14.451 Waiver

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You have 1.5 hours. You must answer all questions. No books or notes are allowed.

Question 1

Consider the neoclassical growth model, in discrete time. There is no exogenous technological change and no population growth; the size of population and labor is 1. The technology is Cobb-Douglas:

$$y_t = f(k_t) = k_t^\alpha$$

where k_t denotes the capital stock, y_t denotes output or income, and $\alpha \in (0, 1)$. Note that $y_t = r_t k_t + w_t$, where r_t is the interest rate and w_t is the wage rate.

We introduce a government in the model. The government imposes distortionary income taxation in order to finance an unproductive public good. In particular, households' income is taxed by a time-invariant tax rate $\tau \in [0, 1)$. The government budget at t is

$$g_t = \tau y_t,$$

where g_t denotes per-capita government spending in date t and $\tau \in [0, 1)$ is the tax rate on household income. The representative household's budget, on the other hand, is

$$c_t + i_t = (1 - \tau) y_t,$$

where c_t denotes consumption and i_t denotes investment. The capital stock accumulates according to

$$k_{t+1} = (1 - \delta) k_t + i_t,$$

where $\delta \in (0, 1)$. The household maximizes his lifetime utility,

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c) = \frac{c^{1-\theta}}{1-\theta},$$

where $\beta \in (0, 1)$ and $\theta > 0$.

(a) Write down the Bellman equation for this problem and derive the FOC and Envelope conditions.

(b) What is the resource constraint of the economy? What is the Euler condition that characterizes equilibrium consumption growth? Write down the dynamic system that determines the evolution of c and k.

(c) Solve for the steady-state value of k. How does it depend on τ? Interpret the effect of the tax rate on the capital stock.

(d) Consider now the continuous-time limit of the discrete-time dynamics. Draw the phase diagram for $\tau = 0$ (no taxes) and for $\tau = \tau' > 0$ (positive taxes). Suppose the economy has been for ever in the steady state with $\tau = 0$. Suddenly and unexpectedly, at some time $t = t_0$, the government announces that the tax will increase $\tau = \tau'$ immediately. The tax increase is permanent. Describe the transition of the economy to the new steady state. What happens at $t = t_0$? How do k and c evolve over $t > t_0$?

Question 2

True, false, or uncertain? Provide a brief explanation for your answer.

(a) The last half centuary has experienced a large increase in the cross-country dispersion of per-capita GDP levels, evidence that contradicts the hypothesis of conditional convergence.

(b) The neoclassical growth (Ramsey) model can well explain the observed income and growth differentials across countries.

(c) The elasticity of intertemporal substitution affects neither the steady-state capital stock in the standard neoclassical growth model (Ramsey), nor the long-run growth rate in endogenous growth models (Lucas, Romer, etc).

(d) If the aggregate technology exhibits constant returns with respect to the vector of accumulable factors (different types of capital), then the economy has necessarily a constant growth rate at all times and it is impossible to make sense of conditional convergence.

(e) An increase in the ability to borrow and to insure against idiosyncratic risks unambiguously promotes economic growth.

Good Luck!