PAM: Pulse Amplitude Modulation

or ASK: Amplitude Shift Keying

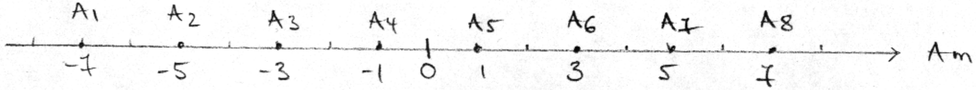
$$u_m(t) = A_m \cdot g(t), \quad A_m \in \{\pm 1, \pm 3, \dots \pm (M-1)\}$$

$$s_m(t) = \mathrm{Re}\{u_m(t)\, e^{j\omega_c t}\}$$

ex. $M=2$ $\quad A_m \in \{\pm 1\}$ binary

$M=4$ $\quad A_m \in \{\pm 1, \pm 3\}$ quaternary

$M=8$ $\quad A_m \in \{\pm 1, \pm 3, \pm 5, \pm 7\}$ $\qquad A_m = -M + 2m - 1, \quad m = 1, \dots M$



→ Most likely errors at receiver are between adjacent amplitude levels

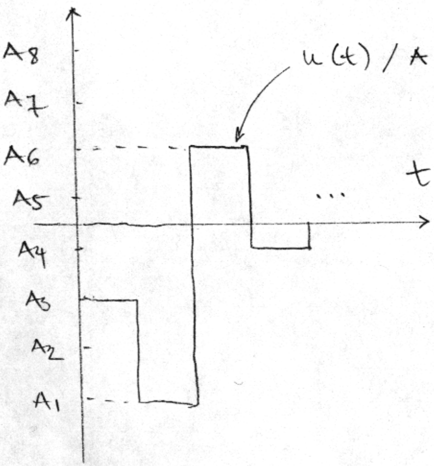
⇒ want adjacent amplitude levels to differ by as few bits as possible (one bit)

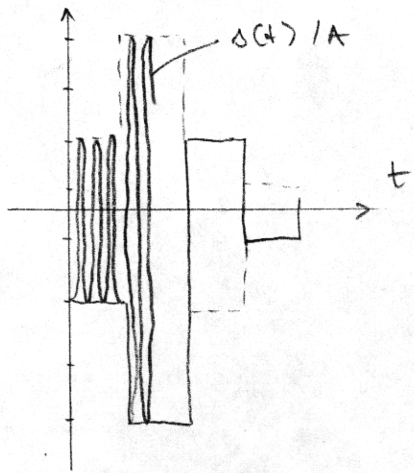
Gray mapping (coding)

$$\left.\begin{array}{l} 000 \rightarrow A_1 \\ 001 \rightarrow A_2 \\ 011 \rightarrow A_3 \\ 010 \rightarrow A_4 \\ 110 \rightarrow A_5 \\ 111 \rightarrow A_6 \\ 101 \rightarrow A_7 \\ 100 \rightarrow A_8 \end{array}\right\} \text{one bit difference}$$

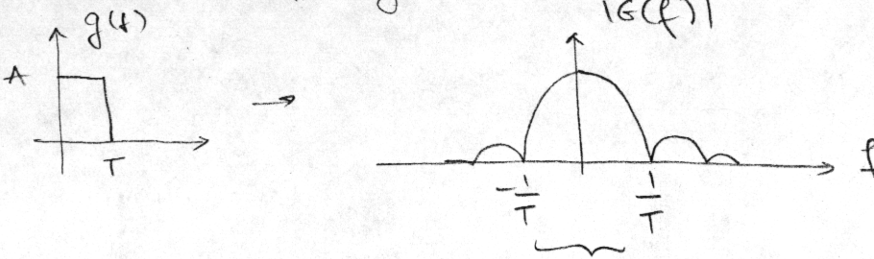
$$\underbrace{011}\,\underbrace{000}\,\underbrace{111}\,\underbrace{010}\cdots = \text{input bit sequence}$$

$$A_3 \quad A_1 \quad A_6 \quad A_4 \qquad = \text{symbol sequence}$$





Bandwidth occupancy [Hz]



also in use:

3 dB bandwidth

99% bandwidth

⋮

$B = 2 \cdot \frac{1}{T}$ first null bandwidth (double-sided)

Bit rate [bps]

