# Problem Set #2

Course 14.454 – Macro IV

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**Due: Tuesday, November 23, 2004 [in class]**

1. **Financial Constraints (via Costly State Verification)**

Consider an economy composed of entrepreneurs and outside investors. Both types are risk neutral and can always invest their wealth in outside capital markets and earn an expected gross return $\bar{R}$. Each entrepreneur has wealth $w$, where $w$ is distributed uniformly between zero and two among the entrepreneurs. Each entrepreneur also has the option to undertake a project that requires an indivisible investment of 1 and has an i.i.d. return of $x \sim U[0, 2\bar{x}]$.

Outside investors have lots of wealth but no access to projects. They are willing to lend money to entrepreneurs if their expected return from lending money is not exceeded by the return to their outside option. However, outside investors cannot verify the returns from the project unless they pay a fixed cost $c$.

Assume that the contract between the investor and entrepreneur takes the form of a debt contract: the entrepreneur pays a return $D$ to the outside investor whenever he can do so. When he cannot afford to pay, the outside investor pays the verification cost $c$ and takes all the profits. That is:

If $x \geq D$, the investor gets $D$ and the entrepreneur gets $x - D$
If $x < D$, the investor gets $x - c$ (which may be negative) and the entrepreneur gets 0

There is no bargaining in this model. Investors are perfectly competitive, so the entrepreneurs will never offer more than necessary to get the financing $(1-w)$ that they need to do the project.

(a) Assume that the entrepreneur is willing to undertake the project, and analyze the problem from the point of view of the outside investor.

   i. First, find the investor's expected gain if she invests in the project. What are the expected verification costs of the investor?
   ii. Graph this expected return as a function of $D$ and show graphically how the equilibrium value $D^*$ will be chosen.
   iii. What is the sign of the derivative $\partial D^* / \partial w$? Interpret
   iv. Under what circumstances will there be no lending?

(b) Taking $D^*$ as given (as seen in part (a, ii), it is a function of the model parameters),

   i. Write down the condition under which the entrepreneur is willing to undertake the project. Call this the entrepreneurs [IR] constraint. Don't substitute $D^*$ out of your equation.
   ii. Now use the equilibrium condition for $D^*$ found in part (a,ii) to express the entrepreneur's [IR] constraint in terms of $\bar{x}$, the expected return of the project, $\bar{R}$, the outside return, and the expected cost of verification for the bank found in part (a,i).

(c) Using your answer from part (b), show that there are projects implemented in an efficient economy that are not implemented here. Which entrepreneurs will not be able to start a project in the economy with positive verification costs?

2. **Amplification and Persistence (via Kiyotaki and Moore)**

Consider an economy with two types of agents: farmers and gatherers. There is a continuum 1 of each type. There are also two goods: an ordinary nondurable product (fruit) and a durable productive asset (land). The total supply of land is equal to $\bar{K}$.

The farmer has constant returns to scale technology: he uses $k_t$ units of time $t$ land to produce $ak_t$ units of time $t+1$ fruit. The farmer is also is also subject to the flow of funds constraint, which implies his investment expenditure is financed by his output and net borrowing:

$$q_t\left(k_t - k_{t-1}\right) + Rb_{t-1} = ak_{t-1} + b_t \qquad (1)$$

where $R$ is one plus the real interest rate, $q_{t+1}$ is the land price in terms of fruit at time $t+1$, and $b_t$ is the value of debt undertaken at time $t$.

For simplicity, you should assume that each farmer is *always* eager to expand (due to their great enjoyment of farming), but faces the following credit constraint:

$$Rb_t \le q_{t+1}k_t \qquad (2)$$

(a) Combine equations (1) and (2) to prove the following condition:

$$k_t = \frac{1}{q_t - \frac{q_{t+1}}{R}}\left[\left(a + q_t\right)k_{t-1} - Rb_{t-1}\right] \qquad (3)$$

i. Why can we interpret $\mu = q_t - q_{t+1}/R$ as the amount of down payment necessary per unit of capital purchased?
ii. How do we interpret the expression inside the bracket?
iii. Why is $\partial k_t / \partial a$ positive?

(b) Consider equation (3), suppose $q_t$ and $q_{t+1}$ increase by 1%. Explain how this changes the necessary down payment and net worth of the farmer and how each change impacts farmers' land demand. Which effect is stronger when $ak_{t-1} < Rb_{t-1}$?

Now consider the gatherers' who use a decreasing returns to scale technology, such that $k_t^{'}$ units of time $t$ land to produce $G(k_t^{'})$ units of time $t+1$ fruit. They do not face any borrowing constraint and will maximizes the expected discounted consumption of fruit with discount factor $1/R < 1$. Land market equilibrium implies $k_t + k_t^{'} = \bar{K}$.

