

16.682: Communications Systems Engineering  
Homework assignment due 5/10/2001

- 1) Suppose that  $X$  and  $Y$  are independent Poisson random variables with rate  $\lambda_1$  and  $\lambda_2$  respectively.
  - A) Show that  $E[X] = \lambda_1$
  - B) Show that  $E[X^2] = \lambda_1 + (\lambda_1)^2$
  - C) Show that  $Z = X + Y$  is Poisson of rate  $\lambda_1 + \lambda_2$
  
- 2) Customers arrive at a fast food restaurant at a rate of 5 per minute. They wait in line for an average of 5 minutes.
  - A) Find the average number of customers waiting in line.
  - B) Suppose half of the customers carry their orders out and the other half eat in the dining room. If a meal requires an average of 20 minutes, what is the average number of customers in the dining room.
  
- 3) Packets arrive at link at an average rate of 10 per second. The transmission rate of the link is 20,000 bits per second and the average packet length is 1000 bits. Assume that packet lengths are Exponentially distributed, and that arrivals are Poisson.
  - A) What is the transmission rate in packets per second?
  - B) What is the average queueing delay?
  - C) What is the average number of packets in the buffer?
  - D) What is the probability that the system is empty?
  - E) Repeat parts B and C assuming that the packets are all the same length.
  - F) Repeat parts B and C assuming that 1/2 the packets are 500 bits and 1/2 are 1500 bits.
  
- 4) A communications satellite company establishes a direct connection between a remote town and the company's central office for providing telephone services. Calls arrive as a Poisson process at a rate of 1 per minute. Call durations are Exponentially distributed with an average of 3 minutes. How many circuits should the company provide to ensure that a blocking probability of less than 1% is maintained?

**Solutions**

- 1)
  - A)  $T=5$  minutes ,  $\lambda=5$  per minute  
 $N = \lambda T = 25$  customers.

B) Now only half the customer go to the dinning room so  $\lambda=2.5$ , and  $T=20$ . So,

$N = 2.5*20 = 50$  customers in dinning room.

2)

A) rate is  $1/\text{delay} = 20$  packets/second

B)  $\mu=20$ ,  $\lambda=10$  so,  $D = 1/(20-10) = 0.1$  seconds. (total delay in the system). Since the transmission delay is  $1/20 = 0.05$  seconds, the queueing delay is equal to the total delay - the transmission delay =  $0.1-0.05 = 0.05$ .

C) use littles law and  $N = \lambda D = 1$  packet (in the system) . With  $\lambda/\mu=0.5$ , the average number in the buffer is  $N-0.5 = 0.5$ .

D)  $P(0) = 1 - \lambda/\mu = 1 - 10/20 = 0.5$ .