3 bits / symbol $\Leftrightarrow$ $R_b = \frac{\mathrm{ld}\, M}{T}$ $\frac{\text{bits}}{\text{second}}$

Symbol rate [sps] $R = \frac{1}{T}$ $\frac{\text{symbols}}{\text{second}}$

Bandwidth efficiency [bps / Hz]

"bits per second per Hz (of occupied bandwidth)"

$\frac{R_b}{B} = \frac{1}{2}\, \mathrm{ld}\, M \ \frac{\text{bps}}{\text{Hz}}$: increases with M

$M = 2$: $T = T_b$, $R_b = \frac{1}{T_b}$, $B = 2 \cdot \frac{1}{T_b}$, $R_b = \frac{B}{2}$ ..

Average Energy per Bit

bit sequence is random, and so is symbol sequence

$$u_m(t) = A_m\, g(t) \Rightarrow E_{u_m} = \int_{-\infty}^{+\infty} u_m^2(t)\, dt = A_m^2 E_g$$

$$E_u = \overline{A_m^2}\, E_g$$

for equally likely symbols: $\overline{A_m^2} = \frac{1}{M}\sum_{1}^{M} A_m^2 = \frac{M^2-1}{3}$

$$\Rightarrow E_u = \frac{M^2-1}{3} E_g$$

$E_s = \frac{1}{2} E_u$: average transmitted energy (actual)

$$E_s = E\left\{ \int_{-\infty}^{+\infty} s_m^2(t)\, dt \right\}$$

Each transmitted waveform carries $k = \mathrm{ld}\, M$ bits of information

$$\Rightarrow E_s = \mathrm{ld}\, M \cdot E_b$$

$$E_b = \frac{1}{\mathrm{ld} M} E_s = \frac{1}{\mathrm{ld} M} \cdot \frac{1}{2} E_u = \frac{1}{\mathrm{ld} M} \cdot \frac{1}{2} \cdot \frac{M^2-1}{3} E_g$$

ex. $M = 2$: $E_b = \frac{E_g}{2}$

QAM : Quadrature Amplitude Modulation

$u_m(t) = A_m\, g(t)$

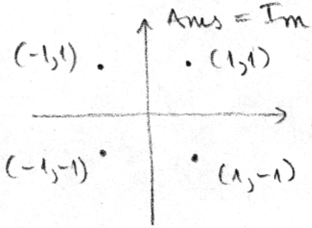
$\downarrow$ complex-valued symbols

$A_m = A_{mc} + j A_{ms}$, $A_{mc,s} \in \{ \pm 1, \pm 3, \dots \pm (\sqrt{M}-1) \}$

ex.

4-QAM

$A_{ms} = \mathrm{Im}\{A_m\}$ $\qquad$ $M=4$



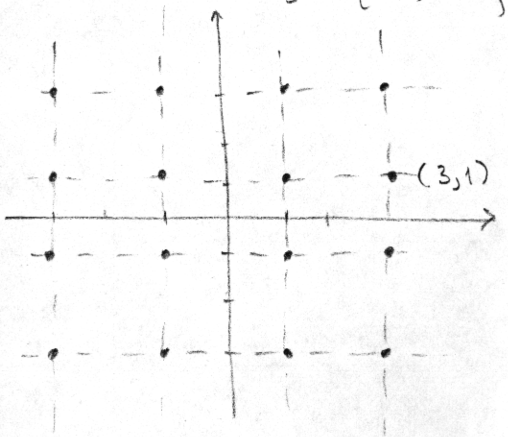
$A_{mc} = \mathrm{Re}\{A_m\}$

two-dimensional "constellation"

16-QAM

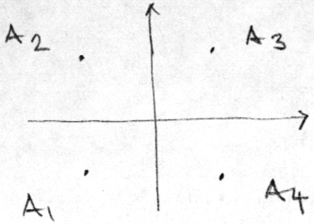
$A_{ms} \in \{ \pm 1, \pm 3 \}$

$M = 16$
$(= 4 \times 4)$



$A_{mc} \in \{ \pm 1, \pm 3 \}$

4 QAM



$0 \to -1$
$1 \to +1$
(not unique)

