

2.165 Robotics  
Spring 2006

Problem Set #4

Issued : Thu 04/20/2006  
Due : Tue 05/02/2006

**Problem 1:** Set-point Control in Image Space

In this problem set, we examine the set-point control of position of a robot end-effector in the image space.

Consider our usual two degree-of-freedom manipulator in the vertical plane ( $z = 0$ ) with two rotational joints. The dynamic equations of this manipulator are given by:

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{D}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}$$

where

$$\mathbf{H} = \begin{bmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} -h\dot{q}_2 & -h\dot{q}_1 - h\dot{q}_2 \\ h\dot{q}_1 & 0 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} d_1|\dot{q}_1| & 0 \\ 0 & d_2|\dot{q}_2| \end{bmatrix}, \mathbf{g} = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$$

$$\begin{aligned} H_{11} &= m_1 l_{c_1}^2 + I_1 + m_2(l_1^2 + l_{c_2}^2 + 2l_1 l_{c_2} c_2) + I_2 \\ H_{22} &= m_2 l_{c_2}^2 + I_2 \\ H_{12} &= m_2 l_1 l_{c_2} c_2 + m_2 l_{c_2}^2 + I_2 \\ h &= m_2 l_1 l_{c_2} s_2 \\ G_1 &= m_1 l_{c_1} g c_1 + m_2 l_{c_2} g c_{12} + m_2 l_1 g c_1 \\ G_2 &= m_2 l_{c_2} g c_{12} \end{aligned}$$

and  $d_1, d_2$  are positive constants.

An imaging system is set up using a stationary CCD camera to capture the image of the end-effector on a  $640 \times 480$  pixel detector plane. The lens of the camera has a focal length of 100 mm and is placed at a distance  $z = 1$  m from the plane  $z = 0$  of

the manipulator. The image plane has a uniform spatial resolution of about  $10\ \mu\text{m}$  per pixel along its length and breadth.

To achieve set-point control of the end-effector in the image space, the following control law is proposed for the joint torques of the manipulator:

$$\tau = -\mathbf{J}^T \mathbf{J}_i^T \mathbf{K}_p \tilde{\mathbf{x}}_i - \mathbf{K}_d \dot{\mathbf{q}} + \mathbf{g}(\mathbf{q})$$

where  $\mathbf{K}_p$ ,  $\mathbf{K}_d$  are both constant symmetric positive-definite matrices,  $\mathbf{J}$  is the manipulator Jacobian, and the image Jacobian  $\mathbf{J}_i$  is a constant matrix defined between the end-effector image-space velocity  $\dot{\tilde{\mathbf{x}}}_i$  and task-space velocity  $\dot{\mathbf{x}}$  by the relation  $\dot{\tilde{\mathbf{x}}}_i = \mathbf{J}_i \dot{\mathbf{x}}$

- Derive the expression for the image Jacobian  $\mathbf{J}_i$  that we obtained in class. State any assumptions needed as part of this derivation. Evaluate the image Jacobian using the parameters of the given imaging system.
- Draw a schematic feedback control block diagram of the closed-loop system. Show clearly the plant, sensors, controllers, and the desired and actual signals.
- Using the following Lyapunov functional candidate:

$$V = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{H} \dot{\mathbf{q}} + \frac{1}{2} \tilde{\mathbf{x}}_i^T \mathbf{K}_p \tilde{\mathbf{x}}_i$$

Analyze the system behavior under the proposed control law. Determine the resultant steady state error. *Note:* Do not use MATLAB in this part of the problem.

- Let us assume that the desired final position of the image of the end-effector  $\mathbf{x}_{id}$  is the center of the image plane, i.e.  $(320, 240)$ . Simulate the system response using MATLAB with the following initial conditions:  $q_1(0) = \pi/6$ ,  $q_2(0) = \pi/4$ ,  $\dot{q}_1(0) = 0$ ,  $\dot{q}_2(0) = 0$ ,  $x_{i_1}(0) = 600$ ,  $x_{i_2}(0) = 400$ . Assume that the links of the manipulator are uniform slender rods, each of mass 1 kg and length 1 m.

Plot the variation of joint torques  $\tau$  and image space error signal  $\tilde{\mathbf{x}}_i$  as functions of time. In order to perform the simulation, do you have to know the value of the desired joint angles  $\mathbf{q}_d$ ?

- Analyze the system behavior for the case when gravity compensation is not included in the given control law for the torque. Predict the steady state error for this case. *Note:* Do not use MATLAB in this part of the problem.

- (f) Use MATLAB to simulate the system response under the control law of Part (e). Use the same initial conditions as in Part (d). Compare the value of the steady state error that you obtain from the simulations with the value that you predicted in Part (e).

**Problem 2:** (optional)

Consider two Fitzhugh Nagumo neurons as discussed in class, and show how to couple them so that they globally synchronize. Illustrate your results in simulations. Recall that the dynamics of an uncoupled Fitzhugh-Nagumo oscillator are given by the following equations:

$$\begin{cases} \dot{v} = c(v + w - \frac{1}{3}v^3 + I) \\ \dot{w} = -\frac{1}{c}(v - a + bw) \end{cases}$$

Hint: You can use the following metric:

$$\Theta = \begin{pmatrix} 1 & 0 \\ 0 & c \end{pmatrix}$$