# Graph Theory Applications in FOCUS-compliant Ship Designs

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*Abstract***— Recent developments in the storage of system data in the Navy's data repository, LEAPS, using the FOCUS product meta-model have opened the doors to graph-theory applications in the design of Navy ship systems in the early stages of design. In this paper, we demonstrate the ability to extract graphs from ship data and present pertinent applications of such graphs including a vulnerability metric for early-stage design, an equipment-sizing algorithm for automated system design, and a network design process that includes vulnerability assessment with preliminary ship arrangements.**

#### *Keywords—graph theory, networks, ship system design, metrics*

## I. INTRODUCTION AND MOTIVATION

The Navy currently employs a common data repository, the Leading Edge Architecture for Prototyping Ships (LEAPS) [1], for the storage of all data related to the early-stage design of ships. The anticipated goal is that LEAPS will become the data repository for all ship-related design data throughout the full life cycle of the ship class. The Formal Object Classification for Understanding Ships (FOCUS) is the product meta-model (PMM) governing the storage of surface ship design data, which delineates the format, location and ontology for pertinent data.

Storing data in an organized manner using a common ontology enables the seamless sharing of data between numerous early-stage design tools, thus eliminating errors caused by data re-entry and ensuring a single, definitive description of the state of a ship design.

The Navy maintains a set of early-stage design tools that are LEAPS-integrated and FOCUS-compliant, including such tools as:

• ASSET: The Advanced Surface Ship Evaluation Tool creates a ship synthesis model based on parametric analysis of previous ships and several input parameters.

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Morpheus: An application for creating and modifying hullform models.

IHDE: The Integrated Hydrodynamics Environment performs hydrodynamic analysis of a ship model using a variety of computational fluid dynamics methods.

SHCP-L: The Ship Hull Characteristics Program – LEAPS performs intact and damaged stability analysis of a ship.

RSDE: The Rapid Ship Design Environment enables design space exploration employing all the above listed tools.

S3D: Smart Ship Systems Design is a ship system design and analysis tool that is currently undergoing integration with LEAPS.

The definition of system connectivity storage within the FOCUS PMM has recently been explored and expanded [2], thus enabling the ability to extract graphs from the stored data. Graph theory can be used to quickly evaluate system performance, enabling many evaluations to be accomplished rapidly on a huge number of ship designs; this is an enabler to the implementation of set-based design and the analysis of broad design spaces.

This paper describes the methodology for extracting a connectivity diagram from a LEAPS database in Section 2, then proceeds to describe several applications that this methodology enables. Specifically, Section 3 describes an equipment sizing algorithm, Section 4 describes the calculation of a vulnerability metric, and Section 5 describes a network design process.

# II. EXTRACTING A CONNECTIVITY DIAGRAM FROM A LEAPS DATABASE

The computer code described herein works directly on data stored in a LEAPS database in a FOCUS-compliant manner to extract a connectivity diagram and pertinent properties from ship data. Although a connectivity diagram is not explicitly stored in a LEAPS database, the FOCUS PMM provides all the requisite information through a series of components, nodes, and connections, along with properties denoting such pertinent information as capacity of the connections, lengths of paths and losses in components, locations of equipment, etc.

A FOCUS-compliant system connectivity example is diagrammed in Figure 1 and described in detail below.

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The LEAPS Diagram is the point of departure for determining the connectivity information for a system. The root connection of a Diagram is a System Connection which contains one or more Component Exchange Connections. Component Exchange Connections contain three members: two Components and the Exchange Connection that links them. Thus, from the component exchange connection level one can determine which components are connected to one another in which order; however, one must continue further down into the system description to determine information on the type of connection.

A LEAPS Component may have one or more Ports which are connected to one or more Terminals using Port-Terminal Linkages. A single Port-Terminal Linkage will link a single port to a single terminal. An Exchange Connection links two Port-Terminal Linkages.

The Ports will typically have properties that pertain to the commodity being transferred through the port, whereas the Components will typically have properties that pertain to the component as a whole. For example, an electrical port may have properties for voltage and current and a piping port may have properties for liquid mass flow rate and liquid temperature, whereas the component to which the electrical port and piping port are attached will have properties such as electrical power required, efficiency, heat produced, weight, and physical dimensions.

