Kinematics

$$\Delta\vec{r} = \vec{r} - \vec{r}_0 \qquad \vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \qquad \vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2}$$

$$\text{average speed} = \frac{\text{distance travelled}}{\Delta t}$$

in Cartesian components (3-D):

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$
$$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$$
$$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$$

in plane polar components (2-D):

$$\vec{r} = r\hat{r}$$
$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$
$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta}$$

Angular Kinematics

$$\Delta\theta = \theta - \theta_0 \qquad \omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} \qquad \alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t}$$

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \qquad \alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$

1-D motion (x or θ) with constant acceleration (a or α)

$$v = v_0 + at \qquad \omega = \omega_0 + \alpha t$$
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad \theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$v^2 - v_0^2 = 2a(x - x_0) \qquad \omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$$
$$v_{\text{avg}} = \frac{v + v_0}{2} \qquad \omega_{\text{avg}} = \frac{\omega + \omega_0}{2}$$

Uniform Circular Motion

$$\vec{v} = R\omega\hat{\theta}$$
$$\vec{a} = -R\omega^2\hat{r} = -\frac{v^2}{R}\hat{r}$$

Relative Velocity

$$\vec{v}_{(\text{C relative to A})} = \vec{v}_{(\text{C relative to B})} + \vec{v}_{(\text{B relative to A})}$$
$$\vec{v}_{(\text{B relative to A})} = -\vec{v}_{(\text{A relative to B})}$$

Simple Harmonic Motion

The differential equation

$$\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} + {\omega_0}^2 x(t) = 0$$

represents a simple harmonic oscillator, and has solutions of the form

$$x(t) = A\cos(\omega_0 t + \phi)$$

where:

ω_0 is the angular frequency.

A and ϕ are arbitrary constants that depend on the initial conditions.

$$f = \frac{1}{T} = \frac{\omega_0}{2\pi} \qquad \text{(relation between frequency, period, and angular frequency)}$$

$$\omega_0 = \sqrt{\frac{k}{m}} \qquad \text{(angular frequency of a simple mass-spring oscillator)}$$

Dynamics

$\sum \vec{F} = m\vec{a}$	(Newton's 2nd Law for a single particle)
$F_G = \dfrac{Gm_1m_2}{r^2}$	(gravitational force of attraction between two particles)
$F_G = mg$	(gravitational force on a mass m near the surface of the Earth)
$f = \mu_k N$	(kinetic friction)
$f \leq \mu_s N$	(static friction)
$F = -kx$	(Hooke's Law: linear restoring force)

Math Corner

"Dot" Notation

$$\dot{f} \equiv \frac{\mathrm{d}f}{\mathrm{d}t}$$

Vectors

$$\vec{A}\cdot\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta_{A,B} = AB_{\parallel\text{ to A}} = A_{\parallel\text{ to B}}B$$

$$\left|\vec{A}\times\vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta_{A,B} = AB_{\perp\text{ to A}} = A_{\perp\text{ to B}}B$$

... in Cartesian components

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\left|\vec{A}\right| = \sqrt{{A_x}^2 + {A_y}^2 + {A_z}^2}$$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{\imath}\,(A_y B_z - A_z B_y) - \hat{\jmath}\,(A_x B_z - A_z B_x) + \hat{k}\,(A_x B_y - A_y B_x)$$

Coordinate Conversion

$$x = r\cos\theta \qquad r = \sqrt{x^2 + y^2} \qquad \hat{r} = \hat{\imath}\cos\theta + \hat{\jmath}\sin\theta$$

$$y = r\sin\theta \qquad \theta = \arctan\left(\frac{y}{x}\right) \qquad \hat{\theta} = -\hat{\imath}\sin\theta + \hat{\jmath}\cos\theta$$

Geometry

A sphere of radius R has volume $\frac{4}{3}\pi R^3$ and surface area $4\pi R^2$.

A cylinder of radius R and height h has volume $\pi R^2 h$ and surface area $2\pi R h + 2\pi R^2$ (the first term is the area around the side, the second term is the area of the top and bottom).

Trigonometry

$$\sin^2\theta + \cos^2\theta = 1 \qquad \sin^2\theta = \tfrac{1}{2} - \tfrac{1}{2}\cos(2\theta) \qquad \sin 45^\circ = \cos 45^\circ = \tfrac{1}{\sqrt{2}}$$

$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi \qquad \cos^2\theta = \tfrac{1}{2} + \tfrac{1}{2}\cos(2\theta) \qquad \sin 30^\circ = \cos 60^\circ = \tfrac{1}{2}$$

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi \qquad \sin(2\theta) = 2\sin\theta\cos\theta \qquad \sin 60^\circ = \cos 30^\circ = \tfrac{\sqrt{3}}{2}$$

Quadratic Formula

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.