Survival Guide to Concourse Physics

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Introduction

The purpose of this document is to give you a clear picture of what we expect from you in Concourse Physics. When I was a Concourse student, very little of this was written down anywhere. I picked some things up by talking to the course staff, but I had to learn a lot by trial and error. I wrote the original version of this document in the fall of 1997 as an email to the Concourse '01 class, after I graded their first problem set and realized that they were confused about many of the same issues that had confused me during my own freshman year. I have since rewritten it, added more examples, and included several new pieces of advice that I have found myself dispensing frequently over the past few years. The guide is a bit long, but that's because I have tried to make it as clear as possible. I hope you will find it helpful.

The Prose Requirement: How to Get Credit for Your Solutions

The course syllabus emphasizes that your solutions to problems in the course are to be written up in coherent English. This "prose requirement" is one of the defining characteristics of Concourse physics, and we take it very seriously.

This idea will be new for most of you, because it contrasts sharply with most high school physics classes. In high school, the objective was simply to get the answer (usually by randomly throwing equations at the problem until you find one that "works"). Here at MIT, the objective is to demonstrate to the grader that you understand how to solve the problem. You may assume that the grader is reasonably intelligent and knows physics. However, the grader cannot read your mind, so you must show your work and write prose to explain anything that is not clear about how you solved the problem. The grader should never look at your paper and wonder how you got from step 3 to step 4 – if it is not clear, then you are not doing your job properly.

Sample Problem

A box of mass 2 kg is pushed along a frictionless table with a force of 10 N. Find the acceleration of the box.

Acceptable Answer #1

By Newton's Second Law, the acceleration of the box is $\frac{\text{force on box}}{\text{mass of box}}$. Thus $a = \frac{10\text{ N}}{2\text{ kg}} = 5\frac{\text{m}}{\text{s}^2}$

This is actually more prose than you need for a problem like this one, but it illustrates the kind of thing you might write in order to clarify how you went about solving the problem.

Acceptable Answer #2

$a = \frac{F}{m} = \frac{10\text{ N}}{2\text{ kg}} = 5\frac{\text{m}}{\text{s}^2}$

This is a more efficient solution to the same problem. It contains no actual English prose at all, but since the first step $\left(a = \frac{F}{m}\right)$ is a well-known equation of physics, anyone reading the solution can easily tell that the problem is being solved using Newton's Second Law. No further explanation is necessary, because nothing is unclear.

An Insufficient Answer

$a = \frac{10\ \mathrm{N}}{2\ \mathrm{kg}} = 5\frac{\mathrm{m}}{\mathrm{s}^2}$

Here, it is not clear how the problem is being solved. A grader can justifiably ask, "How do you know that $a = \frac{10\ \mathrm{N}}{2\ \mathrm{kg}}$? Where did you get that from?" Of course, in a simple problem such as this one, a grader could easily use his or her own problem-solving skills to figure out where that equation came from, but the point is that the grader should never have to figure out what you did; it should be obvious from reading your solution.

Distinguishing Physics from Mathematics

From our physics-centric point of view, mathematics is a tool (actually, a large collection of tools) that we use to help us solve physics problems. The solution to a physics problem usually involves three distinct phases:

1. Translate the problem from physical language (physical quantities, verbal descriptions of what's happening in the problem, pictures, etc) to mathematical language (variables and equations).
2. Use mathematics to manipulate the equations and solve for the values of any unknown variables.
3. Translate the answer from mathematical language (the value of a variable) back to physical language (a verbal statement about the physical situation).

Phase 2 of this process involves only mathematics; it does not contain any physics at all. All of the physics happens in phases 1 and 3. This doesn't mean that you won't spend a significant amount of time on phase 2; in fact, for many problems you'll spend most of your time on phase 2. What it does mean is that you need to make a conscious effort to really pay attention to what goes on in phases 1 and 3 and try to understand it; the art of problem solving is subtle, and many students allow themselves to get so bogged down in the details of the mathematics that they completely lose sight of the physics. Try not to let this happen to you.