An adjacency matrix is created by tracing the connectivity information at the component exchange connection level, leading to a matrix in which entry  $(a,b)$  is equal to 1 if component a is connected to component b, and 0 otherwise. The LEAPS system description does not specifically denote directionality; however, directionality can either be parsed from information about the Component or can be specifically stored within the LEAPS database as a property of the port. In addition to the adjacency matrix, a list of pointers to the Components and a list of pointers to the Exchange Connections is created. These pointers can be used to extract pertinent data from the Components and the Ports to determine all information needed for further analysis.

Since an adjacency matrix for a shipboard distribution system is typically sparse, an adjacency list provides a more streamlined method for storing and accessing the data. Such an adjacency list provides, for each vertex, a list of vertices connected to that vertex via an edge, with flow traveling



Figure 1. Example FOCUS-compliant connectivity diagram

outward from the primary vertex. Thus, directional information is recorded. Directional edges are called arcs.

## III. APPLICATION: EQUIPMENT SIZING

Chalfant et al. [3] present a method for automation of the system design process appropriate to the early stages of design under the RSDE paradigm. One step of the process is the initial sizing of distribution system components, which is accomplished using a graph of the system extracted from LEAPS. As an example, we examine the design of an electrical distribution system. Given a set of generators, a set of electrical loads, and a fully connected distribution system to move the power from the generators to the loads, the sizing of each piece of distribution equipment depends upon the flow of power in any normal possible operational arrangement. See, for example, [Figure 2](#page-1-0) from [4], in which generators (PGMs) provide power to loads (EMRG, Radars, ACLCs, IPNCs) via distribution system components (Bus Nodes, PCMs, switches and cables). If the electrical power rating of the generators and loads is known or assumed, then the rating of the distribution equipment can be determined through observation of the power flow under each possible configuration.

One algorithm for determining the size of the components in a distribution system follows. Note that other algorithms are possible depending on the desired end state of the system; for example, another algorithm could balance the percent of full load power provided by each generator.

- a) Create the adjacency matrix for the system in the baseline configuration.
- b) Add a connection from each source (generator) to a single supersource.
- c) Assign a cost to each edge proportional to the losses incurred traveling from one vertex to the next via that edge.
- d) For each load, determine the minimum-cost path through the network from the load to the supersource.
- e) Following the minimum-cost path, add the rated power of the load, increased by any losses in each



<span id="page-1-0"></span>Figure 2. Sample electrical distribution system

node traversed, to a capacity matrix representation of the network.

- f) If the capacity of a generator is exceeded, delete the connection between the generator and the supersource, subtract the excess demand from the path, and repeat the min-cost path process with the atcapacity generator removed from the system.
- g) After completing this process for each load, compare the capacity of each connection in the network to a stored capacity matrix, saving the maximum capacity required in any vertex.
- h) Repeat this algorithm for every possible operational alignment.

The typical distribution system, however, has enough possible states that an exhaustive search of every combination is prohibitive. Further observation of [Figure 2](#page-1-0) reveals somewhat obvious clustering that would allow a reasonable search of the possible configurations; graph theoretic algorithms can be employed to take advantage of such clustering completely autonomously, making this a tractable process despite complex networks.

This process depends on a significant simplification of a complex interrelated system of electrical components. In the very early stages of design, however, this is a reasonable approach due to several factors. The linearized problem statement facilitates a deterministic answer that can be automatically computed much more quickly than when using load-flow or other equivalent methods. The inaccuracies induced through the linearization are acceptable in this first-cut sizing algorithm for several reasons. Many of the components in the system include inherent ability to accommodate overpower situations for short periods of time; e.g. operating cabling at higher than rated steady-state current generates extra heat, but this is acceptable as long as the amount of excess heat is not too great and the duration is short so the heat can be dissipated without significant damage to the cable. Further, it is traditional to assign a margin to equipment ratings in the early stages of design to account for uncertainties in the design; this margin can account for the inaccuracies here.

The algorithm described here can be used as described to size the equipment. As design progresses and more accuracy is desired, the algorithm can also be used to determine the most stressing configurations which can then be evaluated in a higher-fidelity manner, using a load-flow methodology or full dynamic analysis.

# IV. APPLICATION: VULNERABILITY METRIC

Chalfant et al. [5] present a simple vulnerability metric applicable to the very early stages of design to analyze the impact of ship system design on overall survivability of the vessel. The two-part metric measures two distinct effects of damage: the first part sums the value of all loads that can be serviced, indicating an overall ability to provide and distribute power in the face of damage; whereas the second part indicates the highest priority load that cannot be filled while satisfying all higher priority loads, providing an indication of the severity of the impact of lost loads.

