

# Problem Set #1 Solutions

Course 14.454 – Macro IV

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## 1. Imperfect Monitoring in Labor Markets

Consider an economy similar a la Shapiro and Stiglitz with a continuum of measure 1 of infinitely lived workers with linear preferences. Time is discrete. The workers' discount factor is  $\beta = 1/(1+r)$ . Denote the wage by  $w_t$ . Each period a worker can decide whether to work or shirk. In the first case, he receives the wage  $w_t$ , spends effort  $e$ , and is fired if the job is exogenously terminated, which happens with probability  $b$  at the end of period  $t$ . In the second case, he receives the wage  $w_t$  but with probability  $q$  he is caught shirking at the end of the period  $t$  and is fired. When unemployed, the worker gets a zero utility flow, and he is hired with probability  $a_{t+1}$  at the beginning of next period. This probability is determined endogenously. There is a continuum of measure 1 of plots of land on which a firm can open a "shop". Each shop is identical and produces  $y > e$  units of consumption good using one unit of labor and one unit of land.

- (a) Derive expressions for the expected present value at time  $t$  of being unemployed ( $V_t^U$ ), employed shirking ( $V_t^S$ ), and employed not-shirking ( $V_t^{NS}$ ) in terms of the parameters and the expected present values at time  $t+1$ .

$$V_t^U = \beta(a_{t+1}V_{t+1}^{NS} + (1-a_{t+1})V_{t+1}^U) \quad (1.1)$$

$$V_t^{NS} = w_t - e + \beta(bV_{t+1}^U + (1-b)V_{t+1}^{NS}) \quad (1.2)$$

$$V_t^S = w_t + \beta((b+q)V_{t+1}^U + (1-b-q)V_{t+1}^S) \quad (1.3)$$

- (b) Introduce the no shirking condition as an equality  $V_t^{NS} = V_t^S$  and use it to solve for the employment premium  $V_t^{NS} - V_t^U$  as a function of  $e$ ,  $r$ , and  $q$ .

The no shirking condition (NSC),  $V_t^{NS} = V_t^S$ , is just to ensure that workers are exerting effort. Technically, the condition is  $V_t^{NS} \geq V_t^S \forall t$ , but we know that firms will always choose the wage such that the condition binds with equality for all  $t$ . Using the (NSC) and equations (1.2) and (1.3), we have:

$$\begin{aligned} -e + \beta(bV_{t+1}^U + (1-b)V_{t+1}^{NS}) &= \beta((b+q)V_{t+1}^U + (1-b-q)V_{t+1}^S) \\ -e &= q\beta(V_{t+1}^U - V_{t+1}^S) \\ -e &= q\beta(V_{t+1}^U - V_{t+1}^{NS}) \\ (V_{t+1}^{NS} - V_{t+1}^U) &= \frac{e(1+r)}{q} \end{aligned} \quad (1.4)$$

- (c) Combine the expressions for  $V_t^U$  and  $V_t^{NS}$  so as to find an expression for the wage  $w_t$  as a function only of the parameters and of the time varying probability  $a_{t+1}$ . How does an increase in  $a_{t+1}$  affect  $w_t$ ? Explain briefly.

Subtracting (1.1) from (1.2), we have:

$$\begin{aligned} V_t^{NS} - V_t^U &= w_t - e + \beta(bV_{t+1}^U + (1-b)V_{t+1}^{NS}) - \beta(a_{t+1}V_t^{NS} + (1-a_{t+1})V_t^U) \\ V_t^{NS} - V_t^U &= w_t - e + \frac{1}{1+r}(1-a_{t+1}-b)(V_{t+1}^{NS} - V_{t+1}^U) \end{aligned} \quad (1.5)$$

However, we know from our (NSC) in part (a) that:

$$(V_t^{NS} - V_t^U) = \frac{e(1+r)}{q} \forall t$$

Plugging this into (1.5), we have:

$$\begin{aligned} \frac{e(1+r)}{q} &= w_t - e + \frac{1}{1+r}(1-a_{t+1}-b)\frac{e(1+r)}{q} \\ \frac{er}{q} &= w_t - (q + a_{t+1} + b)\frac{e}{q} \\ w_t &= (r + q + a_{t+1} + b)\frac{e}{q} \end{aligned} \quad (1.6)$$

An increase in  $a_{t+1}$  increases the wage. Since individuals exit unemployment in less time (i.e. being unemployed isn't so bad now), it is necessary for the wage to rise in order to ensure that the NSC still holds.

