)
- Queuing theory: Little's law



Thomas Magnanti

- Large-scale optimization



Richard Van Slyke

- Large-scale optimization



Roger Wets

- Major contributor in SO

Methodological links

Deterministic optimization methods & problems

Linear programming

- Developed during WWII
- Maximize cx , subject to $Ax \leq b$ and $x \geq 0$
- Simplex algorithm
- Ellipsoid and integer point algorithms

Nonlinear programming

- Involves non-linear equation
- Applicable to many real world problems

Integer programming

- Integer variable requirements
- NP-hard problems

Methodological links

Deterministic optimization methods & problems

Linear programming

Nonlinear programming

Integer programming

Combinatorial optimization methods & problems

Combinatorial optimization

- Finding least-cost solution
- Large set of discrete solutions
- Many are NP-hard
- Development of SO methods

Famous NP-hard problems

- **Traveling salesman**: finding shortest possible path
- **Knapsack**: determining number of items to maximize the total value
- **K-minimum spanning tree**: finding a tree spanning a subset of graph with minimum weight

Methodological links

Deterministic optimization methods & problems

Linear programming

Nonlinear programming

Integer programming

Combinatorial optimization methods & problems

Stochastic optimization methods & problems

Monte Carlo methods

- Computational algorithms that generate results from repeated random sampling
- Often used when unfeasible or impossible to find exact result by deterministic algorithms (Hubbard, 2007)
- **Metropolis-Hastings algorithm**: a Markov chain Monte Carlo method for obtaining a sequence of random samples when direct sampling is difficult

Methodological links

Deterministic optimization methods & problems

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Monte Carlo methods

Simulated annealing

- Metaheuristic for approximation of global optimum
- Tests random mutations on an individual solution
- Inspired by annealing in metallurgy
- Kirkpatrick, Gelatt and Vecchi in 1983

Genetic algorithm

- A general heuristics approach in search problems
- Works with a population of multiple solutions
- Inspired by natural evolution
- Practical application from the late 1980s

Stochastic tunneling

- Global optimization approach w/ accelerated process
- Attempt to overcome difficulties of SA
- 'Hopping' to solutions, 'tunneling' through barriers
- Hamacher and Wenzel in 1999

Simultaneous perturbation stochastic approximation

- Applies to large-scale optimization problems
- Changing multiple parameters simultaneously
- Efficient and often guarantees global optimum
- Spall in 1992

Traveling salesman problem (TSP)

- **Description**

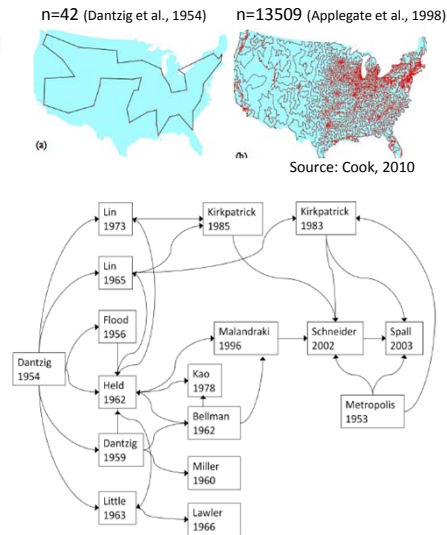
- Minimizing the distance (cost) traveled when visiting n cities exactly once and then returning home
- One of the most studied NP-hard problems

- **Early history**

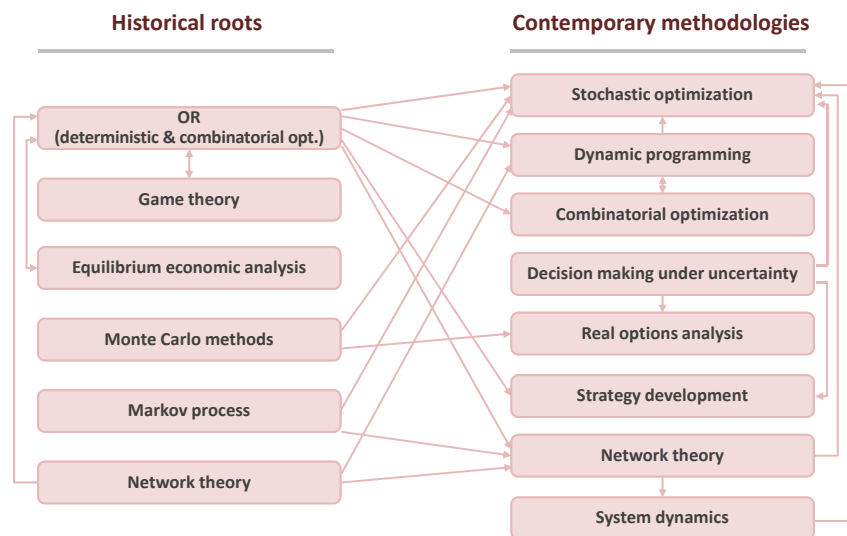
- Began to be popularized in the late 1930s by Flood and Whitney
- Since 1954, TSP has become a test problem for new methods

