Dynamical catalysis

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If a biomolecular chemical reaction is mass transport limited, the reaction rate can be accelerated by embedding one of the reaction centers in a molecular framework. This framework acts as a lower dimensionality surface along which the other reaction partner can travel. The rate enhancement can be identified with an increase in entropy of activation. A model for this effect is constructed and applied to a reaction center embedded in three molecular framework geometries: a linear chain, a disk, and a polar reactive patch on a sphere. The new result for the reactive patch on a sphere is relevant for enzymatic systems where substrates must find their way to a small reactive region.

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I. INTRODUCTION

In the conventional picture of bimolecular reaction kinetics, molecular species A and B reversibly form a reaction complex (A/B) that passes through a transition state or intermediate to form product. The reaction mechanism is

\[ A + B \overset{k_1}{\underset{k_2}{\rightleftharpoons}} (AB) \rightarrow \text{product}. \]  

(1.1)

After making the steady state approximation for the reaction complex intermediate, \( d[(AB)]/dt = 0 \), the effective bimolecular rate coefficient \( k_{\text{eff}} \) is found to be

\[ k_{\text{eff}} = \frac{k_1 k_2}{k_{-1} + k_2}. \]  

(1.2)

Ordinarily the rate-limiting step is the second step in the reaction mechanism Eq. (1.1) because the chemistry of making and breaking bonds is usually slow compared to the dynamics of reactants approaching one another. In this limit \( k_{-1} \gg k_2 \) and

\[ k_{\text{eff}} \rightarrow \frac{k_1}{k_{-1} k_2}. \]  

(1.3)

The rate coefficient for the chemical step is assumed to have the form:

\[ k_2 = k_2^0 \exp\left[-\Delta F^f/k_b T\right], \]  

(1.4)

where \( k_2^0 \) is equal to \( k_b T/\hbar \) according to transition state theory, \( k_b \) is Boltzmann’s constant, \( T \) is the absolute temperature, and \( \Delta F^f \) is the free energy difference between the reaction complex \( (AB) \) and the transition state.

Catalytic activity occurs through a lowering of the transition state free energy relative to the reaction complex intermediate either by a change in the transition state enthalpy \( \Delta H^f \) or entropy \( \Delta S^f \) difference.

In the opposing limit, when mass transport is rate limiting, \( k_{-1} \gg k_2 \) and

\[ k_{\text{eff}} \rightarrow k_1. \]  

(1.5)

Evidently, if a means is found to modify the rate of approach, \( k_1 \rightarrow k_1^* \), then there effectively will be catalytic activity in this mass transport rate determining limit.

In this paper we examine a general mechanism that can increase the rate of encounter and separation of A and B,

\[ k_1^+ = k_1 (1 + g), \quad k_{-1}^- = k_{-1} (1 + g), \]  

(1.6)

where the quantity \( g \) is the increment to the forward and backward rates of encounter. The effective rate coefficient becomes

\[ k_{\text{eff}} = \frac{k_1^+ k_2}{k_{-1}^- + k_2}. \]  

(1.7)

Formally, the enhancement to the rate coefficient can be expressed as an entropy of activation that we denote as

\[ \Delta S_d = k_B \ln(1 + g). \]  

(1.8)

Note that the pre-equilibrium constant

\[ K = \frac{k_{-1}^+}{k_1^-} = \frac{k_1^+}{k_{-1}^-}. \]  

(1.9)

is unaffected by the enhancement in the forward and reverse rates. This rate enhancement has many of the attributes of a catalytic effect so that we term it a dynamical catalytic effect. This dynamical effect is only present when mass transport is the rate-limiting step and, in principle, it involves no modification of the transition state.

The mechanism for enhancing the rate of encounter arises when a reactive partner is embedded in a polymer, a two-dimensional framework, or on the surface of a sphere. The passive framework acts as a lower dimensionality network that more efficiently transports the reacting substrate to the reaction center.