- (d) Write the flow equation that links  $L_t$  and  $L_{t+1}$ , and use it to find  $a_{t+1}$ . *[Hint: Use the following assumption about timing:  $L_t$  is the number of shops active at time  $t$ . At the end of the period  $t$ , a proportion  $b$  of the existing shops are closed, the workers that were in these shops stay in the unemployed pool for one period, and can be hired at the beginning of period  $t+2$ . The plots of land freed upon termination are rented by new firms opening shops at the beginning of period  $t+1$ , which hire only workers who were in the pool of unemployed during time  $t$ .]*

At time  $t+1$ , there will be  $(1-b)L_t$  shops that are carried over from time  $t$ . Additionally, we know that a fraction  $a_{t+1}$  of the  $1-L_t$  unemployed people at time  $t$  will also be hired into new shops at time  $t+1$ . So, in addition to the old shops, we must have  $a_{t+1}(1-L_t)$  new shops. Combining this, we have:

$$L_{t+1} = (1-b)L_t + a_{t+1}(1-L_t) \quad (1.7)$$

Rewriting this equation, we quickly see that:

$$a_{t+1} = \frac{L_{t+1} - (1-b)L_t}{(1-L_t)} \quad (1.8)$$

- (e) Derive the labor demand assuming that the market for land plots is competitive. Using your answers to (c) and (d), derive the steady state level of employment. How does each of the parameters  $q$ ,  $b$ ,  $e$ , and  $y$  affect steady state employment?

The marginal product of labor in this economy is simply  $y$ . Thus, if the wage is less than  $y$  every owner of a plot of land will wish to hire a worker, and demand will equal 1. If the wage is greater than  $y$ , however, no plot of land will wish to hire a worker, and the demand for labor will be zero. In other words, labor demand,  $L_t^D$ , at time  $t$  can be expressed as:

$$L_t^D = \begin{cases} 1 & \text{if } y > w_t \\ [0,1] & \text{if } y = w_t \\ 0 & \text{if } y < w_t \end{cases} \quad (1.9)$$

To solve for the steady state labor and wage, we need to equate our labor demand with the labor supply schedule. In this model, the quasi-labor supply curve is given by the NSC which implied the wage of equation (1.6). Combining (1.6) and (1.9), we have:

$$y = (r + q + a_{t+1} + b) \frac{e}{q}$$

$$y = \left( r + q + \frac{L_{t+1} - (1-b)L_t}{(1-L_t)} + b \right) \frac{e}{q}$$

But, at steady state, it must be the case that  $L_{t+1} = L_t = L^{SS}$ . So,

$$y = \left( r + q + \frac{bL^{SS}}{(1-L^{SS})} + b \right) \frac{e}{q}$$

$$y = \left( r + q + \frac{b}{(1-L^{SS})} \right) \frac{e}{q} \quad (1.10)$$

$$L^{SS} = \left( \frac{qy - e(q+r+b)}{qy - e(q+r)} \right)$$

$$L^{SS} = \left( \frac{q(y-e) - e(r+b)}{q(y-e) - er} \right)$$

Also notice that  $L^{SS} < 1$ , and with an assumption that  $qy > e(q+r+b)$ , we also have that  $L^{SS} > 0$ .

