Problem Set #3

Course 14.454 – Macro IV

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This problem set does not need to be turned in

Question #1: Stock Prices, Dividends and Bubbles

Assume you are in an economy where the stock price, p_t, is given by the standard arbitrage equation (1) and the process for dividends at time t, d_t, is given by equation (2) below:

$$p_t = \frac{1}{1+r}E_t[p_{t+1}] + \frac{1}{1+r}E_t[d_{t+1}] \tag{1}$$

$$d_t - \bar{d} = \rho\left(d_{t-1} - \bar{d}\right) + \varepsilon_t, \quad \varepsilon_t \text{ } i.i.d. \text{ and } E_{t-1}[\varepsilon_t] = 0 \tag{2}$$

(a) Use iterated expectations to solve for the price, p_t, as a function of ONLY future expected dividends. What assumption do you implicitly need to do this?

(b) Assume that $\rho < 1/(1+r)$. Use iterated expectations to find an expression for the expectation (as of time t) for dividends at time $t+i$, $E_t[d_{t+i}]$, that is a function of only $\bar{d}$, ρ, and d_t.

(c) Use your answers from (a) and (b) to find an expression for p_t, as a function of $\bar{d}$, ρ, and d_t. Call this solution to the arbitrage equation the fundamental price, p_t^*.

(d) Now assume the price of the stock has a bubble component, b_t, where $b_t = (1+r)^t b_0$ and $b_0 > 0$. Prove that the price $p_t = p_t^* + b_t$ is also a solution to the arbitrage condition (1) and that our assumption from part (a) is no longer necessary.

(e) Why are individuals willing to pay a higher price, p_t, for the stock than the fundamental price corresponding to the present value of the dividends, p_t^*?

Question #2: Markups via Sticky Prices in the Goods Market

This question is based heavily on sections 1, 4.1, and 4.3 of Rotemberg and Woodford's Handbook chapter. You may find it very helpful to read these sections of the chapter before proceeding with this question.

PART 1: -- Markups: What are they and why do they matter?

Consider a continuum 1 of imperfectly competitive firms. Let the marginal cost of each firm i be given by $Pc(y_i)$, where y_i is the quantity supplied, P is the general price level, and $c'(y) > 0$. Since $c'(y) > 0$, an increase in the quantity supplied by industry i, will be associated with an increase in marginal cost. Because of imperfect competition, each firm faces a downward sloping demand for their good, and can charge a price greater than marginal cost. The markup, μ, of a firm is simply given as its price over marginal cost. In this example, the markup by firm i is given by $\mu_i = P_i/[Pc(y_i)]$. So, if a firm wishes to increase its output and maintain a constant markup, it will need to raise its relative price (P_i/P). However, in a symmetric equilibrium, we know that:

$$\frac{1}{\mu} = c(Y) \tag{3}$$

where the common level of output (and hence aggregate) will be given by Y, and μ is the common (and hence average) markup.

(a) Analyze equation (3). What is the intuition for why average markups and aggregate output move in opposite directions in this equation? If we want to propagate/amplify the business cycle, what type of movement in markups must our models generate?

Again, assume monopolistic competition among a large number of suppliers of differentiated goods. Each firm i faces a downward-sloping demand curve for its product of the form:

$$Y_t^i = D\left(\frac{P_t^i}{P_t}\right)Y_t \tag{4}$$

where P_t^i is the price of firm i at time t, P_t is an aggregate price index, Y_t is an index of aggregate sales at time t, and D is a decreasing function. Assume a constant elasticity of demand, $\varepsilon_D = -xD'(x)/D(x) > 1$, and assume each firm faces the same level of (nominal) marginal costs C_t in a given period. Neglecting fixed costs, profits of firm i at time t are given by:

$$\Pi_t^i = \left(P_t^i - C_t\right)D\left(\frac{P_t^i}{P_t}\right)Y_t \tag{5}$$

(b) Assume completely flexible prices: Maximize the firm's profits to find its optimal markup, μ^*, as a function of the elasticity of demand. Is the markup an increasing or decreasing function of the elasticity? Explain the intuition of this result.

