Kinematics

$$\Delta\vec{r} = \vec{r} - \vec{r}_0 \qquad \vec{v}_{\rm avg} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{a}_{\rm avg} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

$$\text{average speed} = \frac{\text{distance travelled}}{\Delta t}$$

in Cartesian components (3-D):

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$
$$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$$
$$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$$

in plane polar components (2-D):

$$\vec{r} = r\hat{r}$$
$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$
$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta}$$

Angular Kinematics

$$\Delta\theta = \theta - \theta_0 \qquad \omega_{\rm avg} = \frac{\Delta\theta}{\Delta t} \qquad \alpha_{\rm avg} = \frac{\Delta\omega}{\Delta t}$$

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

1-D motion (x or θ) with constant acceleration (a or α)

$$v = v_0 + at$$
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$
$$v^2 - v_0^2 = 2a(x - x_0)$$
$$v_{\rm avg} = \frac{v + v_0}{2}$$

$$\omega = \omega_0 + \alpha t$$
$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$$
$$\omega_{\rm avg} = \frac{\omega + \omega_0}{2}$$

Uniform Circular Motion

$$\vec{v} = R\omega\hat{\theta}$$
$$\vec{a} = -R\omega^2\hat{r} = -\frac{v^2}{R}\hat{r}$$

Math Corner

"Dot" Notation

$$\dot{f} \equiv \frac{df}{dt}$$

Vectors

$$\vec{A} \cdot \vec{B} = \left|\vec{A}\right| \left|\vec{B}\right| \cos\theta_{A,B} = AB_{\parallel \text{ to A}} = A_{\parallel \text{ to B}}B$$

$$\left|\vec{A} \times \vec{B}\right| = \left|\vec{A}\right| \left|\vec{B}\right| \sin\theta_{A,B} = AB_{\perp \text{ to A}} = A_{\perp \text{ to B}}B$$

... in Cartesian components

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\left|\vec{A}\right| = \sqrt{{A_x}^2 + {A_y}^2 + {A_z}^2}$$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{\imath}\,(A_yB_z - A_zB_y) - \hat{\jmath}\,(A_xB_z - A_zB_x) + \hat{k}\,(A_xB_y - A_yB_x)$$

Coordinate Conversion

$$x = r\cos\theta \qquad r = \sqrt{x^2 + y^2} \qquad \hat{r} = \hat{\imath}\cos\theta + \hat{\jmath}\sin\theta$$

$$y = r\sin\theta \qquad \theta = \arctan\left(\frac{y}{x}\right) \qquad \hat{\theta} = -\hat{\imath}\sin\theta + \hat{\jmath}\cos\theta$$

Geometry

A sphere of radius R has volume $\frac{4}{3}\pi R^3$ and surface area $4\pi R^2$.

A cylinder of radius R and height h has volume $\pi R^2 h$ and surface area $2\pi Rh + 2\pi R^2$ (the first term is the area around the side, the second term is the area of the top and bottom).

Trigonometry

$$\sin^2\theta + \cos^2\theta = 1 \qquad \sin^2\theta = \tfrac{1}{2} - \tfrac{1}{2}\cos(2\theta) \qquad \sin 45^\circ = \cos 45^\circ = \tfrac{1}{\sqrt{2}}$$

$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi \qquad \cos^2\theta = \tfrac{1}{2} + \tfrac{1}{2}\cos(2\theta) \qquad \sin 30^\circ = \cos 60^\circ = \tfrac{1}{2}$$

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi \qquad \sin(2\theta) = 2\sin\theta\cos\theta \qquad \sin 60^\circ = \cos 30^\circ = \tfrac{\sqrt{3}}{2}$$

Quadratic Formula

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.