Please note that the prose requirement does not require you to explain every mathematical step of your solution – only the steps that require logical clarification. We assume that you understand algebra by now, and it would be a colossal waste of everyone's time (yours and the grader's) to have you write out prose fragments such as "now multiply both sides of the equation by 3." Once you have written down an equation, you may algebraically transform it as much as you like without any further explanation. Just be sure that you adequately explain how you got the *first* equation; that's the physics part of the problem.

Example

A boulder is falling from a high cliff (neglect air resistance). At time t = 0, its velocity is 7 m/s downward and it is 500 m above the ground. Find the time t at which the boulder is 150 m above the ground.

Solution

Let y be the height of the boulder above the ground.

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \tag{A}$$

$$(150\ \mathrm{m}) = (500\ \mathrm{m}) + (-7\ \tfrac{\mathrm{m}}{\mathrm{s}})t + \frac{1}{2}(-9.8\ \tfrac{\mathrm{m}}{\mathrm{s}^2})t^2 \tag{B}$$

$$t = 7.8\ \mathrm{s} \tag{C}$$

So the boulder is 150 m above the ground at time t = 7.8 seconds.

First we write down the general form of the equation we want to use as our mathematical model (A). Next, we incorporate the details of the physical problem into the model (B). Using algebra, we solve this equation for the variable that represents the quantity we're interested in (C). Finally, we translate our mathematical solution into a physical statement that answers the question posed by the problem.

You've probably noticed that this solution does not include any of the intermediate algebraic steps needed to get from (B) to (C). Including these intermediate steps in your own solutions is probably a good idea, because it will greatly reduce the frequency of careless mistakes in your answers; however, the omitted steps are not strictly necessary from a physics point of view, so the sample solution is acceptable as written.

When you use a more advanced mathematical technique, such as taking a derivative or an integral, we do not expect you to explain the details of how you calculated the derivative or integral, but your reason for doing it in the first place should be clear (for example, "now we use calculus to find the time when the height of the cannonball is at a maximum").

Defining variables and coordinate systems

Before you write down an equation containing a bunch of variables such as x, y, v_1, v_2, and a, make sure that the reader will understand what these variables represent. Don't forget to define reference points for quantities that need them (knowing that a car is at position $x = 5$ doesn't tell you very much if you don't know where the origin of coordinates is). For example:

Let x = position of car, measured from starting line.
Let v_1 = velocity of car
Let y = height of boulder above ground
Let v_2 = velocity of boulder
Let a = acceleration of boulder

If there's only one moving object in the problem, you can generally get away with using v, a, and x or y to represent that object's velocity, acceleration, and position *without* having to write out explicit definitions; this is a standard physics convention. Also, unless you state otherwise, we will assume by convention that the "positive directions" of your coordinate system are upward (for vertically-oriented quantities), to the right (for horizontally-oriented quantities), outward (for radially-oriented rotational quantities), and counterclockwise (for tangentially-oriented rotational quantities).