PART 2 – Generating movement in Markups via Sticky Prices, Deriving the Math

Now, we are going to look at sticky price model that will generate movements in the markups charged by firms over the business cycle. Now assume that in each period t, a fraction $(1-\alpha)$ of firms are able to change their prices while the rest must keep their prices constant. A firm that changes its price, chooses it in order to maximize:

$$E_t \sum_{j=0}^{\infty} \alpha^j \beta^j \frac{\Pi_{t+j}}{P_{t+j}} \tag{6}$$

where β is the discount factor per period of time and α^j represents the probability that this prices will still apply *j* periods later. The profit function is the same as in equation (5).

(c) Denote X_t as the new price chosen at date t by any firms that choose then. Prove that the first order condition for their optimization problem is:

$$E_t \sum (\alpha\beta)^j \frac{Y_{t+j}}{P_{t+j}} D'\left(\frac{X_t}{P_{t+j}}\right) \frac{X_t}{P_{t+j}} \left[1 - \frac{1}{\varepsilon_D} - \frac{C_{t+j}}{X_t}\right] = 0 \tag{7}$$

NOTE: *Part (d) asks you to do a log-linearization. If you unfamiliar with this type of calculation, please go to the course web-site and download the file "Log-Linearization Handout" found on the webpage where you can download the problem sets. This handout will help familiarize you with log-linearization.*

(d) We now want to take a log-linear approximation of the first-order condition (7) around a steady state in which all prices are constant over time and equal to one another, marginal cost is similarly constant, and the constant ratio of price to marginal cost equals μ^*. Let $\hat{x}_t$, $\hat{\pi}_t$, and $\hat{c}_t$ denote the percentage deviations of the variables X_t / P_t, P_t / P_{t-1} and C_t / P_t, respectively, from their steady state values. Do the log linearization of equation (7) to get equation (8) below, and interpret this equation.

$$E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left\{ \left[\hat{x}_t - \sum_{k=1}^{j} \hat{\pi}_{t+k} \right] - \hat{c}_{t+j} \right\} = 0 \tag{8}$$

(e) Equation (8) can be solved for the relative price $\hat{x}_t$ of firms that have just changed their price as a function of future inflation and real marginal costs. Assuming $\alpha\beta < 1$, use a quasi-difference of this relation to prove the following:

$$\hat{x}_t = \alpha\beta E_t \hat{\pi}_{t+1} + (1-\alpha\beta)\hat{c}_t + \alpha\beta E_t \hat{x}_{t+1} \tag{9}$$

(f) Now, given a few assumptions, we can show that the rate of increase of the price index satisfies the following relation in our log-linear approximation:

$$\hat{\pi}_t = \left(\frac{1-\alpha}{\alpha}\right) \hat{x}_t \tag{10}$$

(Please see page 1115-6 if you want more details of where this equation comes from.) Substitute (10) into (9) and use the fact that $\hat{\mu}_t = -\hat{c}_t$, where $\hat{\mu}_t$ denotes the percentage deviation of the average markup $\mu_t = P_t / C_t$ from its steady state value of μ^*, to show that:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \kappa \hat{\mu}_t \tag{11}$$

where $\kappa \equiv (1-\alpha\beta)(1-\alpha)/\alpha$

PART 3 – Interpreting the impact of Sticky Prices on Markups

Okay, now the math part of this question is over. Now, that you have an idea where these results come from, you will need to analyze the intuition and implications of these results. The rest of this question does not need any math calculations.

(g) Holding future expectations of inflation constant, consider a positive shock to aggregate demand. Using equation (11), how will this shock to aggregate demand affect the average markup? Are markups pro- or counter-cyclical in this model? What is the intuition for this result? *[Hint: You should first consider how a positive shock to aggregate demand will affect average prices and output. Then, holding future expectations of prices constant, you can analyze the impact on the average markups using equation (11)]*

(h) How will an increase in α affect the magnitude of movements in the markup in response to a shock to the economy? (Again, hold future expectations constant to do your analysis). Please interpret this result.

(i) Now suppose that the elasticity of demand isn't constant. Rather, assume that elasticity of demand is an INCREASING function of the firm's relative price.

   i. How does an average firm's relative price move immediately following a positive shock to aggregate demand?

   ii. What will happen to an average firm's desired markup during an economic boom? [Hint: Use your answer to part (b) above.]

   iii. Would allowing the elasticity of demand to vary as described above reduce or increase amplification in this model?