

## Practice Question #4 Solutions

Course 14.454 – Macro IV, Fall 2004

### 1. Financial Constraints and the Labor Market

We now introduce a labor market into the CSV model you solved in problem set #2. (You need to use the same setup given in that question to solve this practice question). The labor market is competitive, with a given upward-sloping labor supply curve. Each project now also requires one worker to operate and returns a profit  $\pi = x - y$ , where  $y$  is the equilibrium wage and  $x$  is the project's productivity (again distributed uniformly over the interval  $[0, 2\bar{x}]$ ). The labor market clears and wages are paid *before* project productivities are realized. Also, now assume that all entrepreneurs are endowed with the same amount of wealth,  $w$ .

(a) **Again, assume the entrepreneur is willing to undertake the project, and analyze the project from the point of view of the outside investor.**

i. **Find the investor's expected gain if she invests in the project.**

$$R(D) = \int_{D+y}^{2\bar{x}} \frac{D}{2\bar{x}} dx + \int_0^{D+y} \frac{x - y - c}{2\bar{x}} dx$$

$$R(D) = \left( D - \frac{D(D+y)}{2\bar{x}} \right) + \left( \frac{(D+y)^2}{4\bar{x}} - \frac{(y+c)(D+y)}{2\bar{x}} \right)$$

$$R(D) = D - \frac{(D+y)^2}{4\bar{x}} - \frac{c(D+y)}{2\bar{x}}$$

ii. **Write out the expression that determines the  $D^*$  that will be chosen.**

$$R(D^*) = \bar{R}(1-w)$$

$$D^* - \frac{(D^* + y)^2}{4\bar{x}} - \frac{c(D^* + y)}{2\bar{x}} = \bar{R}(1-w)$$

iii. **What are the expected verification costs for the investor? How and why does a positive wage affect the expected verification costs?**

The expected verification costs are:

$$\frac{c(D^* + y)}{2\bar{x}}$$

They are increasing in the wage because a higher wage implies there is a greater chance that the net profits after paying out wages will be insufficient and the investor will have to verify the returns.

iv. **Taking  $D^*$  as given, write down the condition in which the entrepreneur is willing to undertake the project. Use the equilibrium condition for  $D^*$  to rewrite this condition in terms of the wage, expected return of the project, outside return  $\bar{R}$ , and the expected verification costs for the investor.**

The expected return of the entrepreneur is:

$$\int_{D^*+y}^{2\bar{x}} \frac{x-y-D^*}{2\bar{x}} dx$$

$$\left( \frac{x^2 - 2(y+D^*)x}{4\bar{x}} \right) \Big|_{D^*+y}^{2\bar{x}}$$

$$\bar{x} - (D^* + y) + \frac{(y+D^*)^2}{4\bar{x}}$$

The entrepreneur will invest if and only if:

$$\bar{x} - (D^* + y) + \frac{(y+D^*)^2}{4\bar{x}} > \bar{R}w$$

Combining this with the equilibrium condition for  $D^*$  found in part (ii), we have the following condition:

$$\bar{x} > \bar{R} + y + \frac{c(D^* + y)}{2\bar{x}}$$

- (b) Compare your answer in part (a) to your answers for question 1 in the second problem set. How does the inclusion of a labor market affect the number of projects that will be undertaken?

The inclusion of a labor market will reduce the number of projects undertaken. The need to hire a worker and pay a positive wage increases the threshold that the expected return must achieve in order to both pay the worker and provide the entrepreneur with a return greater than his opportunity cost. Additionally, a positive wage also increases the expected verification costs of the investor which further reduces the number of projects undertaken.

- (c) For simplicity, assume there is a continuum 1 of entrepreneurs. Explain how the labor demand curve can be derived. What is the effect of financial constraints on wages and employment? *[Hint: To see the effect of the financial constraints, compare the equilibrium wage and employment level to what they would be under an efficient equilibrium].*

Consider  $y < \bar{x} - \bar{R} - \frac{c(D^* + y)}{2\bar{x}}$

In this case, all entrepreneurs will be willing to hire labor and demand = 1.

If  $y > \bar{x} - \bar{R} - \frac{c(D^* + y)}{2\bar{x}}$ , however, no entrepreneur will demand labor.

The demand for labor is perfectly elastic at  $y = \bar{x} - \bar{R} - \frac{c(D^* + y)}{2\bar{x}}$

In an efficient equilibrium, it will be the case that  $\bar{x} = \bar{R} + y$ . It is then immediately clear that wages and employment will be lower in the economy with financial constraints (assuming an upward sloping labor supply curve and an interior solution) since the labor demand curve has been shifted downward by the amount of the verification cost. If the labor supply curve is completely inelastic, however, only the wage will fall and not employment levels. Or if we are at full employment in both cases (i.e. a corner solution), then nothing will change.