Adam and Dlbruck\(^1\) were the first to point out that there could be a rate enhancement over free diffusion if the mass transport proceeds in successive stages of lower dimensionality. Considerable work has been done on the case of a reactive site embedded in a linear polymer\(^2–6\) with the work of von Hippel and co-workers on the translocation of proteins on DNA indicating the biological relevance of this problem.

The case of a reactive site on a spherical macromolecule is perhaps the one of greatest interest because of the relevance to reactions at globular enzymes. For many years it
has been recognized that there is a considerable difference between the diffusion-controlled rate to a spherical target molecule that is equally reactive over its entire surface and a spherical target molecule that has only a reactive patch on its surface. Alberty and Hammes\(^7\) first pointed out the significance of this difference for the diffusion rate to an enzyme molecule and to its active site. The kinetics of diffusion-controlled reactions between chemically asymmetric molecules when one or both of the translating and rotating reacting partners possesses an axially symmetric reactive region on a sphere has been investigated by Solc and Stockmayer,\(^8\) Schmitz and Schurr,\(^9,10\) and Temkin and Yakobson.\(^11\)

When the added complications of diffusion to a buried active site\(^12\) are included, it becomes clear that there must be some means beyond free diffusion that permits a small substrate to find the reacting center of an enzyme. The possibility of free diffusion to the surface of the enzyme followed by sliding on the lower dimensionality spherical surface to the active site has been suggested\(^13\) and the role of intermolecular interactions in such a mechanism has been investigated.\(^14\)

II. THE MODEL

The means for realizing this dynamic rate enhancement is to place one of the reacting partners, say B, in a molecular network that accelerates the frequency of encounters. The enhancement will be accomplished if (1) the mobile A species finds the molecular network faster (because it is larger) than the free reacting B center, and (2) the mobile species rapidly diffuse along the molecular network to the reacting center. In sum, the molecular network serves as a lower dimensional pathway for reaction partners to find each other.

A. Reactive center at one end of a linear polymer

In this model the reactive center B is located at one end of a polymer chain of N homogeneous units. Species A binds reversibly from solution to each polymer site \((n = 1,...,N)\) and, if bound, A can shuttle to adjoining sites with rate q. If A reaches the end site \(P_0\) where B is located it may react irreversibly to form product. For simplicity, we assume that the on and off rates for A onto polymer sites is the same as the rate of association and deassociation of A and B to the encounter complex (AB) when A and B are free in solution; we also assume that the \(k_2\) for A reacting with B when B is bound in the polymer is the same as when B is free in solution. See Fig. 1.

The kinetic equations for this reaction mechanism are

\[
\frac{dP_0}{dt} = k_1AP + qP_1 - (q + k_{-1} + k_2)P_0, \quad (2.1a)
\]

\[
\frac{dP_n}{dt} = k_1AP + q(P_{n-1} + P_{n+1}) - (2q + k_{-1})P_n, \quad n = 1,...,N-1, \quad (2.1b)
\]

\[
\frac{dP_N}{dt} = k_1AP + qP_{N-1} - (q + k_{-1})P_N. \quad (2.1c)
\]

In Eq. (2.1) \(P_n\) is the concentration of sites at position on the polymer chain that have bound \(A\), \(k_1AP\) is the flux of \(A\) from solution that binds to each site, \(k_{-1}P_n\) is the dissociation rate of \(A\) from site \(n\), and \(q\) denotes the shuttling rate between adjacent sites. The quantity \(P\) is the concentration of each free site \(n\) available for binding \(A\) and this is proportional to concentration of \(B\) since, by assumption, there is one \(B\) per chain located at position \(n = 0\); thus \(P_0 = B\). Equation (2.1c) can be transformed into the general form of Eq. (2.1b) by introducing the auxiliary quantity \(P_{N+1}\) and adopting the boundary condition

\[
P_{N+1} = P_N. \quad (2.1d)
\]