- **Link between deterministic and stochastic optimization**

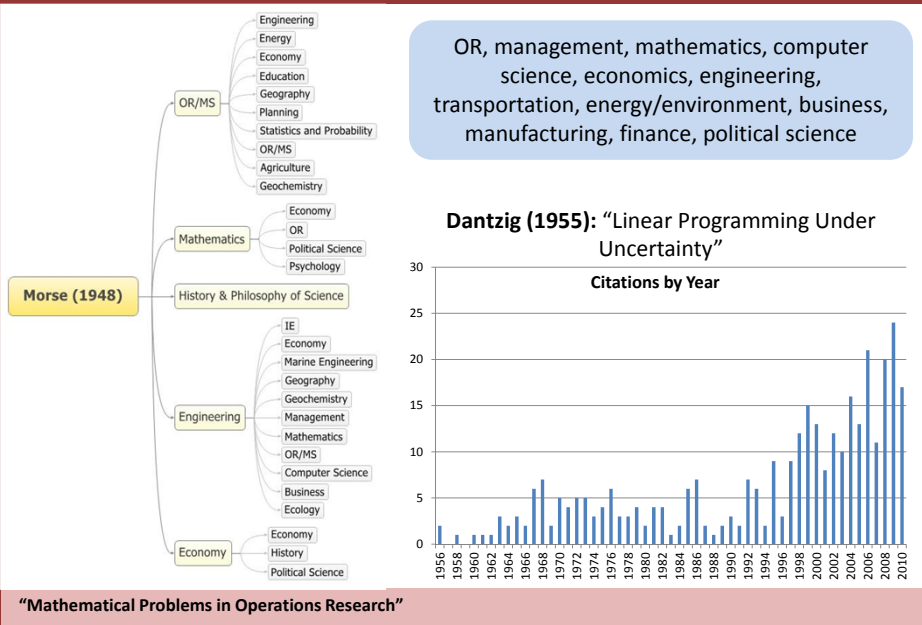
- Originally cast as a deterministic optimization problem
- Recast to incorporate uncertainty in testing new stochastic methods



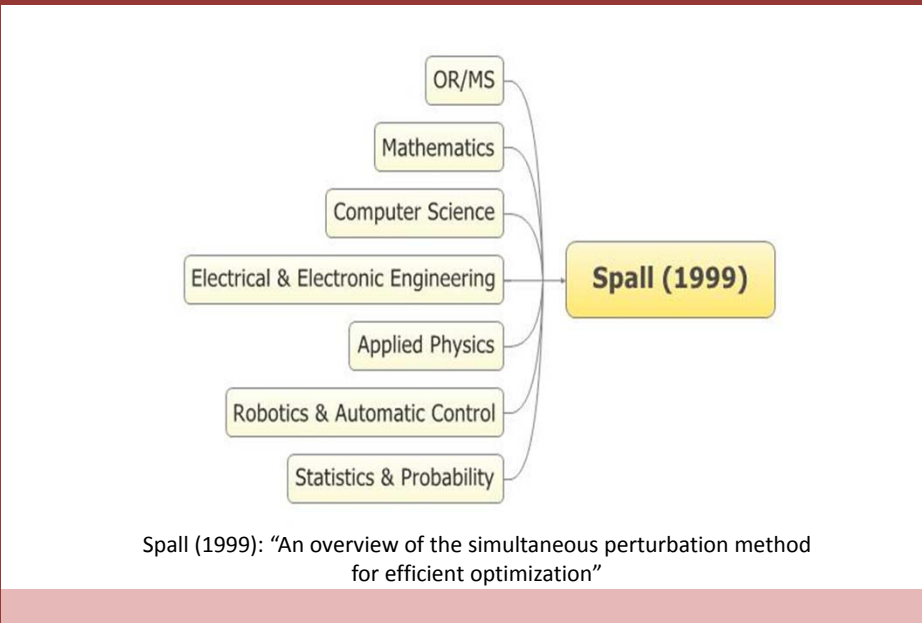
Related methodologies



Citation analysis: forward casting

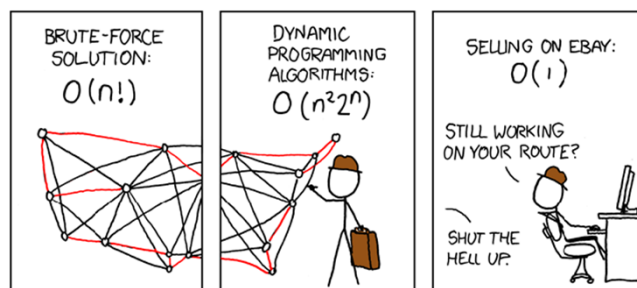


Citation analysis: backward casting



Conclusion

- OR and stochastic optimization intimately linked
 - SO developed out of early OR methods of mathematical programming
 - Modern OR textbooks place SO as one of several OR methods
 - SO has become methodology in own right
- Both used in numerous fields, related to many historical and modern methodologies, and applied to wide range of domains and problems
- Citation analysis demonstrates relationships
- Computer technology has been key to both methodologies
 - Still hit computing limits and need to advance methods to overcome “Curse of Dimensionality”



Source: <http://xkcd.com/399/>

Thank you :)