$00 \to A_1$
$01 \to A_2$
$11 \to A_3$
$10 \to A_4$

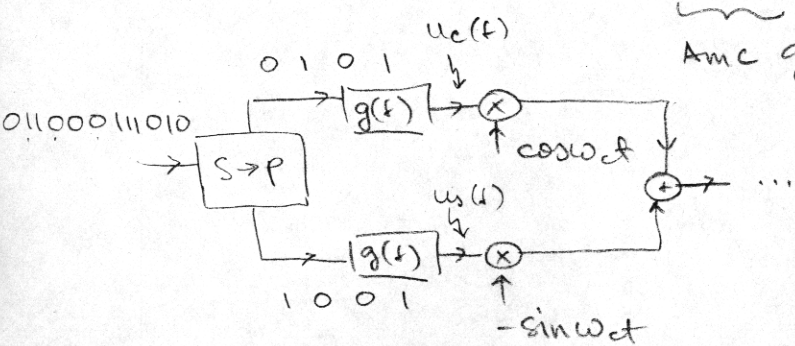
} Gray mapping: adjacent symbols differ by one bit only

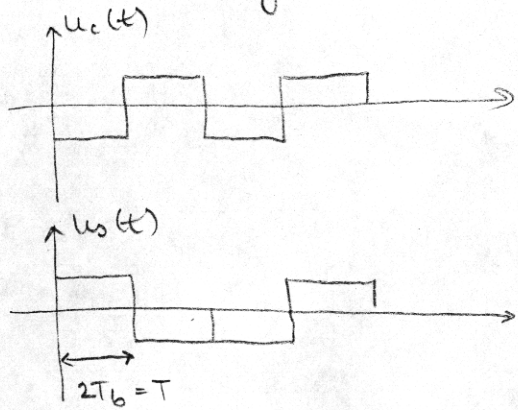
ex. $\underbrace{01}\,\underbrace{10}\,\underbrace{00}\,\underbrace{11}\;1\,010 \ldots$

$A_2 \;\; A_4 \;\; A_1 \;\; A_3 \;\ldots$

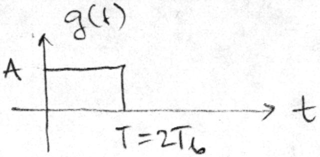
Implementation: $s_m(t) = Re\{u_m(t)\, e^{j\omega_c t}\} =$

$= \underbrace{u_{mc}(t)}_{A_{mc}\, g(t)} \cos\omega_c t - \underbrace{u_{ms}(t)}_{A_{ms}\, g(t)} \sin\omega_c t$

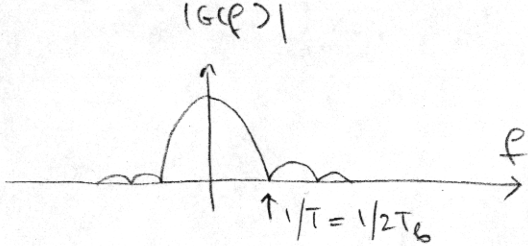




Bandwidth occupancy



$\rightarrow$



twice the pulse duration $\Rightarrow$ half the bandwidth for same information throughput as compared to bin. PAM

(bin. PAM in phase plus bin. PAM in quadrature)

$$B = 2 \cdot \frac{1}{T} = \frac{1}{T_b}$$

Bit rate $\quad R_b = \frac{2 \text{ bits}}{T} = \frac{1}{T_b}$

Bandwidth efficiency $\quad \frac{R_b}{B} = 1.$

In general (M-QAM) : $\quad T = kT_b$ one symbol duration

$B = 2 \cdot \frac{1}{T}$ bandwidth is determined by symbol duration

$$R_b = \frac{k \text{ bits}}{T} = \frac{1}{T_b}$$

Average Energy per Bit M-QAM

$u_m(t) = u_{mc}(t) + j u_{ms}(t)$

$$E_{u_m} = \int_{-\infty}^{+\infty} |u_m^2(t)|^2 \, dt = \int_{-\infty}^{+\infty} [u_{mc}^2(t) + u_{ms}^2(t)] \, dt =$$

$$= (A_{mc}^2 + A_{ms}^2) \cdot E_g = |A_m^2| \cdot E_g$$

$$E_u = \overline{|A_m^2|} \cdot E_g$$

$$\overline{A_{mc}^2} = \overline{A_{ms}^2} = \frac{M-1}{3} \Rightarrow \overline{|A_m^2|} = 2 \cdot \frac{M-1}{3}$$

$$E_u = E_{uc} + E_{us} = 2 \cdot \frac{M-1}{3} E_g$$

$$E_s = \frac{1}{2} E_{uc} + \frac{1}{2} E_{us} = E_{uc} = E_{us} = \frac{1}{2} E_u = \frac{M-1}{3} E_g$$