Rather than solve the set of finite difference equations, we pass to a continuous description of the concentration of \(A\) bound along the chain \(c(x)\) assuming that the polymer sites are separated by equal distance \(\delta\). In addition we introduce the steady state approximation \(dP_n/dt = 0\). Thus Eq. (2.1b) becomes

\[
0 = k_1AP + Dc''(x) - k_{-1}c(x) \quad (2.2)
\]

with \(D = q\delta^2\) for \(0 < x < L = \delta N\). The boundary condition at the end of the chain Eq. (2.1d) is simply

\[
c'(L) = 0. \quad (2.3)
\]

The boundary condition at the origin where the reactive center \(B\) is located, Eq. (2.1a) becomes

\[
0 = k_1AP + \frac{D}{\delta} c'(0) - (k_{-1} + k_2)c(0), \quad (2.4)
\]

which states the steady state flux balance at \(B\): Free \(A\) arriving from the solution plus bound \(A\) arriving from the polymer chain either reacts or dissociates back into solution.

The general solution to Eq. (2.2) which satisfies the boundary condition Eq. (2.3) is

\[
c(x) = a \cosh[(x-L)\lambda] + k_1AP/k_{-1}, \quad (2.5)
\]

where \(\lambda = \sqrt{k_{-1}/D}\). The constant \('a'\) is determined from the boundary condition Eq. (2.3); one finds

\[
a \cosh(\lambda L) = \frac{-k_2k_1AP/k_{-1}}{k_2 + k_{-1} + \frac{D}{\delta} \lambda \tanh(\lambda L)}. \quad (2.6)
\]

The appearance of product is given by

\[
\frac{d[\text{product}]}{dt} = k_2c(0) = k_2 \left[ a \cosh(\lambda L) + \frac{k_1AP}{k_{-1}} \right]. \quad (2.7)
\]
This result should be compared with the kinetic equation:

\[
d\frac{[\text{product}]}{dt} = k_{\text{eff}} \text{A} \text{B}.
\]

(2.8)

Substitution of Eq. (2.6) into Eq. (2.7) and use of the fact that the concentration of \( P \) is proportional to \( B \), leads to an expression for \( k_{\text{eff}} \) of the form of Eqs. (1.6) and (1.7) with

\[
g = \frac{L}{\delta} \tanh(\lambda L),
\]

(2.9)

where \( \lambda = \sqrt{k_{-1}/D} \).

In the limit of rapid diffusion along the chain, the maximum enhancement is exhibited. In this limit \( \lambda \to 0 \), \( g \to (L/\delta) = N \), and thus \( k_{+} = k_{1}(1 + N) \). Diffusion along the chain is so rapid that each polymer site effectively presents a higher concentration of \( A \) to the reactive site located at the end of the chain.

In the opposite limit of slow diffusion, \( \lambda \to \infty \), \( g \to 0 \), and \( k_{+} \to k_{1} \). As expected, the presence of the polymer chain makes no difference to the rate of reaction.

B. Reactive center internal to a linear polymer chain

In this model the reactive center \( B \) is located at position \( n = 0 \), which is attached to two polymer chains of length \( N_{1} \) and \( N_{2} \). See Fig. 2. In the continuous limit the appropriate steady state equations for each chain are

\[
0 = k_{1}AP + Dc'_{i}(x) - k_{-1}c_{i}(x), \quad i = 1, 2,
\]

(2.10)

with the boundary condition at the chain ends \( c'_{i}(L_{i}) = 0, i = 1, 2 \). At the origin, where the reactive site is located, the boundary condition is the balance between incoming and outgoing flux:

\[
0 = k_{1}AP + \frac{D}{\delta} c'_{1}(0) + \frac{D}{\delta} c'_{2}(0) - (k_{-1} + k_{2})c_{1}(0)
\]

(2.11)

with the auxiliary condition \( c_{1}(0) = c_{2}(0) \) (Fig. 3).