This method can be used for any type of service or a combination of services, e.g. electrical power, cooling water, and control data. The metric operates at whatever level of fidelity the ship system is modeled in LEAPS.

The calculation process for the vulnerability metric is summarized below:

- Begin with a prioritized, weighted list of loads, a list of sources, and a description of the distribution system connecting them.
- Create connectivity and capacity matrices describing the network.
- Run the connectivity analysis on an undamaged version of the system and calculate the maximum possible survivability score.
- Impose damage upon the ship and remove damaged equipment from the directed graph; this damage can consist of single or multiple simultaneous blasts. A blast is modeled as a sphere of impact; any piece of equipment that intersects with the sphere is considered damaged.
- Conduct the prioritized electrical analysis to determine which loads are filled and calculate the as-damaged survivability score and priority score.
- Repeat the damage process for a large number of blasts, with centroid locations determined stochastically. Average both metric scores (max and total) over a large number of blasts.

Applying blast centroids in a stochastic manner allows the metric to remain unclassified and allows very rapid calculation. In addition, using a stochastic method prevents gaming the ship design to meet the test criteria, e.g. positioning equipment at the exact spot required to avoid damage. The blasts are distributed evenly to port and starboard, evenly from baseline to the top of the ship, and distributed in a Gaussian curve from forward to aft, thus concentrating blasts in the section of the ship that is most likely to be impacted. See Figure 3 for full



Figure 3. A few blast damage spheres shown at full size (top), and a full complement of damage locations, shown at 1/10 size for clarity [5]

size blast spheres (top) and a full test case of blast spheres shown at reduced size for clarity (bottom).

The vulnerability metric uses two standard graph-theory algorithms. Dijkstra's algorithm calculates the lowest cost path from a single node to each other node in a directed, weighted graph. A max-flow, min-cost algorithm determines the maximum flow that can pass through a network at the minimum cost without exceeding the capacity of any one connection in a graph. Using these two algorithms, we are able to determine for each damage scenario not only whether sufficient supply exists for the loads that remain, but also whether the loads are connected to the sources and whether those connections have sufficient capacity for the configuration. For details of the algorithm along with test cases and further analysis, see [5].

# V. APPLICATION: ARCHITECTURE FLOW OPTIMIZATION AND VULNERABILITY ANALYSIS

Network theory is a subset of graph theory where edges or arcs are assigned values enabling quantitative network theory analyses of various types. This can be used to advantage with large multiplex systems as shown in Figure 4. Brown et al. are extending work by Trapp [6] to optimize the architecture of a multiplex (multisystem) ship system including interdependent combat, mechanical, electrical and thermal subsystems. Indirectly this approach can also perform basic vital component and transmission sizing.

 Connectivity data is extracted from LEAPS, put into a network form visualized in Figure 4, and restructured as input into a linear programing (LP) optimizer for performing a variation of Trapp's Non-Simultaneous Multi-Commodity Flow (NSMCF) method that we call a Non-Simultaneous Multi-Constraint Parallel-Commodity Flow (NSMCPCF) method or more simply an architecture flow optimization (AFO). In this optimization, vertices or nodes represent vital system components and arcs represent (one-way) transmission components including piping (thermal fluid systems), cable (electrical power systems and control), and shafting (mechanical systems). Each arc carries only one commodity in one direction (non-simultaneous), but nodes may have multiple ports, arcs and commodities coming and going (parallel-commodity).

Multiple non-simultaneous constraint sets in a LP formulation are applied in the optimization. Each set represents specific operating and damage conditions to be considered. Component models for vital component nodes are linear continuity, efficiency, capacity and performance relationships for the incoming and outgoing commodities. A combatant ship multiplex system may have more than a thousand nodes and many thousands of arcs. Individual plexes (subsystems) may be of various types. They may have only a few direct point to point paths through the system as in a mechanical drive propulsion system, may require closed-loop paths for operability as in lube oil and chilled water systems, or may allow many redundant paths through a complex mesh restorable network as in an electrical distribution system.

Dependencies between plexes may also be extensive as with electric power and cooling. Redundant nodes and arcs may be specified and systematically reduced in the cost optimization to a more affordable sufficient set. The minimum cost objective includes both sized components and transmission.