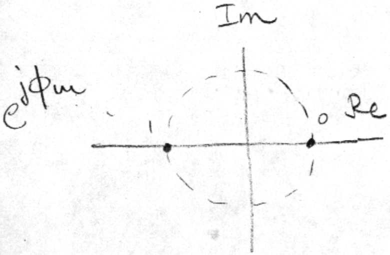
k bits per symbol

$$E_b = \frac{1}{\text{ld} M} \cdot E_s = \frac{1}{\text{ld} M} \cdot \frac{M-1}{3} E_g$$

ex $M = 4$: $E_b = \frac{E_g}{2}$ → compare with Binary PAM.

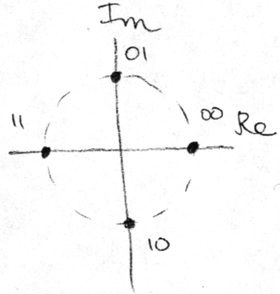
PSK Phase Shift Keying

$$u_m(t) = e^{j\phi_m} \cdot g(t), \quad \phi_m \in \left\{ 0, \frac{2\pi}{M}, 2 \cdot \frac{2\pi}{M}, \dots (M-1) \cdot \frac{2\pi}{M} \right\} \qquad \text{M-PSK}$$



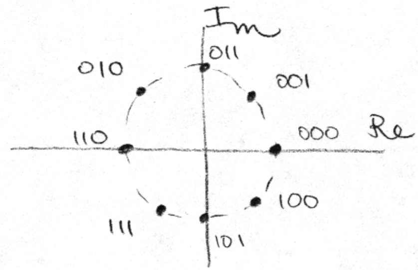
B-PSK

(= bin. PAM)



QPSK

(~ 4-QAM)



8-PSK

$$s_m(t) = \mathrm{Re}\left\{ \underbrace{e^{j\phi_m}}_{d_m} g(t)\, e^{j\omega_c t} \right\} = g(t) \cdot \cos[\omega_c t + \phi_m] =$$

$$= \underbrace{\cos\phi_m}_{\mathrm{Re}\{d_m\}} \cdot g(t) \cos\omega_c t - \underbrace{\sin\phi_m}_{\mathrm{Im}\{d_m\}} g(t) \sin\omega_c t$$

Bandwidth occupancy $B \sim 2 \cdot \frac{1}{T}$, $T = \mathrm{ld}\,M \cdot T_b$

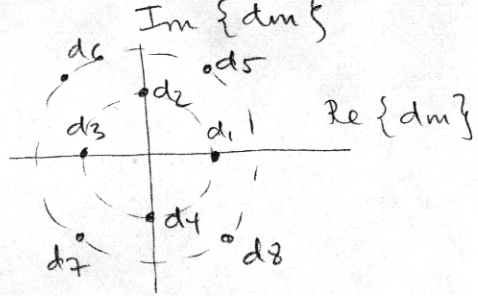
Bit rate: $R_b = \frac{1}{T_b} = \frac{\mathrm{ld}\,M}{T}$

Bandwidth efficiency $\frac{R_b}{B} \sim \mathrm{ld}\,M$

Energy per bit: $E_{u_m} = E_g \Rightarrow E_u = E_g \Rightarrow E_s = \frac{1}{2} E_g \Rightarrow E_b = \frac{1}{\mathrm{ld}\,M} \cdot \frac{1}{2} E_g$

Rectangular pulse → constant amplitude signal

Combination: AM/PM



$M=8$

$$u_m(t) = d_m \cdot g(t)$$

$$s_m(t) = \mathrm{Re}\{u_m(t)\, e^{j\omega_c t}\}$$

Summary: linear modulation methods

$$u(t) = \sum_n d(n)\, g(t-nT)$$

PAM $\quad d(n) \in \{A_m\}_1^M \quad A_m = (2m-1)-M \quad , \quad m=1,\dots,M$

QAM $\quad d(n) \in \{A_{mc}+jA_{ms}\} \quad A_{mc,s} = (2m-1)-\sqrt{M}$

PSK $\quad d(n) \in \{e^{j\phi_m}\} \quad \phi_m = (m-1)\cdot\frac{2\pi}{M}$