The general solution for the concentration of \( A \) particles on each chain is of the following form:

\[
c_{i}(x) = a_{i} \cosh[(x - L_{i})\lambda] + k_{1}AP/k_{-1}.
\]

(2.12)

The constants \( a_{1} \) and \( a_{2} \) are determined from the boundary condition at the origin, Eq. (2.11) in a manner exactly similar to the procedure used in the previous section. One obtains a result for \( k_{\text{eff}} \) of the general form Eqs. (1.6) and (1.7) with

\[
g = \frac{L_{1}}{\delta} \tanh(\lambda L_{1}) + \frac{L_{2}}{\delta} \tanh(\lambda L_{2}).
\]

(2.13)

Again the maximum enhancement is found for rapid diffusion along the chain. In this limit \( \lambda \to 0 \), \( g \to (L_{1} + L_{2})/\delta \), and \( k_{+} = k_{1}(1 + N_{1} + N_{2}) \).

C. Reactive site at the center of a disk

The dynamical catalytic effect will be exhibited when the reactive site is embedded in two- or three-dimensional structures which facilitate mass transport. As a preliminary example we consider a circular reactive region of radius \( b \) containing \( B \) embedded in a disk \( b < r < R \) that contains particles \( P \). The substrate \( A \) binds reversibly to \( P \) (on both sides) and may diffuse to the central reactive region. See Fig. 3.

The diffusion equation governing the distribution of \( A \) at steady state on the disk is

\[
0 = k_{1}AP + \frac{D}{r} \frac{d}{dr} \left[ r \frac{dc(r)}{dr} - c_{i}(r) \right], \quad b < r < R,
\]

(2.14)

with a reflecting boundary condition at the outer edge \( c'(R) = 0 \).

At the inner boundary the total flux of arriving \( A \) particles is balanced by the flux of \( A \) particles that depart due to reaction or disassociation back to the solution:

\[
0 = \pi b^{2}k_{1}AP + 2\pi bDc'(b) - (k_{-1} + k_{2})c(b)C_{\text{P}}.
\]

(2.15)

For simplicity we have assumed that the concentration of \( A \) in the reactive region is uniform and equal to the value at the boundary between the reactive region and the disk.

The solution to the diffusion equation [Eq. (2.14)] that satisfies the reflecting boundary condition at \( r = R \) is

\[
c(r) = a \left[ I_{0}(\lambda r) - \frac{L_{1}(\lambda R)}{I_{0}(\lambda R)} K_{0}(\lambda r) \right] + \frac{k_{1}AP}{k_{-1}},
\]

(2.16)

where \( K_{0}(z) \) and \( I_{0}(z) \) are modified Bessel functions and, as before, \( \lambda = \sqrt{k_{-1}/D} \). The constant ‘‘\( a \)’’ is determined by the condition at the reactive boundary \( r = b \), Eq. (2.15) and
Fig. 4. Kinetic mechanism for the reactive region \( \theta \) on the sphere. Diffusion of species \( A \) absorbed on the surface to the reactive region.

\( k_{\text{eff}} \) is identified from the reactive term \( k_2 c(b) \). One finds \( k_{\text{eff}} \) of the form given in Eqs. (1.6) and (1.7) with

\[
g = -\frac{2D\lambda}{b k_{-1}} \left[ I_0(\lambda b)K_0'(\lambda b) - I_0'(\lambda b)K_0(\lambda b) \right] - \frac{1}{2}\frac{1}{\lambda} I_0(\lambda b)K_0'(\lambda b) - \frac{1}{2}\frac{1}{\lambda} I_0(\lambda b)K_0(\lambda b)
\]

In the limit of rapid diffusion \( \lambda \to 0 \) and the modified Bessel functions approach

\[
I_0(z) \to 1 + \frac{z^2}{4}, \quad K_0(z) \to -\ln(z).
\]

