Propulsion Devices for Locomotion at Low-Reynolds Number

by

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Abstract

We have built a peristaltic swimmer uses lubrication pressures underneath a flexible, sinusoidally waving boundary to generate thrust. Robosnail 1 was found to move at a speed of roughly half the wave speed of the foot (measured with respect to the snail), a result consistent for wave speeds between 0 and 2 cm/s.

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Chapter 1

Robosnail 1 (Waving Sheet Lubrication Swimmer)

1.1 Introduction

The moving boundaries separated by a lubricating layer of viscous fluid can create high pressures as in the case of oil-filled bearings. In the case of the digestive tract of vertebrates, peristaltic motions create similar pressures to transport contents through the intestine [?]. In nature, several species of holothuroid (sea cucumber) are found to use peristalsis as a means of locomotion [?]. It is possible that aquatic snails may also generate lubrication pressures to aid locomotion. Periwinkles (*Littorina* sp.) and several aquatic snails are seen to generate backward propagating (retrograde) waves while in motion. Of the three land snails observed, all used forward (direct) waves in locomotion. It will be shown in the following sections that locomotion using lubrication pressures makes use of retrograde waves. The fact that aquatic snails use retrograde waves to move may be an indication that lubrication pressures play a significant role in their propulsion. It is possible that aquatic snails are able use the ambient water as a lubricating fluid, while land-dwelling species lack sufficient amounts water to move in this way.

A deformable, moving boundary creates lubrication pressures which act on the sloped boundary, resulting in a nonzero horizontal component of force, which can be used as a means of propulsion. One of the simplest forms of a steady, deforming surface is the sine wave. A sinusoidal traveling wave surface situated near a flat, solid boundary will generate high pressures to one side of the wave trough where the fluid is being squeezed, and low pressure on the other side of the wave, where the local volume is being expanded. This means of locomotion bears much similarity to the peristaltic pumping of fluids through flexible channels. The difference is that the force applied by the boundary is transmitted through the fluid to the substrate boundary, rather than being used to pump the fluid.
Figure 1-1: Pressure forces generated in fluid under a sinusoidal waving sheet. Pressure was calculated numerically by solving the lubrication equations for the periodic case of a sinusoidal waving sheet. There is a high-pressure zone immediately in front of the lowest point in the wave, where the fluid is being squeezed, and a low pressure zone immediately behind, where the fluid is being pulled apart.

It was found that marine snails often exhibit retrograde waves that travel from head to tail, unlike land snails, which generally use direct waves, starting at the tail and ending at the head. There is a possibility that marine snails and other aquatic creatures such as flatworms use backwards waving deformations of the body to generate thrust forces.

1.2 Theory

The fluid within the fluid layer then follows the lubrication equation:

\[
\frac{dp}{dx} = \mu \frac{d^2 u}{dy^2}
\]  

(1.1)

Pressure varies only in the x-direction

\[
\frac{dp}{dx} = f(x)
\]

Integration once with respect to y gives

\[
\frac{du}{dy} = \frac{1}{\mu} \frac{dp}{dx} y + A
\]

which shows that velocity profile is parabolic.
\[ u = \frac{1}{2\mu} \frac{dp}{dx} y^2 + Ay + B \]

where A and B are constants, which can be solved for by considering the boundary conditions. Analysis of the flow is greatly simplified by considering a frame of reference in relation to the wave crests.

The velocities at the foot surface and the substrate are known:

\[ u(h) = -v_w \]

\[ u(0) = -v_w + v_s \]

where \( v_w \) is the wave velocity of the snail, defined in the frame of the snail and \( v_s \) is the snail velocity defined in the lab frame. The constants are solved from the boundary conditions:

\[ B = u(0) = -v_w + v_s \]

\[ u(h) = -v_w = \frac{1}{2\mu} \frac{dp}{dx} h^2 + Ah - v_w + v_s \]

\[ 0 = \frac{1}{2\mu} \frac{dp}{dx} h^2 + Ah + v_s \]

\[ A = -\frac{1}{2\mu} \frac{dp}{dx} h - \frac{v_s}{h} \]