What is the steady state wage? Well, it should be immediately clear that  $w^{SS} = y$ . But, if you don't believe this, just take the steady state labor supply found in (1.10) and plug it into our wage equation (1.6) using the fact that:

$$a^{SS} = \frac{bL^{SS}}{1 - L^{SS}}$$

Since the wage is stuck at  $y$  in this economy, the only margin for increasing or decreasing the punishment of being caught shirking is to increase the overall level of unemployment in the economy. Using our steady state level of labor described in equation (1.10), we see that a higher probability of catching a shirker (higher  $q$ ) will increase the equilibrium labor supply as firms will be more willing to hire workers if they can easily catch shirkers because this pushes down the NSC.

But, increases in the discount rate,  $r$ , the probability of exogenous firings,  $b$ , or the amount of effort,  $e$ , will all decrease the steady state labor supply. In other words, if individuals value the future less, face a higher chance of being fired exogenously, or need to exert more effort, the level of unemployment will have to rise in order to ensure that the punishment of being caught shirking is high enough to ensure that workers won't shirk.

- (f) **Prove that the steady state level of employment is less than full employment. Would unemployed workers in this economy be willing to work for less than the going wage? Why doesn't the wage fall to accommodate the unemployed?**

From equation (1.10), it should be clear that the steady state level of labor supply is less than one... which implies there isn't full employment. In fact, the structure of this model with exogenous quits  $b > 0$ , ensures this will always be the case. Moreover, since  $V^{NS} > V^U$  by equation (1.4), it is also clear that unemployed workers would be willing to work at less than the going wage. However, the wage in this economy cannot fall to accommodate these unemployed workers because then the NSC would be violated. At any lower wage, an individual cannot convince the firm that it would not shirk if hired.

- (g) **Suppose the government implements a welfare system such that the unemployed receive a payment of  $f$ , and the government finances this with a tax,  $\tau$ , on output such that a firm's output is now only  $(1 - \tau)y$ . How will this policy affect the steady state level of employment? *[Do not resolve the problem... just think through the math and give an intuitive answer].***

There are two effects. First, the welfare payment makes being unemployment less terrible. Hence, to ensure the NSC still holds, it is necessary for the number of unemployed to rise. (This causes the probability of exiting unemployment to fall... which makes being unemployed less attractive again.). Second, reducing in after-tax income shifts down the demand for labor, which also causes an increase in the number of unemployed.

## 2. Labor Markets and Appropriation Problems

Consider an economy as in Caballero and Hammour's "Fundamental Transformation" paper. Suppose that there is a continuum of measure 1 supply of labor and capital. Labor in autarky sector  $U$  (unemployment/home production) has a production function:  $Y_H = 2U - U^2$

Alternatively, there is the joint-production sector where one unit of labor and one unit of capital will produce  $y = 3$  units of the consumption good. Denote  $L$  as the amount of labor in the joint-production sector. Therefore, market clearing in the labor market requires:  $U + L = 1$

There are pre-existing production units that employ  $L_0 = 1/2$  of the labor force (capital used in these units can't be recycled, so just assume that it is not counted in the current unit supply). The productivity of these units is denoted by  $x$  and is distributed uniformly between 1 and 3. Denote  $L_1$  as the labor in newly formed joint-production units and  $L_2$  as labor in pre-existing joint-production units that are not scrapped. Thus,  $L_1 + L_2 = L$ .

The timing is as follows: First, existing units decide whether to separate or not,  $D$  units are destroyed and  $L_0 - D$  workers remain in old joint-production units.<sup>1</sup> Then, separated workers ( $D$ ) and already unemployed workers ( $1 - L_0$ ) look for jobs in the  $L_1$  new units of joint-production. Finally, the workers that do not find a job in the new joint-production units go into home production and capital in the autarky sector (think of this as investment abroad) has a constant return of  $w_k = 1$ .

