

Problem Set 5
Posted: April 25, 2002
Due: May 2, 2002

Please remember to write your TA's name and your section time, below your name, on the front of your solutions

99 Points, 3 points for each question (Part I) or sub-part (Parts II-IV)

NOTE TO GRADERS: ITALICIZED PORTIONS OF ANSWERS ARE NOT REQUIRED, THEY ARE FOR ELUCIDATION ONLY!

Part I. True/False Questions.

Answer “true” or “false”, and justify your answer with a short argument. (Points are awarded based on the *explanation only*.)

1. The savings rate has no effect on living standards.

False. Savings affects the level of output (*and increasing savings up until the optimal level of savings increases consumption, by the “golden rule”.*)

2. Country A has weaker patent laws than Country B. We therefore expect that Country A will have lower per-capita GDP growth than Country B.

False. (*Either explanation sufficient for full credit:* A) Some countries (typically poorer ones) benefit more from copying foreign innovations, and weaker patent laws facilitate this “technology transfer”; B) patent laws can be “too strong”, by making it too difficult to build on previous innovations, weakening competition, etc.

3. In theory, increasing savings will increase consumption in the long-run.

False in theory-- the “golden rule”. (*But true in practice.*) In theory, there is an optimal savings rate (and associated steady-state capital stock), with $0 < s < 1$.

Exceeding this savings rate leads to “oversaving”, with too much of steady-state output devoted to replacing depreciation, and not enough to consumption. (*In practice countries save much less than the golden rule level, so increasing saving does in fact produce consumption increases.*)

4. Per capita income growth never increases along with the savings rate.

False. It does in the (often long) transition period between steady-states. Saving does not affect long-run per capita GDP growth, however.

5. By definition, production per-worker and capital per-worker do not grow in steady-state.

False. With technological change, Y/N and K/N can grow, at a constant rate, in steady-state. Output and capital per effective worker are constant in this steady-state, however.

6. Arbitrage implies that stock prices associated with historically low dividend-price ratios cannot persist.

False. Investors may be rationally anticipating rising dividends or growth in “fundamental value” in the future.

7. In a world without risk, we wouldn't expect to see downward-sloping yield curves in practice, since we always expect short-term rates to exceed long-term rates.

False. The yield curve will slope down when financial markets expect short-term interest rates to decrease in the future. (This can be seen clearly from equation (15.8) in Blanchard: $i_{2t} < i_{1t}$ when the expected 1-year rate is lower than the current rate i_{1t} . E.g., if $i_{2t} = 2.5\%$ and $i_{1t} = 3\%$, then the expected 1-year rate is 2.0% , which is less than 2.5% .)

8. Output and investment are neutral with respect to monetary policy in the medium-run. (Answer this using our new enriched IS-LM model that allows for different nominal and real interest rates).

True, if at our initial equilibrium output was at its natural rate. In this case Y returns to its natural rate in the medium-run. *Nominal* interest rates will change as the result of an increase or decrease in money growth, but real interest rates (and consequently investment, since we have already noted that Y does not change) will not change in the medium-run.

II. Growth (steady-state)

Consider the following description of an economy

- The aggregate production function is: $Y = K^{0.5}NA^{0.5}$, where K is capital, N is the number of workers, and A is the state of technology.
- Remember that you can think of NA as the amount of effective labor in the economy.
- Capital depreciates at rate δ .
- The rate of technological progress is g_A .
- Population and the labor force grow at rate g_N .
- The saving rate is s .
- The economy is closed (i.e., no trade of goods, services, or ideas)

a) Does the aggregate production function exhibit constant returns to scale? Explain your answer, and show mathematically.

Yes. E.g., doubling both inputs (capital and effective labor) doubles output:

$$xY = (xK)^{0.5}(xNA)^{0.5} = xK^{0.5}NA^{0.5}$$

b) Suppose you quadrupled capital, keeping effective labor constant. What would be the effect on output? What does this imply about returns to capital?

Output would only double, implying that the production function has decreasing returns to capital:

$$2Y = (4K)^{0.5}NA^{0.5} = 2K^{0.5}NA^{0.5}$$

c) Rewrite the production function so that you have output per effective worker on the left-hand side. Now solve for investment per effective worker.

$$Y/NA = (K^{0.5}NA^{0.5})/NA = (K/NA)^{0.5}$$

Since the economy is closed, $I = S = sY$

Investment per effective worker is therefore: $I/NA = s(K/NA)^{0.5}$

d) Explain what the “required level of investment” is and solve for it.

The required level of investment is investment needed to maintain a constant level of capital per effective worker; i.e., the amount necessary to offset the effects of capital depreciation and growth.

$$(I/NA)_{required} = (\delta + g_A + g_N)K/NA$$

This result comes from the capital accumulation equation:

$$K_{t+1}/NA - K_t/NA = s(K_t/NA)^{0.5} - (\delta + g_A + g_N)K_t/NA$$

In the steady-state the level of capital per effective worker is not changing. So the LHS is equal to zero, and investment per effective worker is equal to required investment per effective worker.

e) Solve for the steady-state *levels* of capital per effective worker and output per effective worker.

$$K_{t+1}/NA - K_t/NA = 0 = s(K_t/NA)^{0.5} - (\delta + g_A + g_N)K_t/NA$$

Therefore,

$$s(K_t/NA)^{0.5} = (\delta + g_A + g_N)K_t/NA$$

Square both sides to get:

$$s^2(K/NA) = (\delta + g_A + g_N)^2(K/NA)^2$$

Therefore

$$(K/NA) = s^2 / (\delta + g_A + g_N)^2 ; \text{ and } (Y/NA) = s / (\delta + g_A + g_N)$$

f) What are the steady-state *growth rates* of capital per effective worker and output per effective worker?