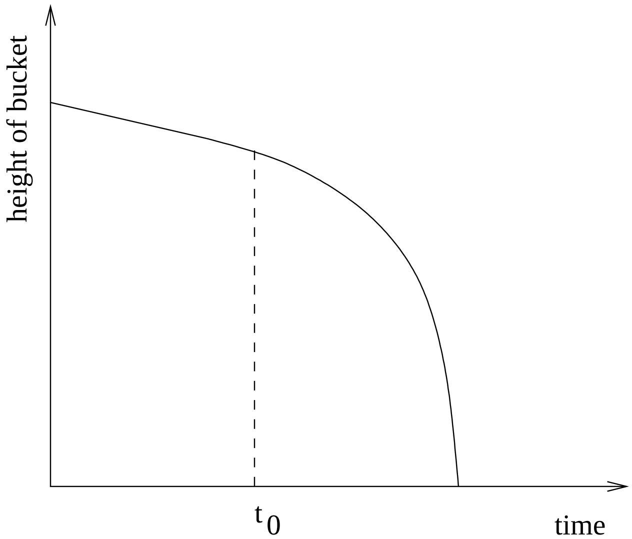
Clarifying Graphs

The same philosophy of clarification that applies to other problems also applies to drawing graphs, but there are a few extra things to be aware of. Whenever you draw a graph, you should explain the important features of your graph and the reasoning behind them, and label any points of special interest. This allows us to evaluate your solution based on what you *meant* to draw, instead of relying on your artistic skills (which vary widely from person to person). You must also label each axis of your graph with the physical quantity (or variable) that it represents.

Example

A bucket on a rope is being lowered into a well with constant speed. At time t_0, the rope is cut and the bucket begins to fall freely (neglect air resistance). Qualitatively sketch a graph showing the height of the bucket as a function of time.

Solution



At first, the height of the bucket decreases at a constant rate (linearly) because the bucket is being lowered with a constant speed. After the rope is cut at time t_0, the bucket experiences a constant downward acceleration, so its height decreases at an increasing rate (quadratically, concave down).

Note: the phrases shown in parentheses ("linearly", "quadratically", "concave down") are often very convenient because they provide a concise way to describe graph shapes; however, it's always acceptable to just write out what you mean in plain English (e.g. "its height decreases at an increasing rate").

Important Mathematical Concepts

A few notes on some important mathematical concepts you will encounter...

Vectors

Although vectors are technically a mathematical concept, many students see them for the first time while taking physics. Consequently, we do expect those parts of your solution that rely on vector mathematics to be clear and complete. In particular, please be careful to distinguish between vector quantities and scalar quantities. To refer to an object's velocity, for example, we might use the vector variable $\vec{v}$. The arrow indicates that this symbol represents a vector quantity. To refer to that same object's speed (which is the magnitude of $\vec{v}$), we would use the scalar variable v. Be aware that some textbooks such as K&K will indicate vector quantities with bold type ($\mathbf{v}$) rather than an arrow ($\vec{v}$), but please use arrows in your own solutions. Unit vectors (vectors of length 1) should be written with a "hat" instead of an arrow: $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$, and so forth.

The most common mistakes are made during the transition between vector mathematics and scalar mathematics.

Example

A raft floats southward down a river with a speed of 1 m/s. A man runs from the west side of the raft to the east side of the raft. His speed with respect to the raft is 2 m/s; what is his velocity with respect to the ground?

Incorrect Solution

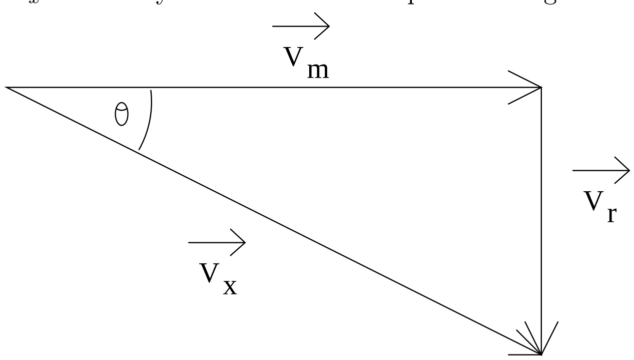
Let $\vec{v}_m$ = velocity of the man with respect to the raft, $\vec{v}_r$ = velocity of the raft with respect to the ground, and $\vec{v}_x$ = velocity of the man with respect to the ground (which we're trying to find).

$$\vec{v}_x = \vec{v}_m + \vec{v}_r$$
$$v_x = 2\ \frac{\mathrm{m}}{\mathrm{s}} + 1\ \frac{\mathrm{m}}{\mathrm{s}}$$
$$v_x = 3\ \frac{\mathrm{m}}{\mathrm{s}}$$

The vector equation $\vec{v}_x = \vec{v}_m + \vec{v}_r$ is true, but the scalar equation $v_x = v_m + v_r$ is false (the scalar equation does not follow from the vector equation because $\vec{v}_m$ and $\vec{v}_r$ are in different directions). The magnitude and direction of $\vec{v}_x$ must be determined using vector addition, as shown in the correct solution below. Notice that the incorrect solution never determines a direction for $\vec{v}_x$ at all; this should be a big hint that something is wrong!

