

# Problem Set #3

Course 14.451 – Macro I

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**Due: Wednesday, March 9, 2005 [in class]**

## 1. Fiscal Policy in the Ramsey Model

Consider the standard Ramsey model of infinite-horizon households with the following set of preferences.

$$U = \int_0^{\infty} u[c(t)]e^{-\rho t} dt$$
$$u(c) = \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

The population growth rate is  $n$ , there is the typical neoclassical production function, and technology grows at rate  $x$ . The government now purchases goods and services of quantity  $G$ , imposes lump-sum taxes in the amount  $T$ , and has an outstanding quantity  $B$  of government bonds. The quantities  $G, T$ , and  $B$  -- which can vary over time -- are all measured in units of goods, and  $B$  starts at a given value,  $B(0)$ . Bonds are of infinitesimal maturity, pay the interest rate  $r$ , and are viewed by individual households as perfect substitutes for claims on capital or internal loans. (Assume that the government never defaults on its debts.) The government may provide public services that relate to the path of  $G$ , but the path of  $G$  is held fixed in this problem. Finally, for this question, please make sure you derive and express all of your solutions in continuous time.

- (a) What is the government's budget constraint? [Hint: You should write this in the form of a law of motion for  $B$ , and then write out the expression for  $\dot{b}$ , where  $b = B/L$ ].
- (b) What is the representative household's budget constraint? [Hint: Remember that the households take wages,  $w$ , and the interest rate,  $r$ , as given, where  $r$  is net of depreciation, such that  $r = f'(k) - \delta$ . Also make sure to express your constraint in per capita terms.]
- (c) Combine the government's per capita budget constraint from part (a) with the individual's budget constraint in part (b) to find the law of motion for  $k$ . [Hint: Use the fact that  $w + (r + \delta)k = f(k)$  in the competitive economy to substitute out  $w, r$ ].
  - (i) How does additional government spending change the growth of capital and its steady state value in the economy? Why?
  - (ii) Does it matter whether the government uses taxes or bonds to finance its expenditures?
- (d) Re-express the household's budget constraint from part (b) in terms of total per capita assets,  $a$ , where  $a = b + k$ , and solve the household's optimization problem, and derive the growth rate of consumption in this economy.

- (i) Why can we re-write and solve the problem using just total assets?
  - (ii) How does the government affect the growth rate of consumption?
- (e) Using your answers above, briefly describe (in words) how differences in  $B(0)$  or in the path of  $B$  and  $T$  affect the transitional dynamics and steady-state values of the variables  $c$ ,  $k$ ,  $y$ , and  $r$ ? What about the effect of  $G$ ?

## 2. Government and Growth in the Ramsey Model

Consider the household-production version of the Ramsey model. The government taxes output at the rate  $t_Y$ , taxes labor at the rate  $t_L$  (a lump-sum tax), provides per capita lump-sum transfers in the amount  $v$ , and purchases goods and services in the per capita amount  $g$ . The production function is Cobb-Douglas, (i.e.  $Y = AK^a L^{1-a}$ ) and there is no technological progress. Thus, households maximize utility given by:

$$U = \int_0^{\infty} u[c(t)]e^{-\rho t} dt$$

$$u(c) = \frac{c^{1-\frac{1}{q}}}{1-\frac{1}{q}}$$

subject to the budget constraint,

$$\dot{k} = (1-t_Y)Ak^a - t_L - c - (n+d)k + v$$

where  $k(0)$  is given. Suppose that the government uses its goods and services to blow up Pacific islands, actions that neither provide utility nor enhance productivity. All of the government's remaining tax revenues are remitted to households as lump-sum transfers.

- (a) What is the government's budget constraint (in per capita terms)?
- (b) What are the household's first-order optimization conditions, assuming that the representative household takes  $t_Y, t_L, v$  and  $g$  as given?
- (c) Use the household's FOCs to solve for the growth rate of consumption. What is the steady state level of  $k$  in this economy?
- (d) Use a phase-diagram in  $(k, c)$  space to show how the paths of  $k$  and  $c$  change when the government surprises people by permanently raising the values of  $t_Y$  and  $g$  (without changing  $t_L$  and  $v$ ). What will be the immediate impact on consumption? What happens to the steady state values of  $k$  and  $c$ ?
- (e) Redo part (d) for the case in which the government raises  $t_L$  and  $g$  (without changing  $t_Y$  or  $v$ ). What happens to the steady state value of  $k$ ? Explain in 1-2 why the effect of this policy change is different from what we saw in part (d).
- (f) Redo part (d) for the case in which the government raises  $t_L$  and  $v$  (without changing  $t_Y$  and  $g$ ). What happens to the steady-state value of  $k$ ? Explain why this policy change is different from those in parts (d) and (e).