By definition, both capital and output per effective worker are constant in steady-state, and therefore their growth rates are equal to zero.

Now consider two different countries that share the above characteristics, but differ in the following ways:

- Country X has $g_N = 3\%$, and $g_A = 0$
- Country Z has $g_A = 2\%$, and $g_N = 0$

g) What is the steady-state growth rate of output in Country X? What is the steady-state growth-rate of output *per worker* in Country X?

In steady state, Y/NA is constant. So if NA is growing at 3%, then Y must be growing at 3% as well. Y/N is constant (since both Y and N are growing at 3%); i.e., it has a zero growth rate.

h) What is the steady-state growth rate of output in Country Z? What is the steady-state growth-rate of output *per worker* in Country Z?

Again Y/NA is constant, so since NA grows at 2% then Y must grow at 2%. But N is not growing (NA 's growth is entirely due to A 's growth), so Y/N also grows at 2%.

i) If X and Z initially have the same output per worker, which country will end up with a higher standard of living in the long-run? (Assume that the savings rate is below the “golden rule” level.)

Country Z, since it has growth in output per worker and X doesn't.

III. Growth (with dynamics)

Consider the aggregate production function $y = F(xK, xNA)$

a) Graph production, investment, and required investment in effective worker units; i.e., in (Y/NA) and (K/NA) space. Assume that investment per effective worker equals required investment initially.

See figure 12-2 in Blanchard, p. 231.

Now use the graph to help develop answers to parts b)–f).

b) Illustrate the effects of a reduction in the saving rate.

See figure 12-3 in Blanchard, p. 233 (this figure shows an *increase* in the savings rate, but pretend we start at s_1 and move down to s_0 , and reverse the direction of the arrows).

c) Explain what happens to K/NA and Y/NA over time

The lower saving rate causes the investment curve to shift down. I/NA is now less than required investment. K/NA and Y/NA will fall over time until the new steady state is reached at point B.

d) What are the long-run effects on the levels of Y/NA and K/NA ?

Both will be permanently lower.

e) Prior to the drop in s , what was the rate of growth in output per worker?

Y/N grew at the rate of technological progress.

f) What are the effects of this reduction in s on the growth rate of output per worker, *during the transition between steady-states?*

During the adjustment, the growth rate of Y/N declines.

g) Once the new steady state is reached, what is the rate of growth in output per worker?

Y/N once again grows at the rate of technological progress (just as in the previous steady-state).

IV. Financial Markets: Short Problems

1. Prices, Yields, and Interest Rates

Consider bonds that make \$1000 payments upon maturity. The one-year interest rate is 10%, and the expected one-year rate for next year is 8%.

Note: these problems require only simple, one-equation calculations!

a) What is the price of a one-year bond today?

Present of value of \$1000 tomorrow, today = $\$1000/1.1 = \909.09

b) What is the price of a two-year bond today?

Now we need to discount over two years.... what rate to use for the second? Think of the two-year bond as two one-year bonds; therefore we use the expected one year rate to discount the second period: $\$1000/[(1.1)(1.08)] = \841.75

c) What is the price of a two-year bond one year from now?

In one year, the two-year bond is then a one-year bond: $\$1,000/(\$1.08) = \$925.53$

2. The Yield Curve

Suppose the yield curve is initially upward sloping. Use IS-LM to help explain what effect each of the following events will have on the shape of the yield curve (i.e., “steeper”, “flatter”, or “unchanged”):

Hints:

i) Consider the effect of the given events on the future one-year rate.

ii) You should be able to answer these in 3 short sentences or less!

a) financial markets expect a future Fed monetary expansion

The expected future one-year rate will fall, causing the yield curve to become flatter.

b) financial markets expect a future tax cut

The expected future one-year rate will increase, causing the yield curve to become steeper.

c) financial markets expect a future reduction in consumer confidence which results in a reduction in consumer spending

The expected future one-year rate will fall, causing the yield curve to become flatter.

3. Policy and Stock Prices

Use the IS-LM model to determine the impact of each of the following scenarios on stock prices. If the effect is ambiguous, explain what additional information would be needed to reach a conclusion.

Note: You should be able to answer these in 3 sentences or less!

a) An unexpected expansionary monetary policy with no change in fiscal policy.

Surprise shift down of LM curve implies (in the short-run) unexpected fall in interest rates and unexpected increase in Y. Stock prices increase.

b) A fully expected expansionary monetary policy with no change in fiscal policy.

Fully anticipated policy or economic changes do not change stock prices.

c) A fully expected expansionary monetary policy with unexpected expansionary fiscal policy.

Ambiguous effect on stock prices. Unexpected expansionary fiscal policy leads to a higher interest rate and higher output than expected. The interest rate effect tends to reduce stock prices; the output effect tends to increase them.