Correct Solution

Let $\vec{v}_m$ = velocity of the man with respect to the raft, $\vec{v}_r$ = velocity of the raft with respect to the ground, and $\vec{v}_x$ = velocity of the man with respect to the ground (which we're trying to find).



$$\vec{v}_x = \vec{v}_m + \vec{v}_r$$
$$v_x = \sqrt{(1\ \tfrac{\mathrm{m}}{\mathrm{s}})^2 + (2\ \tfrac{\mathrm{m}}{\mathrm{s}})^2} = \sqrt{5\ \tfrac{\mathrm{m}^2}{\mathrm{s}^2}} \approx 2.24\ \tfrac{\mathrm{m}}{\mathrm{s}}$$
$$\theta = \arctan\left(\frac{1}{2}\right) \approx 26.57^\circ$$

The velocity of the man with respect to the ground is 2.24 m/s, directed 26.57° south of due east.

Proofs

There is often a great deal of confusion about questions that require you to "prove" or "show" something. For example, an astonishing number of people every year turn in roughly the same incorrect answer to the following problem (which usually appears on the first 8.01 problem set).

Problem

Show that if $|\vec{A} - \vec{B}| = |\vec{A} + \vec{B}|$, then $\vec{A}$ is perpendicular to $\vec{B}$.

A Popular Incorrect Answer

Let $\vec{A}$ be (1,1) and $\vec{B}$ be (1,-1).
Then $\vec{A} + \vec{B}$ is (2,0), which has magnitude 2, and $\vec{A} - \vec{B}$ is (0,2), which also has magnitude 2.
Thus $|\vec{A} - \vec{B}| = |\vec{A} + \vec{B}|$.
Since $\vec{A} \cdot \vec{B} = 0$, we know that $\vec{A}$ and $\vec{B}$ are perpendicular.

This is not a valid proof of the theorem "If $|\vec{A}-\vec{B}| = |\vec{A}+\vec{B}|$, then $\vec{A}$ is perpendicular to $\vec{B}$", because it has shown only that the statement is true for the case where $\vec{A}$ is (1,1) and $\vec{B}$ is (1,-1). A correct solution must show that the statement is *always* true, regardless of the values of $\vec{A}$ and $\vec{B}$.

Checking your answers

When you finish solving a physics problem, go back and reread the problem to make sure you have answered the correct question. Always specify the units of all numerical answers (and don't give your answer in meters if the problem says "Find the length of the rod in centimeters"). If the question asks for a vector quantity (such as velocity), make sure you have given both the magnitude and direction. If the problem asks you to prove something, your final answer should be a restatement of what you have proven; this makes it clear to the reader that your proof is finished. If you are asked to symbolically derive the value of some quantity (i.e. find its value in terms of other quantities), be sure that the only variables appearing in your final answer are those which were given to you in the problem. If you don't think that the problem gives you enough information, ask a tutor (or the proctor, if it's an exam question) whether you are allowed to use any other quantities in your answer.

This is not a math class; the main point of solving physics problems is to get the physics right. Consequently, if you make a purely mathematical mistake that causes your final answer to be wrong, but you did all of the physics correctly, you will still get almost all of the points for the problem. However, part of the philosophy we teach is that you should always look at your final answer carefully and think about what it means; if your answer is *obviously* wrong from a physics point of view, it will count as a physics mistake even if the actual error you made was a mathematical one. Here are some techniques for detecting "obviously wrong" answers.

