

18.099b Problem Set 3a

Due: Thursday, March 3rd (in class or before).

Suppose $X \subseteq \mathbb{R}$ is such that every infinite subset of X contains an infinite convergent sequence. That is, if $F \subseteq X$ is infinite then there exists an infinite sequence $\{a_n\}$ with $a_n \in F$ for all n , and a number $a \in \mathbb{R}$, such that $\lim_{n \rightarrow \infty} a_n = a$.

Prove that X is bounded from above and below.

Hint: Assume, for a contradiction, that the conclusion fails. Construct an infinite set $\{p_1, p_2, \dots\}$ in X such that for any $i \neq j$ $|p_i - p_j| \geq 1$. Use this to contradict our assumption on X .