- (a) **Derive the efficient allocation of labor and capital in each sector, efficient destruction margin,  $\hat{x}$ , for existing joint-production units, and total production of the economy. [Hint: Setup an optimization problem to maximize the sum of total production in the old joint-production units, new joint-production units, home production, and the return on investment abroad. Then, derive the optimal destruction margin  $\hat{x}$ , optimal creation  $L_1$ , and optimal unemployment  $U$ ]**

First, let's write out the production of each sector in this economy:

The production in old joint-production units is:

$$L_2 \int_{\hat{x}}^3 \left( \frac{x}{3 - \hat{x}} \right) dx \quad (1.11)$$

The production in the new production units is:

$$3L_1 \quad (1.12)$$

The production at home is given as:

$$2U - U^2 \quad (1.13)$$

Finally, the return on investment from abroad (which is simply the autarky return on capital that is not part of joint production) is:

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<sup>1</sup> Hint: You can easily show that there will be a cutoff rule  $\hat{x}$  such that  $D = L_0 \int_1^{\hat{x}} \frac{1}{3 - x} dx$

$$1 - L_1 \tag{1.14}$$

Now, I list some of the constraints of the problem:

$$\begin{aligned} L_2 &= L_0 - D \\ D &= L_0 \int_1^{\hat{x}} \frac{1}{3-x} dx = L_0 \left( \frac{\hat{x}-1}{2} \right) \end{aligned} \tag{1.15}$$

Plugging these two definitions into our equations (1.11)-(1.14), and taking their summation, we can express total output of the economy as:

$$Y = \left( L_0 - L_0 \left( \frac{\hat{x}-1}{2} \right) \right) \int_1^{\hat{x}} \left( \frac{x}{3-x} \right) dx + 3L_1 + 2U - U^2 + 1 - L_1$$

And, with a little algebra (WALA), we can write this as:

$$Y = \left( \frac{9}{4} - \frac{\hat{x}^2}{4} \right) L_0 + 2L_1 + 2U - U^2 + 1 \tag{1.16}$$

The only constraint we haven't addressed yet is  $U + L = 1$ . This can be rewritten:

$$\begin{aligned} U + L_1 + L_2 &= 1 \\ U + L_1 + (L_0 - D) &= 1 \\ U + L_1 + \left( L_0 - L_0 \left( \frac{\hat{x}-1}{2} \right) \right) &= 1 \end{aligned} \tag{1.17}$$

Now, we can write the maximization problem as:

$$\begin{aligned} \max_{\hat{x}, L_1, U} & \left( \frac{9}{4} - \frac{\hat{x}^2}{4} \right) L_0 + 2L_1 + 2U - U^2 + 1 \\ \text{s.t.} & \quad U + L_1 + \left( L_0 - L_0 \left( \frac{\hat{x}-1}{2} \right) \right) = 1 \end{aligned}$$

Solving this maximization problem and using  $L_0 = \frac{1}{2}$ , we quickly find:

$$\begin{aligned} U &= 0 \\ L_1 &= \frac{3}{4} \\ \hat{x} &= 2 \end{aligned}$$

From these, we see that total output,  $Y = \frac{25}{8}$

- (b) Consider now the economy with appropriability problems as described in Caballero & Hammour. Write the ex-ante participation constraint for capital. Derive the wage in the autarky sector and the wage in the joint-production sector. Derive the probability of finding a job in the joint-production sector as a function of the cutoff productivity for pre-existing units  $\hat{x}$ . Derive the cutoff productivity for pre-existing units, equilibrium  $L_1$ , unemployment  $U$  and total production.

Since we are in a model of Caballero and Hammour, each input is only guaranteed their ex-post opportunity cost after joint-production. They then will split the remaining surplus evenly under Nash-bargaining. So, in this setup, we have the usual surplus  $s$ , as given by:

$$s = y - w_l, \quad (1.18)$$

where the autarky wages of labor is denoted by  $w_l$ . Thus, the joint-production wage of capital will be half of this surplus, and this must exceed its ex-ante opportunity cost. i.e.