Numerical sanity checks

If your final answer is numerical, think about whether it makes intuitive sense to you. If you determine that the mass of the car is 87,000,000 kg, you probably made a mistake.

Direction sanity checks

Many problems contain vector quantities whose directions are intuitively obvious from physical intuition. For example, if a boulder is falling, it should have a downward velocity (not an upward one). Make sure that the answer you calculate using mathematics agrees with your basic intuitions about the problem; if it doesn't, go back and check your math again.

Dimensional Analysis

Always check your equations to make sure that the dimensions (units) of both sides are the same. For example, suppose you're solving a problem that asks you to calculate g, and you arrive at the solution $g = \frac{8h^2}{(T_a^2 - T_b^2)}$. The dimensions of h are meters, since h represents a height. The units of T_a and T_b (representing times) are seconds, and the 8 is a dimensionless constant. Therefore, the whole right side of the equation has dimensions of $\frac{\mathrm{m}^2}{\mathrm{s}^2}$. The left side of the equation, g, has units of $\frac{\mathrm{m}}{\mathrm{s}^2}$ (because g is an acceleration), so your equation is dimensionally inconsistent (and therefore wrong). Realizing this, you go back to check your algebra and discover that you copied an extra h by mistake; your new answer is $g = \frac{8h}{(T_a^2 - T_b^2)}$. Now both sides of the equation are in $\frac{\mathrm{m}}{\mathrm{s}^2}$. As with the other methods, this doesn't necessarily guarantee that your answer is correct (dimensional analysis could never tell you, for example, whether that 8 should actually be a 4), but it will help you avoid certain types of errors.

Comparing Answers With Your Classmates

This technique should not be used on exams, but it works very well for problem sets (especially for problems with numerical answers).

Problem Set Etiquette

Please follow these guidelines when writing your solutions to the problem sets and exams. They are mostly about little things, but little things can have an amazing influence on the smoothness of the grading process. It is a well-known fact that happy graders give more partial credit.

Really basic stuff:

- Always write your name, the course number (8.01 or 8.02), and the problem set number at the top of the first page. Attach pages together by stapling them in the upper-left-hand corner. If there are any special instructions for handing in the problem set, read them and follow them.
- Label each part of each problem.

  We don't know why, but some people refuse to do this. Please don't be one of those people. Also, make sure your final answer to each part of the problem is easy to find (in most cases, drawing a little box around each final answer is very helpful).
- Do the problems in order.

  Graders waste an astonishing amount of time just looking through out-of-order problem sets to find the answer to a particular problem so that it can be graded. If for some reason you must put your answer to problem 1 part d between your answers to problems 5 and 6, write a little note to that effect in the place where problem 1 part d would normally appear (i.e. right after problem 1, part c).
- Write legibly!

  This is the most important rule of the bunch because it can affect your grade in a very real way - you can't possibly convince the grader that you understand how to solve the problem if the grader can't read your answer. Along the same lines, use a pen or pencil that shows up clearly on the paper.

The Tutors

Know that you will get the most out of evening tutorials and review sessions if you come prepared. Before you come to tutorial and start asking questions about the problem set, you should really make an effort to solve each problem by yourself. Yes, the Concourse environment theoretically makes it possible to do all of your problem sets just by asking the tutors for hint after hint after hint until you get to the end of the problem and then writing up the solutions... but *you won't be able to pass the exams unless you get used to solving problems on your own.* Before review sessions, go through the material on your own to find out which concepts you understand and which ones need some clarification, and come up with some questions to ask at the review session. The more you take the initiative to ask specific questions about the things you don't understand, the more useful we can be in helping you to understand them.

So, if you ever have questions about anything, please don't hesitate to ask the tutors. That's what we're here for. You can find us at tutorials, or send questions by email (you should all have a contact list for the course staff). We'll do our best to clear up anything that is bothering you.