Recognizing Conservation Problems

The most commonly encountered conservation problem is a collision between two objects, but the technique described here can be applied to any interaction that has a clear beginning and end, and where the goal of the problem is to establish a relationship between the *initial state* of the system (before the interaction begins) and the *final state* of the system (after the interaction is over), without necessarily establishing anything about what happens in between. In fact, the real power of this method is its undiminished effectiveness in situations where the details of the interaction are *not* well understood, because those problems are extremely hard to solve by other means.

Basic Technique

1. Choose your system.
2. Draw two pictures: a **before picture** representing the state of the system before the interaction, and an **after picture** representing the state of the system after the interaction. Each picture should include the mass and velocity of every object in the system.
    - You may need to define new variables for masses and velocities that are not known.
3. Determine which physical quantities, if any, are conserved during the interaction.

    e.g. if you know that there is no net external force on the system during the interaction, then you can conclude that the momentum of the system will be conserved.
    - It is important to think carefully and justify your conclusions with logical reasoning. *Never assume* that any quantity will be conserved, unless the problem you're solving specifically tells you to do so.
    - Remember that the interaction occurs during the time interval *between* the before picture and the after picture, so looking at those pictures won't help you with this step. You may find it useful to draw one or more force diagrams to assist you in your reasoning.
4. Write a **conservation constraint equation** for each conserved quantity.

    e.g. if you determine that the momentum of the system is conserved,

    (a) write an expression for the momentum of the system before the interaction, in terms of the masses and velocities shown in the before picture

    (b) write an expression for the momentum of the system after the interaction, in terms of the masses and velocities shown in the after picture

    (c) equate these two expressions to indicate that the momentum of the system is the same before and after the interaction.
5. Use these constraint equations to solve for the desired unknown quantities.

Advanced Variations

- You may need to use other tools (e.g. kinematics and dynamics) to analyze what happens before the before picture and/or after the after picture.
- Some problems go through several stages of interactions, and must be analyzed using a sequence of before/after pictures instead of just two. The trick in this case is to find a way of relating each consecutive picture to the one before it, until eventually you have a chain of equations linking the final picture all the way back to the one at the very beginning.