Thus in this limit one finds \( g \to [(R^2 - b^2)/b^2] \) and accordingly

\[
k_{\text{eff}}^+ = k_1 \left[ 1 + \frac{(R^2 - b^2)/b^2}{R^2} \right].
\]

Here again the maximum rate enhancement can be expressed as \( k_{\text{eff}}^+ = k_1[1 + N_{\text{eff}}] \), where \( N_{\text{eff}} \) is the effective number of passive molecular sites available to transport \( A \) to the reactive region containing \( B \). The effective number \( N_{\text{eff}} \) is simply the ratio of the passive molecular area to the reactive area \( \pi b^2 \) since the sites are assumed to be uniformly distributed.

In the opposite limit of slow diffusion one may show \( g \to 0 \) and \( k_{\text{eff}}^+ \to k_1 \), as expected.

### III. REACTIVE SITE ON A SPHERE

The dynamical catalytic effect under consideration here is the rate enhancement due to sliding along the unreactive portion of the sphere to the chemically reactive region compared to the situation where the unreactive portion of the sphere simply acts as a reflecting barrier.

We treat the diffusion of particles \( A \) to the sphere and dissociation of particles \( A \) from the sphere as a kinetic equilibrium step and focus attention of the diffusion of particles \( A \) on the sphere surface to the reactive region occupied by particles \( B \). See Fig. 4. The combined problem of diffusion to and on the sphere that has a chemically reactive region can be analyzed directly but requires numerical computation.

The steady state diffusion equation for the concentration of particles \( A \) on the surface of the sphere of radius \( R \) is

\[
0 = k_1 A P + \frac{D}{R^2} \frac{1}{\sin \vartheta} \frac{d}{d \vartheta} \sin \vartheta \frac{d c(\vartheta)}{d \vartheta} - k_{-1} c(\vartheta), \quad 0 < \vartheta < \pi,
\]

where the chemically reactive patch occupies the north polar region \( 0 < \vartheta < \vartheta_0 \). In terms of the variable \( x = \cos \vartheta \), Eq. (3.1) is

\[
x_0 < k < -1.
\]

The boundary condition at the south polar region \( x = -1 \) must be

\[
\lim_{x \to -1} (1 - x^2)c'(x) \to 0.
\]

Obtaining a solution for this case is a bit more complicated than the cases previously considered because diffusion is taking place on a curved surface. We find a solution in terms of solutions to the Legendre equation

\[
(1 - x^2)w''_n(x) - 2xw'_n(x) + (\nu + 1)w_n(x) = 0.
\]

This equation has two solutions on the cut \(-1 < x < +1\) that are denoted \( P_n(x) \) and \( Q_n(x) \). Legendre functions of degree \( \nu \) and order zero of the first and second kind. A solution to Eq. (3.2) is obtained as a linear combination of these Legendre functions:

\[
c(x) = a_1(\nu) P_n(x) + a_2(\nu) Q_n(x) + \frac{k_1 A P}{k_{-1}},
\]

provided that

\[
\nu(\nu + 1) = -\frac{k_{-1} R^2}{D}.
\]

The particular linear combination must be taken to satisfy the boundary condition Eq. (3.3). Note that \( \nu \) can be complex. For the region where \( \nu \) is real

\[
0 \leq \frac{k_{-1} R^2}{D} < \frac{1}{4},
\]

we select the positive square root for \( n \):

\[
\nu = -\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{4k_{-1} R^2}{D}}.
\]