$$w_k^{JP} = \frac{s}{2} \geq w_k = 1$$

Plugging in for the surplus, we can write the ex-ante participation constraint for capital as:

$$y \geq 2 - w_l \quad (1.19)$$

Assuming perfect competition, the wage in autarky will be given by its marginal product:

$$w_l = 2 - 2U \quad (1.20)$$

And, wage in joint-production is:

$$w_l^{JP} = \frac{s}{2} + w_l \quad (1.21)$$

Since there are  $1 - L_0 + D$  workers looking for jobs, and only  $L_1$  will get a job, the probability of finding employment,  $p$ , conditional on being unemployed is:

$$p = \frac{L_1}{1 - L_0 + D} \quad (1.22)$$

The destruction of firms is privately efficient. Thus, a firm is destroyed only if its output is less than the combined opportunity costs of its inputs. Capital in the old firms has a zero opportunity cost, and the labor has a probability  $p$  of being hired in a new joint-production unit, and probability  $1 - p$  of being in autarky. Thus, a firm is destroyed if the following is true:

$$x < p w_l^{JP} + (1 - p) w_l = \hat{x} \quad (1.23)$$

Okay... finally, with equations (1.19) - (1.23) we can solve for the equilibrium. Because of competition, we know that the inequality of equation (1.19) must hold with equality. So, we have immediately that  $w_l = 1$ . From equation (1.20), this implies then that  $U = 1/2$ , and equation (1.18) now gives us  $s = 2$ . Thus, from (1.21) we have  $w_l^{JP} = 2$ .

The next step is to realize that  $p$  can be written as a function of  $\hat{x}$ . Use (1.17) and  $U = 1/2$  to rewrite  $L_1$  as a function of  $\hat{x}$ . Then, use (1.15) to rewrite  $D$  as a function of  $\hat{x}$ . If you do this, you will find the following:

$$p = \frac{\frac{1}{2} + \left(\frac{\hat{x}-3}{2}\right)L_0}{1 + \left(\frac{\hat{x}-3}{2}\right)L_0} \quad (1.24)$$

Plugging (1.24) into (1.23) and using our existing results, we find that  $\hat{x} = 1$ , and this implies that  $L_1 = p = 0$ , and total output is  $11/4$ .

- (c) **Compare your answers from (a) and (b) in terms of the levels of production, creation and destruction. Explain your results.**

From Part A, we have:

$$\begin{aligned} U &= 0 \\ \hat{x} &= 2 \\ w_l &= 2 \\ w_l^{JP} &= 2 \\ D &= 1/4 \\ L_1 &= 3/4 \\ \text{output} &= 25/8 \end{aligned}$$

From Part B, we have:

$$\begin{aligned} U &= 1/2 \\ \hat{x} &= 1 \\ w_l &= 1 \\ w_l^{JP} &= 2 \\ D &= 0 \\ L_1 &= 0 \\ \text{output} &= 22/8 \end{aligned}$$

Because of the appropriation problems, it is now necessary for the surplus of joint-production to be higher so as to guarantee capital a sufficient ex-post return. Since capital is perfectly elastic while labor is not, the wages of labor in autarky are depressed to create this additional surplus. This is done via a higher unemployment rate. However, we see that the wages of labor in joint-production are not affected by the appropriation problem. Because the autarky wage of labor is depressed, the opportunity cost of labor is less and hence it is now efficient to keep open more of the old joint-production units. In fact, we see that all old joint-production units are now kept open. Finally, we see that there will be no joint-production units created in the economy of part (b). Overall, appropriation problems lead to too little creation and destruction *relative* to the efficient outcome.

- (d) **Suppose the government wishes to correct the appropriability problem and return to the first best outcome of (a). To do this, it tries to increase creation in the economy by providing each new joint production unit a lump-sum subsidy,  $\tau = 1$ . Determine the new equilibrium using the same approach as in (b). Has creation returned to its first-best level? What about the level of destruction and output? Explain.**

The subsidy on creation will show up in the surplus of the joint-production unit. So, we can now express the surplus as:

$$s = y + 1 - w_l \quad (1.25)$$

This changes the participation constraint for capital, shown in (1.19) to:

$$y \geq 1 - w_l \quad (1.26)$$

None of our other constraints from part (b) change, and we can now solve for the new equilibrium exactly as we did in part (b). Doing this, we will find:



$$\begin{aligned}
U &= 0 \\
\hat{x} &= 3 \\
w_l &= 2 \\
w_l^{JP} &= 3 \\
D &= 1/2 \\
L_1 &= 1 \\
output &= 3
\end{aligned}$$

The subsidy has succeeded in fostering creation, but now the economy is destroying all of the old joint-production units. This is part of the excess destruction inherent in economies with appropriation problems. See part (e) for more details.

