Using Gauss's Law to Find Electric Fields

(for highly symmetric charge distributions)

1. Use the symmetry of the charge distribution to deduce the direction of $\vec{E}$, and (as much as possible) how its magnitude E depends on position.

2. Choose a *parametrized* Gaussian surface ("parametrized" means that it has variable dimensions like r and h, as opposed to fixed dimensions like 7 and R_0) with the same kind of symmetry as the charge distribution:

   - If the charge distribution has *spherical symmetry*, your Gaussian surface should be a concentric sphere with variable radius (r).
   - If the charge distribution has *cylindrical symmetry*, your Gaussian surface should be a coaxial cylinder with variable radius (r) and height (h).
   - If the charge distribution has *plane symmetry*, your Gaussian surface should be a "pill-box" with variable cross-sectional area (A) which straddles the plane of symmetry, extending a variable length (x) on each side.

3. Write down Gauss's Law for your surface, substituting an appropriate expression for the enclosed charge Q_{enclosed}.

4. If your surface consists of several distinct pieces, then split the surface integral up so that you have one surface integral for each piece. E.g. for a cylinder,

$$\oint_{\text{cylinder}} \vec{E} \cdot \mathrm{d}\vec{A} = \int_{\text{top}} \vec{E} \cdot \mathrm{d}\vec{A} + \int_{\text{bottom}} \vec{E} \cdot \mathrm{d}\vec{A} + \int_{\text{side}} \vec{E} \cdot \mathrm{d}\vec{A}$$

5. *Hopefully*, for each piece S, you know either that $\vec{E}$ is everywhere tangent to S (and therefore $\vec{E} \cdot \mathrm{d}\vec{A} = 0$) or that $\vec{E}$ is everywhere perpendicular to S (and therefore $\vec{E} \cdot \mathrm{d}\vec{A} = E\ \mathrm{d}A$).

   - "Hopefully" in this context means "if not, then go back and try a different Gaussian surface because this one isn't going to help you find E". Remember: Gauss's Law is always *true* (no matter what surface you choose), but it is not always *useful*.

6. *Hopefully*, for each remaining piece S, you know that the magnitude E is the same everywhere on S (and therefore you can factor it out of the integral: $\int_S E\ \mathrm{d}A = E \int_S \mathrm{d}A$).

7. $\int_S \mathrm{d}A$ for any surface S is just the surface area of S.

8. Solve algebraically to find E as a function of position (the position variables come from the parameters of your Gaussian surface).

Remember: Gauss's Law will never, ever tell you the *direction* of $\vec{E}$, only its magnitude. You have to already know the direction of $\vec{E}$ from symmetry arguments in order to use Gauss's Law this way.