Since \( P_n(x) \) and \( Q_n(x) \) are singular \([x] \) as \( x \to -1 \):

\[
P_n(x) \to \frac{\sin(\nu \pi)}{\pi} \left[ \ln \left( \frac{1 + x}{2} \right) + \pi \cot(\nu \pi) + \text{const} \right],
\]

\[
Q_n(x) \to \frac{\cos(\nu \pi)}{\pi} \left[ \ln \left( \frac{1 + x}{2} \right) - \pi \tan(\nu \pi) + \text{const} \right],
\]

one must choose a particular linear combination that cancels the singularity in order to satisfy Eq. (3.3). We choose the particular combination

\[
c(x) = a \left[ \frac{\sin(\nu \pi)}{\nu \pi} Q_n(x) - \frac{\cos(\nu \pi)}{2 \nu} P_n(x) \right] + \frac{k_1 A P}{k_{-1}},
\]

which one can verify satisfies Eq. (3.3) for all \( \nu \neq 0 \).
We must be concerned with $\nu = 0$. The limit $\nu \to 0$ arises when $k_{-1} \to 0$; in this limit,
\[ c(x) \to a \left[ Q_0(x) - \frac{1}{2\nu} P_0(x) - \frac{1}{2} \left( \frac{dP_0(x)}{d\nu} \right)_{\nu=0} \right] + \frac{k_1}{k_{-1}}. \]  
(3.11)

Since $P_0(x) = 1$ and
\[ \left( \frac{dP_0(x)}{d\nu} \right)_{\nu=0} = \ln \left( \frac{1+x}{2} \right) \]  
(3.12)

it is necessary to choose $a = -2k_1APR^2/D$ to cancel the singularity at $n = k_{-1} = 0$. Thus, in the limit $\nu \to 0$, the general solution Eq. (3.10) approaches
\[ c(x) = \frac{k_1APR^2}{D} \left[ \ln \left( \frac{1+x}{2} \right) - 2Q_0(x) \right] \]  
(3.13)

with $Q_0(x) = \frac{1}{2} \ln \left[ (1+x)/(1-x) \right]$ and $c(-1) = 0$. One may verify that Eq. (3.13) is correct by direct integration of the original differential equation for $c(x)$, Eq. (3.2), with $k_{-1}$ set equal to zero.

In the general case when $k_{-1}$ is not equal to zero, the constant $a$ in Eq. (3.10) must be determined from the boundary condition at $x = x_0$. As usual, the condition at the reactive boundary is found by equating the flux of $A$ into the region $0 < x < x_0$ with the flux leaving by reaction and dissociation:
\[ 0 = k_1APS(x_0) - 2\pi D(1 - x_0^2)c'(x_0) \]
\[ - \left( k_{-1} + k_2 \right) c(x_0)S(x_0), \]  
(3.14)

where $S(x_0)$ is the area of the reactive region $S(x_0) = 2\pi R^2(1-x_0)$ and again we have assumed that the concentration of $A$ in the reactive region is uniform and equal to the value at the boundary.

Once the constant “$a$” is determined $c_{eff}$ is identified from the reactive flux $k_{2c}(x_0)$. The effective rate coefficient is of the form of Eqs. (1.6) and (1.7) and the quantity $g$ is
\[ g_1 = \frac{(1+x_0)D}{k_{-1}R^2} \left[ \frac{(2\pi)\tan(\nu \pi)Q_0'(x_0) - P_0'(x_0)}{(2\pi)\tan(\nu \pi)Q_0(x_0) - P_0(x_0)} \right], \]  
(3.15)

with the complex quantity $\nu$ given by Eq. (3.8) in the regime $0 < (k_{-1}R^2/D) < (1/4)$. Alternatively, using Eq. (3.6) one finds
\[ g_1 = \frac{(1+x_0)}{\nu(\nu+1)} \left[ \frac{(2\pi)\tan(\nu \pi)Q_0'(x_0) - P_0'(x_0)}{(2\pi)\tan(\nu \pi)Q_0(x_0) - P_0(x_0)} \right]. \]  
(3.16)

In the limit $k_{-1} \to 0$ ($\nu \to 0$), the quantity $g$ approaches
\[ g_1 \to (1+x_0) \left[ 2Q_0'(x_0) - \frac{1}{1+x_0} \right]. \]  
(3.17)

where use has been made of Eq. (3.11). From the definition of $Q_0(x)$, one finds
\[ g_1 \to \frac{1+x_0}{1-x_0}, \quad k_1 \to k_{-1} \left[ 1 + \frac{1+x_0}{1-x_0} \right]. \]  
(3.18)