The velocity profile is then

\[ u = \frac{1}{2\mu} \frac{dp}{dx} y^2 + \left( \frac{1}{2\mu} \frac{dp}{dx} h - \frac{v_s}{h} \right) y - v_w + v_s \]

which simplifies to

\[ u = \frac{1}{2\mu} \frac{dp}{dx} (y^2 - yh) + v_s (1 - \frac{y}{h}) y - v_w \quad (1.2) \]

The height of the foot \( h \) is a function of time and distance along the foot. In a frame of reference following the wave crests, analysis of the flow is greatly simplified. \( h \) is now independent of \( t \) and only a function of \( x \)

\[ h = h_0 + a\sin(x) \]

where \( h_0 \) is the average height, an unknown constant to be determined by force balance.
Volume flux $Q$:

$$Q = \int_{0}^{h} u dy = \frac{\mu}{2} \frac{dP}{dx} \left( \frac{y^3}{3} - \frac{hy^2}{2} \right) + v_s(y - \frac{y^2}{2h}) - v_w y$$

$$Q = \frac{1}{12\mu} \frac{dP}{dx} h^3 + \left( \frac{v_s}{2} - v_w \right) h$$

Because the height of the waves becomes fixed, the flow is steady and as a result volume flux remains constant. Solving for the gradient of pressure,

$$\frac{dP}{dx} = \frac{12\mu Q}{h^3} + \mu \left( \frac{v_s}{2} - v_w \right) \frac{1}{h^2}$$

Integration along the $x$ direction gives the expression for pressure.

$$p(x) = \int \frac{dP}{dx} dx + p_0$$

$$p(x) = \int \left( \frac{12\mu Q}{h^3} + \mu \left( \frac{v_s}{2} - v_w \right) \frac{1}{h^2} \right) dx + p_0 L$$

$$p(x) = -12\mu Q \int \frac{1}{h^3} dx + 12\mu \left( \frac{v_s}{2} - v_w \right) \int \frac{1}{h^2} dx + p_0 L$$

If $h$ and $x$ are made non-dimensional, $h^* = h/a$ and $x^* = x/L$

$$p(x) = \frac{12\mu L}{a^2} \left( \frac{v_s}{2} - v_w \right) \int \frac{1}{h^2} dx - \frac{12\mu Q L}{a^3} \int \frac{1}{h^3} dx + p_0 L$$

Assuming periodic wave deformation and pressure, we solve for $Q$, the flow rate (henceforth we take $h$ and $x$ to be non-dimensional, dropping * from $h^*$ and $x^*$) The net pressure difference over a

Figure 1-2: Left: Lab reference frame. Right: reference frame of wave crests. In the second frame of reference, the boundaries are fixed and the flow is steady.
wavelength must be zero.

\[ p(L) - p(0) = \int_0^L \frac{dp}{dx} = 0 \]

\[ 0 = -\frac{12\mu Q L}{a^3} \int_0^L \frac{1}{h^3} dx + \frac{12\mu L}{a^2} \left( \frac{v_s}{2} - v_w \right) \int_0^L \frac{1}{h^2} dx \]

The non-dimensional volume flux

\[ q = \frac{Q}{a \left( \frac{v_s}{2} - v_w \right)} = \int_0^L \frac{1}{h^2} dx \]

pressure is then a function of \( h_0/a \)

\[ p(x) = \frac{12\mu L}{a^2} \left( \frac{v_s}{2} - v_w \right) f_1 + p_0 \]

where

\[ f_1 = \int \frac{1}{h^2} dx - q \int \frac{1}{h^3} dx \]

The horizontal force is the sum of the pressure force acting on the sloped sections of the foot and viscous shear forces along the line of motion.

