Problem Set #1

Course 14.451 – Macro I

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1. **Human Capital in the Solow Model (based on Mankiw, Romer & Weil 1992)**

    Assume that the production function is given by:

    $$Y = K^a H^l (AL)^{1-a-l}$$

    where Y is output, K is physical capital, H is human capital, A is the level of technology, and L is labor. Assume $a > 0$, $l > 0$ and $a + l < 1$. L and A grow at constant rates n and g, respectively. Output can be used on a one-for-one basis for consumption or investment in either type of capital. Both types of capital depreciate at the rate d. Assume that gross investment in physical capital is the fraction s_K of output and that gross investment in human capital is the fraction s_H of output.

    (a) Let $k \equiv K / AL$ and $h = H / AL$. Obtain the laws of motion for k and h.

    (b) What are the steady-state values of physical capital, human capital, and output, all per unit of effective labor?

    (c) What is the growth rate of output per capita in steady state?

        (i) If we think of all countries as being at their steady state, can this model explain why income per capita grows at different rates across countries?

        (ii) What if countries are at various distances from their steady state?
    (No math for parts (i) and (ii). Just explain in 2-3 brief sentences).

    (d) This augmented Solow model can be tested empirically with cross-country data if we assume that all countries are in their steady states.

        (i) Derive a log-linear regression equation for output per worker that you could estimate using OLS assuming you have measures for $s_{i,K}, s_{i,H}, d_i, n_i$ for each country i and that g and A_0 are known and constant across countries.

        (ii) Give 1-2 brief examples of some problems that might arise in estimating this equation by OLS.

2. **Embodied Technological Change (from Romer)**

One view of technological progress is that the productivity of capital goods built at time t depends on the state of technology at t and is unaffected by subsequent technological progress. This is known as embodied technological progress (technological progress must be embodied in new capital before it can raise output). This problem asks you to investigate its effects.

(a) First, let us modify the basic Solow model to make technological progress capital augmenting rather than labor augmenting. So that a balanced growth path exists, assume that the production function is Cobb-Douglas of the form:

$$Y(t) = [A(t)K(t)]^a L(t)^{1-a}$$

Assume that population grows at a constant rate n, A grows at rate m [i.e. $\dot{A}(t) = mA(t)$], and capital accumulates as usual:

$$\dot{K}(t) = sY(t) - dK(t)$$

   (i) Show that the economy converges to a balanced growth path.
   [Hint: Show that we can write $Y/A^f L$ as a function of $K/A^f L$, where $f = a/(1-a)$. Then analyze the dynamics of $K/A^f L$].
   (ii) Find the growth rates of Y and K on the balanced growth path.

(b) Now consider embodied technological progress. Specifically, let the production function be the following:

$$Y(t) = J(t)^a L(t)^{1-a}$$

   where $J(t)$ is the effective capital stock, and the dynamics of $J(t)$ are:

$$\dot{J}(t) = sA(t)Y(t) - dJ(t)$$

The presence of the $A(t)$ term in this expression means that the productivity of investment at time t depends on the technology at time t.

   (i) Show that the economy converges to a balanced growth path.
   [Hint: Let $\bar{J}(t) = J(t)/A(t)$. Then use the same approach as in (a), focusing on $\bar{J}/A^f L$ instead of $K/A^f L$].
   (ii) What are the growth rates of Y and J on the balanced growth path?

(c) What is the elasticity of output on the balanced growth path with respect to s?
[Hint: First solve for the steady state level of output per unit of $A^f L$].

(d) In the vicinity of the balanced growth path, how rapidly does the economy converge to the balanced growth path? *[Hint: Use a first order taylor series approximation around the steady state level of output per unit of $A^f L$].*

(e) Compare your results for (c) and (d) with the responding results in the text for the basic Solow model: In the model with embodied technological change, is the speed of convergence faster or slower? Is the elasticity of output with respect to savings higher or lower?