- (e) **Now suppose the government decides to also decrease destruction in the economy by taxing workers who willingly leave old joint-production units by one unit of consumption. What is the new equilibrium of the economy? Why would the government want to increase creation while simultaneously reducing destruction?**

The effect of this policy is to change the private efficient destruction equation given by (1.23). It would now be given as:

$$x < p(w_l^{JP}) + (1 - p)w_l - 1 = \hat{x} \quad (1.27)$$

Resolving the equilibrium as done in part (d), we have:

$$\begin{aligned}
U &= 0 \\
\hat{x} &= 2 \\
w_l &= 2 \\
w_l^{JP} &= 3 \\
D &= 1/4 \\
L_1 &= 3/4 \\
output &= 25/8
\end{aligned}$$

Now, the economy is returned to its first best level as seen in part (a) except that labor in joint-production has a higher wage because of the subsidies. It was necessary to both foster more creation and less destruction because the economy with appropriation problems exhibits too little creation... but given the amount of creation it also exhibits excess destruction. The excess destruction derives from the fact that labor in joint-production units now receive a wage greater than the social shadow wage, and this leads to private decisions to continue the firm that are not socially efficient.

### 3. A Simple Labor Market Search Model

Assume that the labor market is described by the following model. Population is normalized to 1. The unemployment rate is  $u$ ,  $v$  the vacancy rate, and  $w$  the wage. Let  $x$  be the output net of capital costs that is produced by a match between a worker and a job. Workers separate from jobs at the exogenous rate  $s$ ; they are hired at rate  $h = h(u, v)$ , where  $h$  is CRS. The following equations describe the economy.

$$\dot{u} = s(1 - u) - h \quad (1.28)$$

$$h = h(u, v) \quad (1.29)$$

$$w = w\left(\frac{u}{v}\right) \quad (1.30)$$

$$\dot{v} = g(x - w) \quad (1.31)$$

Assume  $g(0) = 0$ .

- (a) Give some intuition for each of the above equations. For equation (1.28) you should explain where each term comes from, and why it differs from Blanchard-Diamond's equation (6). For equation (1.29) you should discuss the values and signs of  $h_u$ ,  $h_v$ ,  $h(0, v)$ , and  $h(u, 0)$ . For equation (1.30), you should discuss a plausible assumption about the sign of  $w'(\cdot)$ . For equation (1.31), discuss a plausible sign for  $g'(\cdot)$  and the assumption that  $g(0) = 0$ .

Equation (1.28) describes the change in unemployment. At every point in time, unemployment increases by the number of workers exogenously fired,  $s(1 - u)$ , and decreases by the number of people hired,  $h$ . The equation is slightly different than that of Blanchard-Diamond since we no longer have to worry about firms exogenously becoming non-productive in the economy.

Looking at equation (1.29): Increases in unemployment or the vacancy rates should increase the number of matches made in the economy. Thus,  $h_u, h_v > 0$ . And, if there aren't any vacancies or people in unemployment, then there must be that zero new people are hired. In other words,  $h(0, v) = h(u, 0) = 0$ .

