16.61 - Aerospace Dynamics Spring 2006
Sample Quiz 2

Version: 1.0
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Problem 1

As shown in Figure 1, a simple pendulum consisting of a mass m and a massless rod of length l is mounted on a support of mass M which is attached to a horizontal spring with force constant k. The horizontal surface on which the support mass M rests is frictionless, and gravity works in the minus y-direction. Use the Lagrangian approach to find the equations of motion of the system taking (x, θ) as generalized coordinates.
References:
[1] Lim, Problems and Solutions on Mechanics, Part II, Section 2, Problem 2051 (Columbia Physics PhD Exam Question), p 577–579, World Scientific, 1994.

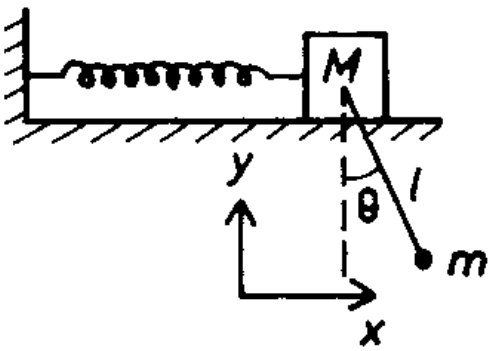

Figure 1: Spring and Mass with Pendulum

Problem 2

As shown in Figure 2, a box of mass m_0 supports a simple pendulum of mass m and length l. A spring of stiffness k forms a horizontal mass-spring system with the box which can slide without friction on a horizontal surface. Use Lagrangian techniques to find the differential equations of motion of the system.

References:

[1] 16.61 OCW, Exam #2, Problem 3, 2003.

[2] Greenwood, Principles of Dynamics, Chapter 6, Problem 6-7, p 276-77, Prentice-Hall, 1965.

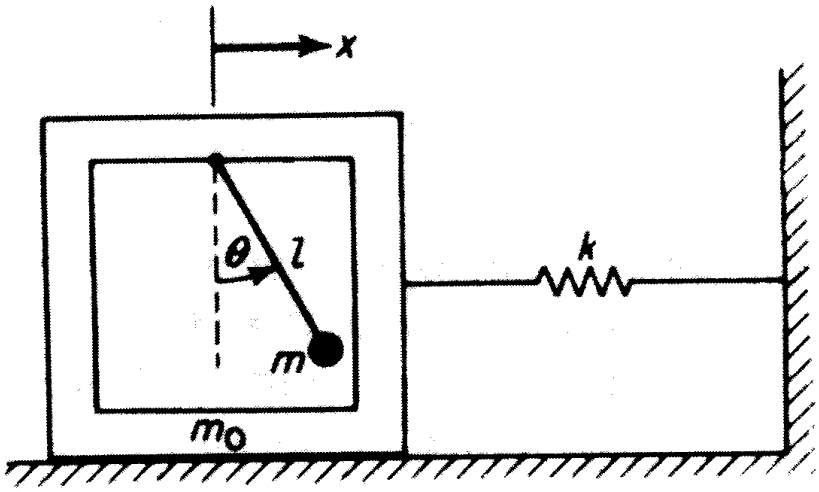

Figure 2: Pendulum in Box

Problem 3

Figure 3 shows a homogeneous circular cylinder of radius r, mass m, and mass moment J about its center of mass. The cylinder rolls without slipping on a curved surface of radius R. Taking θ as the generalized coordinate, use the Lagrangian approach to find the equations of motion for this one-degree-of-freedom system.
Hint: Find the velocity v of the center of mass of the cylinder as a function of $(R, r, \dot{\theta})$, and then using the rolling constraint find the angular velocity of the cylinder.
References:
[1] Shabana, Computational Dynamics, Chapter 3, Example 3.12, p 247–248, Wiley, 1994.

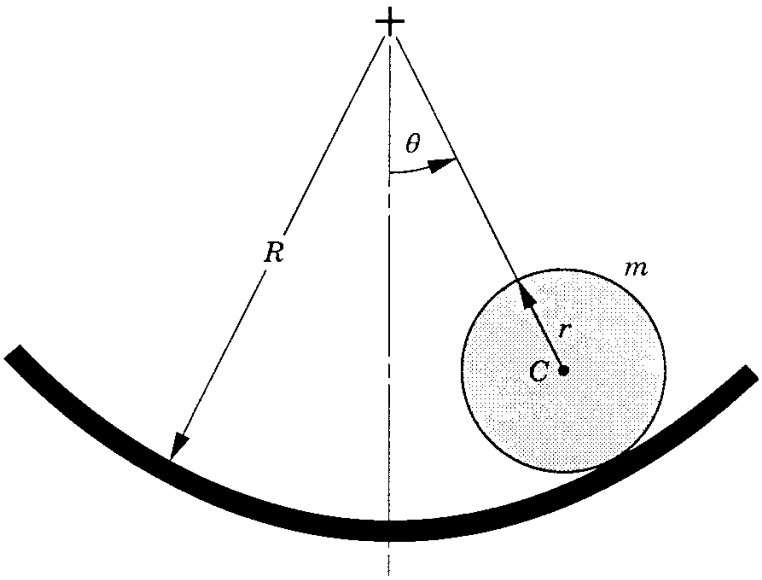

Figure 3: Cylinder Rolling on Curved Surface

Problem 4

As shown in Figure 4, a horizontal turntable rotates at a constant rate ω about a fixed vertical axis through its center O. A particle of mass m can slide in a frictionless circular groove of radius r which is centered at O', which is located a distance $r/3$ from O. Taking θ as defined in the figure as the generalized coordinate, use Lagrangian techniques to find the differential equations of motion.

References:

[1] 16.61 OCW, Exam #2, Problem 1, 2003.

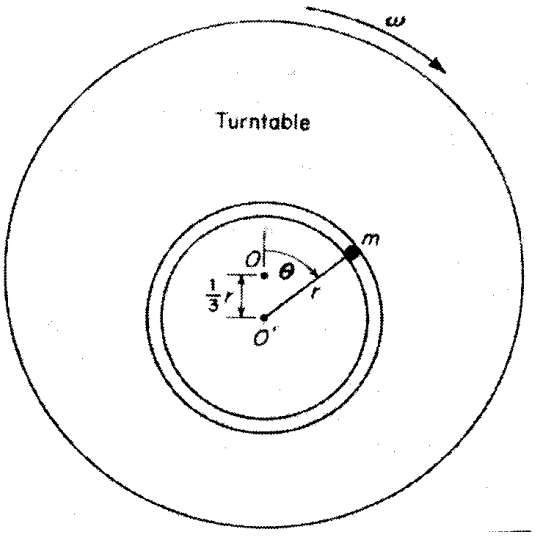

Figure 4: Ball in Track