\[ F_x = F_{x,p} + F_{x,s} = 0 \]

\[ \int_0^L p \frac{dh}{dx} dx + \mu \int_0^L \frac{du}{dy} |_h dx = 0 \]

keeping in mind that the constant \( p_0 \) does not effect the horizontal force on the snail because the integral \( \int_0^L \frac{dp}{dx} dx = h(L) = h(0) \) goes to zero for any periodic \( h(x) \). The shear rate at the foot boundary is found to be

\[ \frac{du}{dy} |_h = \frac{1}{2\mu} \frac{dp}{dx} h + \frac{v_s}{h} \]

Substituting in the equations for pressure, \( \frac{du}{dy} \), \( \frac{dp}{dx} \) and for simplicity’s sake defining the following functions A B C

\[ A = \int \left( \int_0^x \frac{1}{h^2} dx - q \int_0^x \frac{1}{h^3} dx \right) \frac{dh}{dx} dx \quad (1.5) \]

\[ B = \int_0^L \frac{1}{h} - \int_0^L \frac{1}{h^3} dx \]

\[ B = \frac{2\pi}{h^3 \left( 2 + 1/h^2 \right) \sqrt{1 - 1/h^2}} \quad (1.6) \]
\[ C = \int_{0}^{2\pi} \frac{1}{h} dx = \frac{2\pi}{\sqrt{1 - 1/h^2}} \]  

(1.7)

one finds that the ratio \( \frac{v_s}{v_w} \) is only a function of the normalized wave shape. In theory the shape can be any periodic function, and for future versions of Robosnail, can be optimized. However, owing to the simplicity of the actuation mechanism (a rotating helix), the foot boundary of Robosnail 1 follows a simple sinusoid, which in the lab reference frame is: \( h = h_0 + acos(x - (v_w - v_s)t) \) In the wave reference frame, \( h = h_0 + acos(x) \) When \( h \) is in non-dimensional form, \( A, B, \) and \( C \) are all functions of the the shape of the wave and independent of the other variables. When \( h \) is sinusoidal, \( A, B \) and \( C \) are only functions of \( h_{av}/a \).

Solving for \( \frac{v_s}{v_w} \) gives

\[ \frac{v_s}{v_w} = \frac{12A - 6B}{-6A + 3B + C} \]  

(1.8)

using the identities

\[ \oint d\theta \cos \theta + h = \frac{2\pi}{h\sqrt{1 - 1/h^2}} \]

\[ \oint \frac{d\theta}{(\cos \theta + h)^2} = \frac{2\pi}{h^2 (1 - 1/h^2)^{3/2}} \]

\[ \oint \frac{d\theta}{(\cos \theta + h)^3} = \frac{\pi (2 + 1/h^2)}{h^3 (1 - 1/h^2)^{5/2}} \]

A, B, and C can be simplified

\[ A = \int (\int_{0}^{x} \frac{1}{h^2} dx - q \int_{0}^{x} \frac{1}{h^3} dx) \frac{dh}{dx} dx \]  

(1.9)

Integration by parts gives

\[ A = \int_{0}^{L} h \cdot \frac{dp}{dx} dx \]

\[ A = \int_{0}^{L} (\frac{1}{h} + q \frac{1}{h^2}) dx \]

\[ A = -\frac{2\pi}{h^3 (2 + 1/h^2) \sqrt{1 - 1/h^2}} \]  

(1.10)
Figure 1-3: Robosnail ratio of wave speed to snail speed as a function of height for an infinitely long foot, two-dimensional case. Theoretically, the snail speed should be able to surpass the wave speed by 50 percent. In experiments, this does not occur and the snail speed is a fraction of the wave speed.

\[
B = \int_0^L \frac{1}{h} - \int_0^L \frac{1}{h^3} dx
\]

\[
B = \frac{2\pi}{h^3(2 + 1/h^2) \sqrt{1 - 1/h^2}} \quad (1.11)
\]

\[
C = \int_0^{2\pi} \frac{1}{h} dx = \frac{2\pi}{\sqrt{1 - 1/h^2}} \quad (1.12)
\]

The snail speed

\[
\frac{v_s}{v_w} = \frac{12A - 6B}{-6A + 3B + C} \quad (1.13)
\]

simplifies to

\[
\frac{v_s}{v_w} = \frac{9}{(\frac{h_0}{a})^2 + 5} \quad (1.14)
\]

which is graphed in Figure ?? as a function of \(h_0/a\).