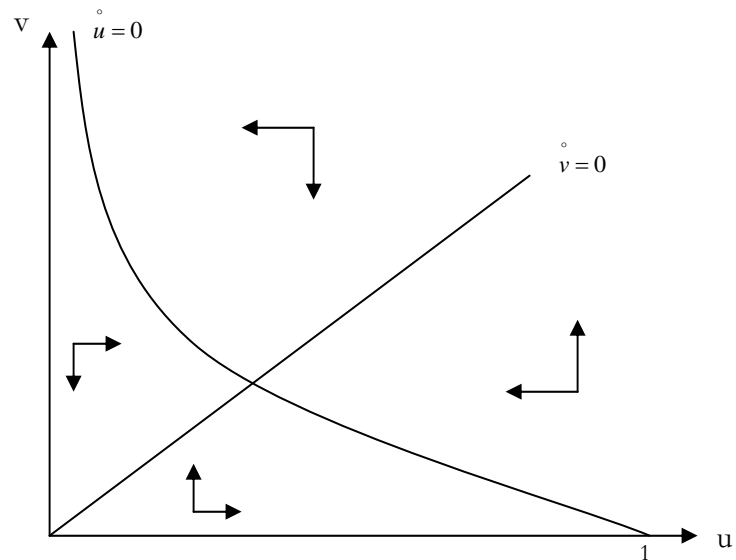
With regards to equation (1.30): This equation tells us that the wages of the economy are a function of unemployment and vacancies. One would expect that an increase in the number of unemployed to vacancies will drive down the wage in the economy. So, it would be plausible to assume  $w'(\cdot) < 0$ .

Finally, we have equation (1.31). This equation simply says that the number of vacancies posted is a function of the gap between output net of capital costs and wages. If the output net of capital costs,  $x$ , increases and wages are fixed, then we would expect that firms would post new vacancies. So, it would be plausible to assume  $g'(\cdot) > 0$ . And,  $g(0) = 0$  means that if output net of capital costs equals the wage, then no new vacancies will be posted.

- (b) In  $(u, v)$  space, show the  $\dot{u} = 0$  and  $\dot{v} = 0$  curves and indicate the directions of movement around these curves.

First, let's analyze the characteristics of the  $\dot{u} = 0$  curve. Notice,  $\dot{u} = 0$  iff  $s = h(u, v) + us$ . But, by CRS,  $s/u = h(1, v/u) + s$  for  $u \neq 0$ . [And, if you notice,  $u = 0$  is not a possibility]. Thus, we can see that for an increase in  $v$ , it must be that  $u$  falls in order for  $\dot{u} = 0$ . Thus, the  $\dot{u} = 0$  curve must be downward sloping. Moreover, given  $s = h(u, v) + us$ ,  $v = 0$  implies  $u = 1$  on the  $\dot{u} = 0$  curve.

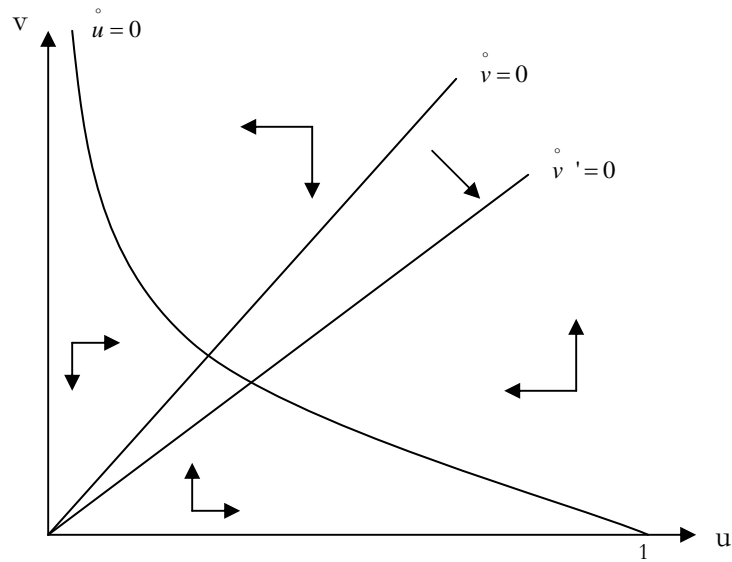
Now, let's look at the  $\dot{v} = 0$  curve. Notice,  $\dot{v} = 0$  iff  $x = w(u/v)$ . So  $u = w^{-1}(x) \times v$  along the  $\dot{v} = 0$  curve. This will be represented by an upward sloping ray from the origin.



