Special Relativity

For any pair of space-time events measured from two inertial reference frames S and S′, where S′ is moving relative to S with constant velocity v in the positive x direction:

$$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta x = \gamma(\Delta x' + v\Delta t') \qquad \Delta x' = \gamma(\Delta x - v\Delta t)$$

$$\Delta y = \Delta y' \qquad \Delta y' = \Delta y$$

$$\Delta z = \Delta z' \qquad \Delta z' = \Delta z$$

$$\Delta t = \gamma(\Delta t' + \frac{v}{c^2}\Delta x') \qquad \Delta t' = \gamma(\Delta t - \frac{v}{c^2}\Delta x)$$

If $\Delta t'$ is the proper time between two events, then the time interval measured from another frame will be

$$\Delta t = \gamma \Delta t'$$

If $\Delta x'$ is the proper length of an object, then the length measured from another frame will be

$$\Delta x = \frac{\Delta x'}{\gamma}$$

For a particle moving with velocity $\vec{u}$ relative to S (and velocity $\vec{u}\,'$ relative to S′),

$$u_x = \frac{u'_x + v}{1 + \frac{v}{c^2}u'_x} \qquad u'_x = \frac{u_x - v}{1 - \frac{v}{c^2}u_x}$$

$$u_y = \frac{u'_y}{\gamma(1 + \frac{v}{c^2}u'_x)} \qquad u'_y = \frac{u_y}{\gamma(1 - \frac{v}{c^2}u_x)}$$

$$u_z = \frac{u'_z}{\gamma(1 + \frac{v}{c^2}u'_x)} \qquad u'_z = \frac{u_z}{\gamma(1 - \frac{v}{c^2}u_x)}$$

Electrostatics

$$\vec{F} = q\vec{E}$$ (electric force on a particle with charge q)

The electric field at point P due to a small element of charge dq is

$$\mathrm{d}\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{\mathrm{d}q}{r^2}\hat{r}$$

where $\vec{r}$ ($= r\hat{r}$) is the displacement vector that points from dq to P.

Known Electric Field Strengths

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \qquad \text{(point charge } Q\text{)}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \qquad \text{(infinite line of uniform charge density } \lambda \text{ C/m)}$$

$$E = \frac{\sigma}{2\epsilon_0} \qquad \text{(infinite sheet of uniform charge density } \sigma \text{ C/m}^2\text{)}$$

Math Corner

Vector Multiplication

$$\vec{A} \cdot \vec{B} = \left|\vec{A}\right| \left|\vec{B}\right| \cos\theta_{A,B} = AB_{\parallel \text{ to A}} = A_{\parallel \text{ to B}}B$$

$$\left|\vec{A} \times \vec{B}\right| = \left|\vec{A}\right| \left|\vec{B}\right| \sin\theta_{A,B} = AB_{\perp \text{ to A}} = A_{\perp \text{ to B}}B$$

Geometry

A sphere of radius R has volume $\frac{4}{3}\pi R^3$ and surface area $4\pi R^2$.

A cylinder of radius R and height h has volume $\pi R^2 h$ and surface area $2\pi Rh + 2\pi R^2$ (the first term is the area around the side, the second term is the area of the top and bottom).

Trigonometry

$$\sin^2\theta + \cos^2\theta = 1 \qquad \sin^2\theta = \tfrac{1}{2} - \tfrac{1}{2}\cos(2\theta) \qquad \sin 45^\circ = \cos 45^\circ = \tfrac{1}{\sqrt{2}}$$

$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi \qquad \cos^2\theta = \tfrac{1}{2} + \tfrac{1}{2}\cos(2\theta) \qquad \sin 30^\circ = \cos 60^\circ = \tfrac{1}{2}$$

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi \qquad \sin(2\theta) = 2\sin\theta\cos\theta \qquad \sin 60^\circ = \cos 30^\circ = \tfrac{\sqrt{3}}{2}$$

Quadratic Formula

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.