One sees that the horizontal speed with respect to the wave velocity is only a function of \(h/a\).
As the mean height approaches a, the amplitude of wave height, the snail velocity approaches 1.5 times the wave velocity. Vertical force:

\[ F_y = \int p(x)dx \]

\[ F_y = \int p(x)dx + p_0L = mg \]  \hfill (1.15)

The vertical force depends on not only the integral of lubrication pressures, but also from the product \( p_0L \), \( p_0 \) being the constant of integration from \( \frac{dp}{dx} \). In theory, \( p_0 \) can be arbitrary, depending on what point \( x \) on the sinusoid one chooses to integrate \( \frac{dp}{dx} \). In practice it depends on the flow characteristics of the foot. More specifically, the end effects at the front and rear of the foot determine the pressure. If the Robosnail is higher in front than in back, there should be high pressure pushing up the snail, similar to Newton’s sliding sheet lubrication problem. If the front is lower than the back, there should be a suction instead.

### 1.3 Design

The Robosnail has a solid polycarbonate body. Its total weight is 1.67 N. Its foot is powered by an external DC power source, capable of supplying 1.5, 3.0 and 4.5 volts. The motor is connected to a variable-speed gear box. A toothed pulley connects the gearbox to a shallow brass helix which passes through an array of aluminum sheets perforated with slots. Each of the sheets is constrained to vertical motion - they ride in equally spaced slots along the body. The bottom edges of the sheets are directly glued onto a flexible foam sheet. When the helix is spun by the motor and gearbox, it causes the plates to translate up and down inside their tracks in a moving sinusoidal wave (evident when seen from the side). The wave is transferred directly to the foam sheet.
The test track of Robosnail 1 (Figure ??) was constructed to be only slightly larger than the width of the snail to minimize the leakage of fluid past the open sides of the foot. A laser, fitted with a lens to emit a plane of light, was fixed at an angle of 45 degrees with respect to the bottom of the clear channel. The line of the laser hitting the foam sole, seen from the underside, reflects the height of the wave with respect to the bottom of the channel, and the profile of the film thickness becomes clearly visible from an underside view (Figure ??). The track was filled with 0.5 mm thick layer of glycerol and the Robosnail was activated on top of the layer. After the motion reached steady state, measurements of wave speed, foot height (revealed by the laser), and snail speed were recorded by video.

1.4 Results

Robosnail 1 was tested using differing speeds on a layer of glycerol. Direction of motion was found to be opposite the direction of wave propagation. The speed of robosnail as a function of wave speed is plotted in Figure ??.

It was found experimentally that Robosnail 1 traveled at a height very close to the minimum \( h/a \approx 1 \). As predicted, the snail speed is approximately linear with respect to wave speed. However, the experimental velocity was found to be about 1/3 the expected value for the infinite two-dimensional case. The great discrepancy in speed is most likely due fluid leakage through the gap between the snail and the walls of the channel. Such leakage will greatly decrease the thrust from
Figure 1-6: Robosnail 1 in motion, still from video. A sheet of laser light shining at an angle on the waving foot shows the foot height.

Figure 1-7: Robosnail 1 setup
lubrication pressures while barely affecting the shear drag that is proportional to the speed of the snail.

Whether or not marine snails use lubrication pressures as an aid to locomotion remains unknown. We have attempted to measure variations in the film thickness of moving periwinkles (*Littorina* sp.) using an angled laser setup similar to the Robosnail 1 experiment. Because the dimensions of the largest periwinkle foot are on the order of 1 cm, and owing to the diffusion of light through the translucent snail foot, the resolution of the angled laser technique was not fine enough to reveal any height variation.
Bibliography