It is clear we will always converge to the steady state.

- (c) Suppose that there is a reduction in the production technology. Show what happens in both the short-run and the long-run. Explain in words.

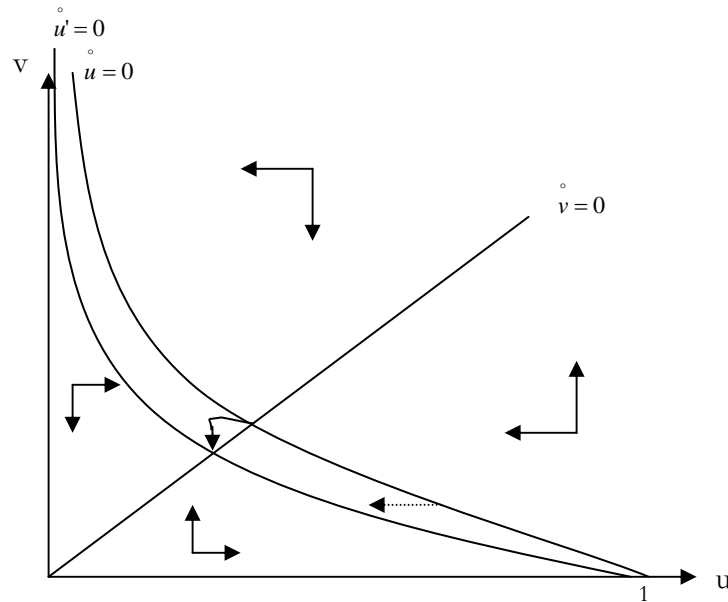
A reduction in the production technology will serve to decrease  $x$ . This will only affect the  $\dot{v} = 0$  curve. Now for a lower output, it must be that wages fall so that  $x = w(u/v)$ . This implies a rise in the  $u/v$  ratio. This is seen as a downward shift in the  $\dot{v} = 0$  curve. Graphically, we now have:



Immediately after the reduction in technology, the number of vacancies in the economy will go down. This will lead to a lower hiring rate, and the unemployment rate will subsequently rise, and wages will go down. In the long run, the economy will settle on a new steady state that has a lower wage, lower vacancy rate, and higher unemployment rate.

**(d) Assume  $h = m\sqrt{uv}$ . What happens if  $m$  increases? Show both in diagrams and words.**

If  $h$  increases, then the  $\dot{u} = 0$  curve will shift in as drawn below. To see this, plug  $h = m\sqrt{uv}$  into your  $\dot{u} = 0$  curve. Notice that if  $m$  increases and you hold  $u$  constant, then it must be that  $v$  falls (except for when  $v = 0$ ).



An increase in  $m$  represents an increase in the matching technology. For a given amount of unemployment and vacancies, the number of new hirings will increase. The immediate effect of the increase in  $m$  will be that unemployment falls. However, as unemployment falls wages will rise causing firms to reduce the number of vacancies offered since output hasn't changed. This will push the wage back down. In the new steady state, the unemployment rate will be less, the vacancy rate will be lower, and the equilibrium wage will be unchanged since we are still on the  $\dot{v} = 0$  curve. Note: The medium run dynamics are more complicated and ambiguous because it is possible we may circle in on the new steady state.

- (e) Look at the below graph<sup>2</sup> of unemployment and vacancy rates for Australia, 1966-1999. With the above model in mind, what kinds of shocks might explain it?

The below graph seems to trace out the  $\dot{u} = 0$  curve of our model. This would lead us to believe the Australian economy is subject to productivity shocks as seen in part (c) rather than shocks to the ability of the economy to produce matches as seen in part (d).

<sup>2</sup> From Groenewold, Nicolaas. "Long-Run Shifts of the Beveridge Curve and the Frictional Unemployment Rate in Australia" <http://www.econs.ecel.uwa.edu.au/economics/Research/2001/DW%2001.09.pdf>

Figure 1: Unemployment and Vacancy Rates, Australia, 1966:Q3-1999:Q1

