

14.451 Waiver

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Spring 2005

You have 1.5 hours. You must answer all questions. No books or notes are allowed.

Question 1 (60%)

Consider the neoclassical growth model, in discrete time. There is no exogenous technological change and no population growth; the size of population and labor is 1. The technology is Cobb-Douglas:

$$y_t = f(k_t) = k_t^\alpha g_t^\gamma$$

where k_t denotes the capital stock, g_t denotes productive services provided by the government, and y_t denotes output or income, and where $0 < \alpha < 1$, $0 < \gamma < 1$, $\alpha + \gamma < 1$. The government finances the productive services g_t with income taxation:

$$g_t = \tau y_t,$$

where $\tau \in [0, 1)$. The representative household's budget is

$$c_t + i_t = (1 - \tau)y_t,$$

where c_t denotes consumption and i_t denotes investment. The capital stock accumulates according to

$$k_{t+1} = (1 - \delta)k_t + i_t,$$

where $\delta \in (0, 1)$. The household maximizes his lifetime utility,

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c) = \frac{c^{1-\theta}}{1-\theta},$$

where $\beta \in (0, 1)$ and $\theta > 0$.

(a) Consider the problem of the household for given τ and given $\{g_t\}_{t=0}^{\infty}$. Write down the Bellman equation for this problem and derive the FOC and Envelope conditions.

(b) What is the resource constraint of the economy? What is the Euler condition that characterizes equilibrium consumption growth? Write the dynamic system of the economy in terms of c and k .

(c) Solve for the steady-state values of k and y . How do they depend on τ ? Interpret the effect of the tax rate on capital and income.

(d) Consider now the continuous-time limit of the discrete-time dynamics. Draw the phase diagrams for $\tau = \tau_L$ and $\tau = \tau_H$, for some given τ_L, τ_H such that $0 < \tau_L < \tau_H < 1$. Suppose the economy has been for ever in the steady state with $\tau = \tau_L$. Suddenly and unexpectedly, at some time $t = t_0$, the government announces that the tax will increase to $\tau = \tau_H$ immediately. The tax increase is permanent. Describe the transition of the economy to the new steady state. What happens at $t = t_0$? How do k and c evolve over $t > t_0$?

(e) Now suppose $\alpha + \gamma = 1$, in which case the economy exhibits a linear growth path. How does the growth rate then depend on τ ? Briefly explain this effect.

Question 2 (40%)

True, false, or uncertain? Provide a brief explanation for your answer.

(a) The last half century has experienced a large increase in the cross-country dispersion of per-capita GDP levels, evidence that contradicts the hypothesis of conditional convergence.

(b) The neoclassical growth (Ramsey) model can well explain the observed income and growth differentials across countries.

(c) Since it is a form of rent-seeking activity, industrial espionage necessarily reduces growth.

(d) An increase in the ability to borrow and to insure against idiosyncratic risks unambiguously promotes economic growth.

Good Luck!