 Since our intention is to consider hundreds of alternative systems each with multiple possible architectures, it is essential to automate the formulation of the LP problem from initial system definition in LEAPS as in templates or patterns to a refined architecture definition and then put the refined data back into LEAPS. This is done for hundreds of system combinations. In order to consider weapon damage vulnerability as a constraint in this optimization, a preliminary arrangement of system vital components within the total ship is also required. This is accomplished as shown in Figure 5.

 In this process, preliminary system options for power, energy and combat systems are identified. This includes lists of vital components assembled in ensembles with different levels of capability. Table 1 is an example (partial) for Anti-Air Warfare option (AAW1). There are 66 vital components in this option: some are collocated groups of components. Each is assigned to a compartment where it is typically found and is represented by specific component capacity, weight, power and space data in a Combat System Equipment List (CSEL). Figure 6 shows the corresponding AAW1-only system network with dependencies. Similar lists and system networks or templates would be extracted from LEAPS for each system option. These systems can then be explored and architecture optimized in various discipline-specific studies including basic vulnerability and dynamic analysis. These discipline-specific studies are similar to those that might be done in parallel as initial explorations of a set-based design. Feasible and nondominated sets can then be brought together with the results of other discipline-specific studies and their intersection used to identify infeasible designs outside of the feasible set.



Figure 4. Simplified Surface Combatant Multiplex System Network



Figure 5. Ship Subsystem Design and Total Ship Integration Process [7]

 After completing a hullform exploration, representative hullforms are created for each system option combination in the design space. For example, if there were 3 AAW options, 3 ASW options, 3 ASUW options and 5 power and energy system options, 135 representative hullforms with subdivision would be created. Each of these hullforms is idealized into subdivision blocks (SDBs) as shown in Figure 7.

 Compartments and their associated vital components (VCs) are then assigned to subdivision blocks using an operability and minimum hit probability algorithm which may be implemented in the first (allocation) phase of the Integrated Ship Arrangements (ISA) software, already LEAPS-compliant. The probability of hit algorithm and subsequent ship capability vulnerability assessment are described in references [7,8,9]. Representative hit probability results are shown in Figure 8 and Preliminary Arrangements are shown in Figure 9.



Figure 6. AAW System Option 1 Network

Once representative ship preliminary arrangements are created for each combination of system options, each representative design is assessed for vulnerability. This requires additional system data in deactivation diagram form.



Figure 7. Hull Subdivision Block [7,10]



Figure 8. Subdivision Block (SDB) probability of hit [7,9,10]

TABLE 1. PARTIAL LIST (31 OF 66 COMPONENTS) - AAW SYSTEM OPTION 1



These additional data may also be extracted from LEAPS system connectivity data and converted to a logical network topology similar to the multiplex architecture shown in Figure 4. This is mapped to a deactivation block diagram form using a Breadth-First Search (BFS) algorithm for traversing or searching tree or graph data structures. This starts at the tree root (or some arbitrary node of a graph, sometimes referred to as a search key) and explores the neighbor nodes first, before moving to the next level neighbors. The output of the algorithm identifies all vertices reachable from the root and results in a deactivation diagram similar to that shown in Figure 10.



Figure 9. Preliminary Compartment and Vital Component Arrangement [10]



Figure 11. Damage extent ellipsoids applied to SDBs [7,9]

Using these system DBDs, hit locations shown in Figure 8 are used as the centroids of damage extent ellipsoids which are sized for the particular threat weapons, ship structure and subdivision [9] and applied to the ship SBDs. Any intersection between ellipsoids and SBDs results in the deactivation of any VCs within the intersected SDBs as shown in Figure 11. These deactivations are rolled up to the ship system level using the system DBDs to determine the resulting loss in ship cababilities and ultimately an overall measure of vulnerability.

Preliminary compartment and vital component locations, optimized system architecture and the resulting vulnerability metric value are passed back to LEAPS and may be used in total ship synthesis in RSDE/ASSET or in S3D for more complete system modeling, assessment and synthesis.

### VI. CONCLUSIONS

We have presented a methodology for extracting graphs from a system that is stored in LEAPS in a FOCUS-compliant manner and demonstrated several applications using these graphs in the areas of equipment sizing, vulnerability calculation, and network design.

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Figure 10. Hybrid Electric Drive Deactivation Block Diagram [8]