(c) Use the land market equilibrium condition and the FOC of the gatherer's maximization problem to prove the following market clearing condition:

$$q_t - \frac{1}{R}q_{t+1} = \frac{1}{R}G'\left(\bar{K} - k_t\right)$$

i. What is the sign of $\partial G'/\partial k_t$? Explain
ii. Given this condition and holding future prices constant, how will today's land prices respond to an increase in $k_t$? (No math)

(d) Now let's put all the pieces together and analyze the impact of a one-time, temporary, upward shock to the productivity of farmers, $a$, at time $t$. Just give 1-2 sentence explanations for each part below.

i. Describe the direct impact on land demanded by farmers at time $t$.
ii. How does the demand change affect land prices and cause amplification?
iii. Why does the shock persist and affect farmer's net worth and demand for land tomorrow (after the shock is gone)?
iv. Why do these future impacts further amplify the shock today?

## 3. Banks and Bank Runs (via Diamond and Dybvig)

Assume there is a continuum 1 of individuals that are each endowed with one unit of currency. There are three time periods, $t = 0,1,2$. At $t = 0$, individuals have two options with regards to how they can invest their money. They can either stuff it in their mattress, where it gets a return equal to 1, or they can invest it in a long-term project that yields a return $R = 4$ in period two. For example, in individual that invests an amount $I$ will receive $4I$ in period two, and have $1-I$ stuffed under the mattress. However, individuals always have the option of withdrawing their money from the long-term project early in period one at a penalty. If they withdraw early, they only receive a return $L = 1/4$ in period 1, rather than the return $R = 4$ in period 2.

At time $t = 1$, a fraction $\pi = 1/2$ of the individuals receive a liquidity shock. These individuals are "impatient" and only value consumption in period one. The fraction $1-\pi$ individuals that do not receive a liquidity shock are "patient" and only value consumption in period two. At time $t = 0$, each individual has an equal chance of being hit by the liquidity shock. Assume that individuals do not discount the future, so that their ex-ante expected utility is given by, $U = \pi u(c_1) + (1-\pi)u(c_2)$, where $c_1$ and $c_2$ is the consumption period 1 and 2 respectively, and $u(c) = -1/c$.

(a) Assume there are no markets available to individuals, so that individuals must simply invest on their own. Given that the individual has invested an amount $I$ at time $t = 0$, what will be the optimal levels of consumption, $c_1$, $c_2$, if:

i. the individual receives a liquidity shock (i.e. is impatient)
ii. the individual does not receive a liquidity shock (i.e. is patient)

(b) What is the optimal level of investment, $I^*$? Given $I^*$, what is the ex-ante expected utility of an individual? Explain in 1-2 sentences why both patient and impatient individuals regret their initial investment decision ex-post in period 1 after their type is realized.

(c) Now suppose an ex-post financial market exists where individuals can trade bonds at time $t = 1$. Each bond costs $p$ units of goods at time $t = 1$, and the bond pays 1 unit of goods at time $t = 2$. Assume all individuals invest an initial amount $I = 1/2$.

i. What is the aggregate demand and supply of bonds at $t = 1$?
ii. What is the equilibrium price $p$?
iii. How much does an "impatient" individual consume in each period?
iv. How much does a "patient" individual consume in each period?

(d) When ex-post financial markets exist, what is the ex-ante expected utility of individuals? Compare this with part (b). Are individuals better off? And, do individuals now have any regrets about their initial investment decision?

(e) Now suppose that when types are revealed in period 1, this information is publicly observable. Suppose there exists a social planner that individual's entrust all of their endowment to at time 0. The social planner will pay impatient individuals $c_1*$ in period 1 and patient individuals $c_2*$ in period 2.

i. Solving the social planner's problem, what is $c_1*$ and $c_2*$?
ii. How much does the social planner invest? (i.e. what is $I$?)
iii. What is an individual's ex-ante expected utility now?
iv. Why is the social planner able to improve the individual's ex-ante utility relative to that found in part (d)?

(f) Now suppose an agent's type is private information, and the social planner can only offer a contract contingent only an individual's announcement of his or her type at time 1. (i.e. she cannot condition the contract on other agents' announcements). Furthermore, at time 1, she meets each agent once with the meeting order randomly determined. If individual's report honestly, can the social planner offer the same contract as in part (e)? Is it optimal for an individual to report honestly when everyone else does? Explain in 1-2 sentences how this planner can be interpreted as a bank.

(g) Suppose all agents fear a bank run, and each agent reports to the bank at time 1 as being impatient. How many individuals will get paid by the bank before it runs out of money in period 1? Given this, explain why this bank run can be an equilibrium… i.e. why is it optimal for a "patient" individual to run on the bank when he/she expects a bank run?

(h) Suppose the bank implements a policy of only paying the first $\pi$ individuals that show up at time 1, and the rest will get paid at time 2. (i.e. it suspends convertibility). Will this eliminate the bank run as an equilibrium?