18.099b Problem Set 3b

Due: Thursday, Match 3rd (in class or before).

Let X be a metric space. Recall that a *neighbourhood* is a subset of X of the form

$$N_r(p) := \{x \in X : d(p, x) < r\}$$

where r is a positive real number and p is a point in X. The number r is called the *radius* of $N_r(p)$.

Suppose every infinite subset of X has a limit point. Prove that for every positive real number $r > 0$ there are finitely many neighbourhoods of radius r that cover X.

Hint: Assume, for a contradiction, that the conclusion fails for some $r > 0$. Construct an infinite set $\{p_1, p_2, \ldots\}$ in X such that for any $i \neq j$ $d(p_i, p_j) \geq r$. Use this to contradict our assumption on X.