Again, the maximum rate enhancement can be expressed in terms of $N_{eff}$, the effective number of passive molecular sites available to transport $A$ to the reactive region occupied by $B$:
\[ N_{eff} = \frac{4\pi R^2 - S(x_0)}{S(x_0)} = \frac{1+x_0}{1-x_0}. \]  
(3.19)

We now seek a solution for the concentration $c(x)$ when $(k_{-1}R^2/D) > (1/4)$ and the value of the degree $\nu$ of the Legendre functions becomes complex. We display the imaginary part of this parameter explicitly:
\[ \nu = \frac{1}{4} \pm i \sqrt{d}, \]

where
\[ k_{-1}R^2/D = 1 + d. \]  
(3.20)

In this regime we choose for $c(x)$ a linear combination of the Legendre functions of the second kind which is real and satisfies the boundary condition Eq. (3.3) that $c(x)$ is not singular at the south pole:
\[ c(x) = a \{ Q_{-1/2+i,0}(x_0) + Q_{-1/2-i,0}(x_0) \} + \frac{k_1APR^2}{k_{-1}}, \]  
(3.21)

The constant “$a$” and $k_{eff}$ is determined exactly as before. The quantity $g_2$, in this region, is found to be
\[ g_2 = \frac{(1+x_0)D}{k_{-1}R^2} \left[ \frac{Q_{-1/2+i,0}'(x_0) + Q_{-1/2-i,0}'(x_0)}{Q_{-1/2+i,0}(x_0) + Q_{-1/2-i,0}(x_0)} \right]. \]  
(3.22)

Accordingly we find for the rate enhancement factor:
\[ g = \begin{cases} \frac{g_1}{g_2} = \text{Eq. (3.15)} & \text{for } (k_{-1}R^2/D) < (1/4) \\ \frac{g_2}{g_1} = \text{Eq. (3.16)} & \text{for } (k_{-1}R^2/D) > (1/4) \end{cases} \]  
(3.23)

\[
\Delta S_d / k_B \text{ versus } k_{-1}R^2/D -- \text{ various } x_0
\]

![Graph showing kinetic rate enhancement expressed as change in entropy $\Delta S_d$ versus the kinetic parameter $k_{-1}R^2/D$ for the case of a sphere with reactive region $S(x_0) = \pi R^2(1-x_0)$ for $x_0 = \cos \theta_0$.}
reactive sites to passive sites available to transport substrate is \( N_{\text{eff}}=100 \). Note the decay is slowest for the linear array because in the lower dimensionality situation, a slower diffusion rate (relative to desorption) will still permit the active species to reach the reactive region.

In this paper we have stressed the analogy of this dynamical effect to catalysis. The analogy is not perfect, however, since the "catalytic" effect arises in comparing the reaction

\[ A + B \rightarrow \text{products} \]

in free solution with the reaction

\[ A + B/\text{network} \rightarrow \text{products}. \]

The reactants are not identical and the products may not separate from the reaction complex at the same rate. Nevertheless, the analogy with catalytic activity is instructive, especially because the situation corresponds closely to an active site embedded in a macromolecular framework that is encountered so often in biological applications.

A situation that is more familiar from the perspective of heterogeneous catalysis is where \( A \) and \( B \) react by one of two pathways: (1) either directly in solution or (2) by first diffusing to a solid network and then diffusing along the network until there is a reactive encounter or dissociation back into the solution.

If the product species easily separate from the network and if diffusion along the network for \( A \) and \( B \) is fast compared to free diffusion in solution, then the system will exhibit a rate enhancement. This catalytic effect is due to diffusion along the chemically passive heterogeneous network which provides the acceleration only through mass transport.  This model is also under study.

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