

Problem Set #1

Issued : Tue 02/14/2006  
Due : Thu 02/23/2006

**Problem 1:**

Consider a two degree-freedom planar manipulator with two rotational joints with link lengths  $l_1 = 5$  and  $l_2 = 3$ . The endpoint velocity is denoted by  $V = [v_x, v_y]^T$ .

- (a) Given a desired endpoint velocity, find joint velocities that produce the desired endpoint velocity.
- (b) Find singular configurations, and determine in which direction the endpoint can't move for each singular configuration.

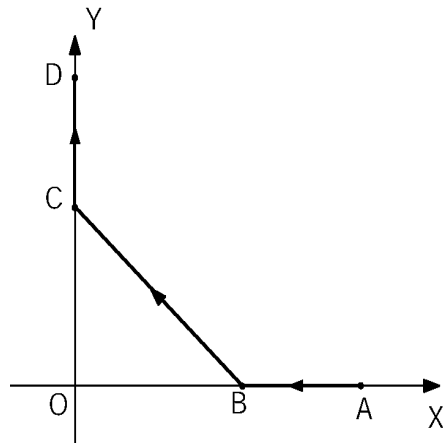


Figure 1: Trajectory for Problem 1. OA=6, OB=4, OC=4, OD=6. These dimensions have been chosen such that the trajectory lies within the workspace of the manipulator.

- (c) Plot profiles of joint velocities when the endpoint is required to track a specified trajectory (shown in Figure 1) at a constant tangential speed.

**Problem 2:**

Consider a two degree-freedom planar manipulator with link lengths  $l_1 = l_2 = 2m$ . Measuring the ratio of joint torque to joint displacement, we identify the stiffness of each joint:

$$k_1 = 3 \times 10^5 Nm/rad \quad k_2 = 2 \times 10^5 Nm/rad$$

- (a) Compute the endpoint compliance matrix for the configuration of  $\theta_1 = 45^\circ$  and  $\theta_2 = 60^\circ$ .
- (b) Find the directions of maximum and minimum compliance at this configuration.
- (c) Plot the maximum and minimum stiffness values as the function of  $\theta_1$  and  $\theta_2$ .

**Problem 3:**

For the same manipulator as above, consider the problem of inverse kinematics using sliding variables as we have discussed in class. Explain why defining  $q_e(t)$  by the equation:

$$\dot{q}_r = \dot{q}_e - \lambda(q - q_e)$$

leads to an explicit inverse kinematics solution for  $q_e$ . Plot your result of  $q_e$  in simulation for  $x_d$  being a circle with radius 1 and constant tangential acceleration.