Kinematics

$$\Delta\vec{r} = \vec{r} - \vec{r}_0 \qquad \vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \qquad \vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2}$$

$$\text{average speed} = \frac{\text{distance travelled}}{\Delta t}$$

in Cartesian components (3-D):

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$
$$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$$
$$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$$

in plane polar components (2-D):

$$\vec{r} = r\hat{r}$$
$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$
$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta}$$

Angular Kinematics

$$\Delta\theta = \theta - \theta_0 \qquad \omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} \qquad \alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t}$$

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \qquad \alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$

1-D motion (x or θ) with constant acceleration (a or α)

$$v = v_0 + at \qquad \omega = \omega_0 + \alpha t$$
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad \theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$v^2 - v_0^2 = 2a(x - x_0) \qquad \omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$$
$$v_{\text{avg}} = \frac{v + v_0}{2} \qquad \omega_{\text{avg}} = \frac{\omega + \omega_0}{2}$$

Uniform Circular Motion

$$\vec{v} = R\omega\hat{\theta}$$
$$\vec{a} = -R\omega^2\hat{r} = -\frac{v^2}{R}\hat{r}$$

Non-uniform Circular Motion

$$\vec{v} = R\omega\hat{\theta}$$
$$\vec{a} = -R\omega^2\hat{r} + R\alpha\hat{\theta}$$

Relative Velocity

$$\vec{v}_{(\text{C relative to A})} = \vec{v}_{(\text{C relative to B})} + \vec{v}_{(\text{B relative to A})}$$
$$\vec{v}_{(\text{B relative to A})} = -\vec{v}_{(\text{A relative to B})}$$

Simple Harmonic Motion

The differential equation

$$\frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} + {\omega_0}^2 x(t) = 0$$

represents a simple harmonic oscillator, and has solutions of the form

$$x(t) = A\cos(\omega_0 t + \phi)$$

where:
ω_0 is the angular frequency.
A and ϕ are arbitrary constants that depend on the initial conditions.

$$f = \frac{1}{T} = \frac{\omega_0}{2\pi} \quad \text{(relation between frequency, period, and angular frequency)}$$

$$\omega_0 = \sqrt{\frac{k}{m}} \quad \text{(angular frequency of a simple mass-spring oscillator)}$$

Rolling Without Slipping

$$V_{\mathrm{cm}} = \pm R\omega$$

Dynamics

$\sum \vec{F} = m\vec{a}$	(Newton's 2nd Law for a single particle)
$F_G = \frac{G m_1 m_2}{r^2}$	(gravitational force of attraction between two particles)
$F_G = mg$	(gravitational force on a mass m near the surface of the Earth)
$f = \mu_k N$	(kinetic friction)
$f \leq \mu_s N$	(static friction)
$F = -kx$	(Hooke's Law: linear restoring force)

Momentum

$\vec{p} = m\vec{v}$	(momentum of a particle)
$\sum \vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$	(Newton's 2nd Law for a single particle, in terms of momentum)
$\vec{F}_{\mathrm{avg}} = \frac{\Delta\vec{p}}{\Delta t}$	(defintion of average [net] force)

Systems of Particles

$$M = \sum_j m_j \qquad \text{(total mass of a system of particles)}$$

$$\vec{P} = \sum_j \vec{p}_j \qquad \text{(momentum of a system of particles)}$$

$$\sum \vec{F}_{\text{ext}} = \frac{\mathrm{d}\vec{P}}{\mathrm{d}t} \qquad \text{(Newton's 2nd Law for a system of particles)}$$

Center of Mass

$$\vec{R}_{\text{cm}} = \frac{\sum_j m_j \vec{r}_j}{\sum_j m_j} \qquad \text{(center of mass of a system of particles)}$$

$$\vec{P} = M\vec{V}_{\text{cm}} = M\frac{\mathrm{d}\vec{R}_{\text{cm}}}{\mathrm{d}t} \qquad \text{(momentum of a system of particles, in terms of CM)}$$

$$\sum \vec{F}_{\text{ext}} = M\vec{A}_{\text{cm}} = M\frac{\mathrm{d}^2\vec{R}_{\text{cm}}}{\mathrm{d}t^2} \qquad \text{(Newton's 2nd Law for a system of particles, in terms of CM)}$$

Thrust Equation

$$M\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \sum \vec{F}_{\text{ext}} + \vec{u}_{\text{rel}}\frac{\mathrm{d}M}{\mathrm{d}t}$$

$\vec{u}_{\text{rel}}$ is the velocity of the entering (or leaving) mass particles relative to the main object.

Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \qquad \text{(angular momentum of a particle about the origin)}$$

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \text{(torque about the origin due to a force } \vec{F}\text{)}$$

$$\sum \vec{\tau} = \frac{\mathrm{d}\vec{L}}{\mathrm{d}t}$$

For a system of particles,

$$\sum \vec{\tau}_{\text{ext}} = \frac{\mathrm{d}\vec{L}}{\mathrm{d}t}$$

Rigid Bodies

$$I = \sum_j m_j R_j^2 \qquad \text{(moment of inertia of a rigid body)}$$

$$I_{\parallel} = I_{\text{cm}} + Md^2 \qquad \text{(Parallel Axis Theorem)}$$

$$I_z = I_x + I_y \qquad \text{(Perpendicular Axis Theorem, for a 2-D object in the } xy \text{ plane)}$$

For a rigid body rotating about the z axis,

$$L_z = I_z\omega$$
$$\sum \tau_z = I_z\alpha$$

Known Moments of Inertia

$$I = MR^2 \quad \text{(thin ring)}$$
$$I = \tfrac{1}{12}ML^2 \quad \text{(uniform thin rod, about axis through center)}$$
$$I = \tfrac{1}{2}MR^2 \quad \text{(uniform disc or cylinder)}$$
$$I = \tfrac{2}{5}MR^2 \quad \text{(uniform solid sphere)}$$

Simultaneous Rotation and Translation

For a rigid body which is both translating and rotating (about an axis through its center of mass, oriented along the z direction),

$$L_z = (\vec{R}_{\text{cm}} \times M\vec{V}_{\text{cm}})_z + I_z\omega$$
$$\sum \tau_z = I_z\alpha \text{ about CM}$$

Work and Energy

$$\sum W = \Delta K \quad \text{(Work-Energy Theorem)}$$
$$W = \int_{\vec{r}_a}^{\vec{r}_b} \vec{F}\cdot \mathrm{d}\vec{r} \quad \text{(work done by } \vec{F} \text{ as particle moves from } \vec{r}_a \text{ to } \vec{r}_b)$$
$$K = \frac{1}{2}mv^2 \quad \text{(kinetic energy of a particle)}$$
$$Pwr = \frac{\mathrm{d}W}{\mathrm{d}t} = \vec{F}\cdot\vec{v} \quad \text{(power)}$$

For a rigid body rotating about the z axis,

$$K = \tfrac{1}{2}I_z\omega^2$$

For a rigid body which is both translating and rotating (about an axis through its center of mass, oriented along the z direction),

$$K = \tfrac{1}{2}MV_{\text{cm}}^2 + \tfrac{1}{2}I\omega_{\text{about cm}}^2$$

Potential Energy

$$\Delta U = -\int_{\vec{r}_a}^{\vec{r}_b} \vec{F} \cdot \mathrm{d}\vec{r}$$ (difference in potential energy due to a conservative force $\vec{F}$)

$$U(\vec{r}) = U(\vec{r}_0) - \int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot \mathrm{d}\vec{r}$$ (potential energy defined with respect to a reference point)

$$F_x = -\frac{\mathrm{d}U}{\mathrm{d}x}$$ (finding force from U, in one dimension)

$$E = K + U$$ (total mechanical energy)

$$\sum W_{\mathrm{NC}} = \Delta E$$ (Work-Energy Theorem restated in terms of total mechanical energy)

$$U = \frac{-Gm_1m_2}{r}$$ (potential energy due to gravity, universal law)

$$U = mgy$$ (potential energy due to gravity, near Earth's surface)

$$U = \frac{1}{2}kx^2$$ (potential energy due to spring force)

Math Corner

"Dot" Notation

$$\dot{f} \equiv \frac{\mathrm{d}f}{\mathrm{d}t}$$

Vectors

$$\vec{A} \cdot \vec{B} = \left|\vec{A}\right| \left|\vec{B}\right| \cos\theta_{A,B} = AB_{\parallel \text{ to A}} = A_{\parallel \text{ to B}}B$$

$$\left|\vec{A} \times \vec{B}\right| = \left|\vec{A}\right| \left|\vec{B}\right| \sin\theta_{A,B} = AB_{\perp \text{ to A}} = A_{\perp \text{ to B}}B$$

... in Cartesian components

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\left|\vec{A}\right| = \sqrt{{A_x}^2 + {A_y}^2 + {A_z}^2}$$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{\imath}\,(A_yB_z - A_zB_y) - \hat{\jmath}\,(A_xB_z - A_zB_x) + \hat{k}\,(A_xB_y - A_yB_x)$$

Coordinate Conversion

$$x = r\cos\theta \qquad r = \sqrt{x^2 + y^2} \qquad \hat{r} = \hat{\imath}\cos\theta + \hat{\jmath}\sin\theta$$

$$y = r\sin\theta \qquad \theta = \arctan\left(\frac{y}{x}\right) \qquad \hat{\theta} = -\hat{\imath}\sin\theta + \hat{\jmath}\cos\theta$$

Geometry

A sphere of radius R has volume $\frac{4}{3}\pi R^3$ and surface area $4\pi R^2$.

A cylinder of radius R and height h has volume $\pi R^2 h$ and surface area $2\pi Rh + 2\pi R^2$ (the first term is the area around the side, the second term is the area of the top and bottom).

Trigonometry

$\sin^2\theta + \cos^2\theta = 1$	$\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos(2\theta)$	$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$
$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$	$\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$	$\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$
$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$	$\sin(2\theta) = 2\sin\theta\cos\theta$	$\sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$

Quadratic Formula

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.