IMPLEMENTATION OF GHOST-CELL IMMERSED BOUNDARY METHOD IN 2.29 FV FRAMEWORK

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Course: 2.29 Numerical Methods in Fluid Mechanics

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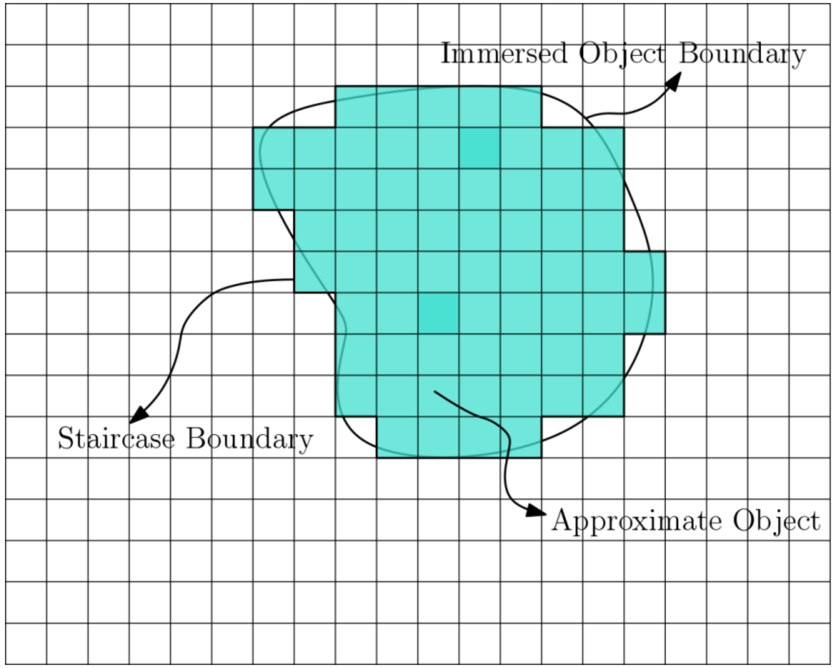
Outline

1. Motivation
2. Literature Review
3. Methodology
4. Results
5. Conclusion

Representation of Complex Shape Boundaries

- Currently 2.29 FV Framework approximates complex geometry of the boundary (mainly of obstacles) with a staircase pattern
- Hence it may require very fine mesh to approximate a geometry as simple as circular obstacle
- Many times create artificial artifacts



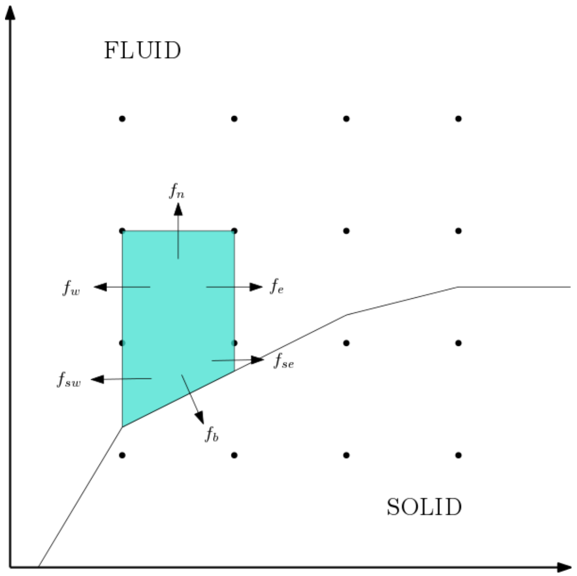
Literature Review

- Key Question: How is complex geometry handled in other CFD codes?
- Broadly we can classify into 3 groups
  - Continuous forcing approach
  - Discrete forcing approach
    - Ghost-cell immersed boundary method
  - Cut-cell finite-volume approach

$$\vec{F}(\vec{x}_b, t) = \alpha \int_0^t \vec{u}(\vec{x}_b, \tau) d\tau + \beta \vec{u}(\vec{x}_b, t)$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \mathrm{RHS}_i + f_i$$

$$f_i = -RHS_i + \frac{V_i^{n+1} - u_i^n}{\Delta t}$$

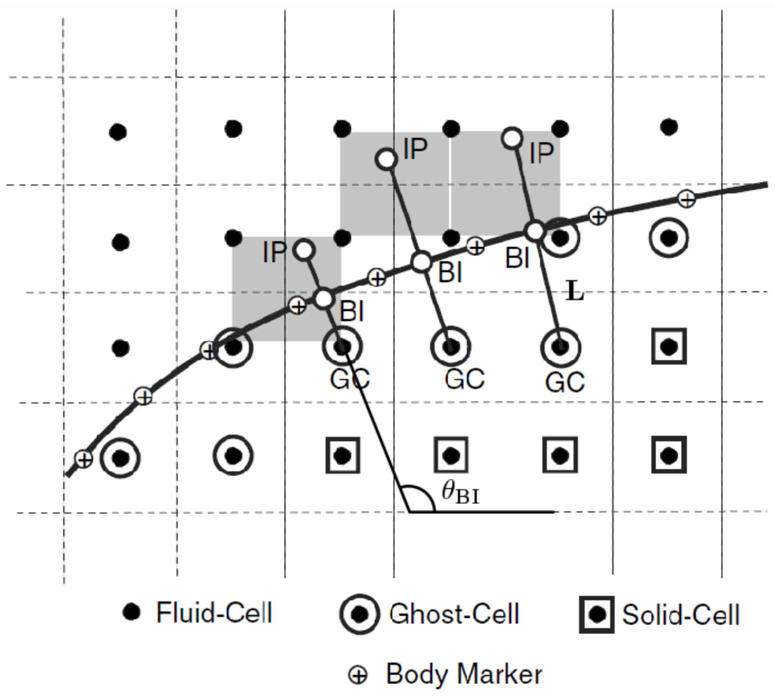


Ghost-Cell Immersed Boundary Method

Bilinear Interpolation: $\phi = a_0 + a_1 x + a_2 y + a_3 xy$

$$B = \begin{bmatrix} 1 & x_0 & y_0 & x_0 y_0 \\ 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_2 & y_2 & x_2 y_2 \\ 1 & x_3 & y_3 & x_3 y_3 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 & x_0 & y_0 & x_0 y_0 \\ 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_2 & y_2 & x_2 y_2 \\ 1 & x_3 & y_3 & x_3 y_3 \end{bmatrix}^{-1} \begin{bmatrix} \phi_0 \\ \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}$$



$$\phi_{IP} = c_0\phi_0 + c_1\phi_1 + c_2\phi_2 + c_3\phi_3$$

$$\phi_{IP} = \begin{bmatrix} 1 & x_{IP} & y_{IP} & x_{IP} y_{IP} \end{bmatrix} \begin{bmatrix} 1 & x_0 & y_0 & x_0 y_0 \\ 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_2 & y_2 & x_2 y_2 \\ 1 & x_3 & y_3 & x_3 y_3 \end{bmatrix}^{-1} \begin{bmatrix} \phi_0 \\ \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}$$

Interpolation

Neumann BC

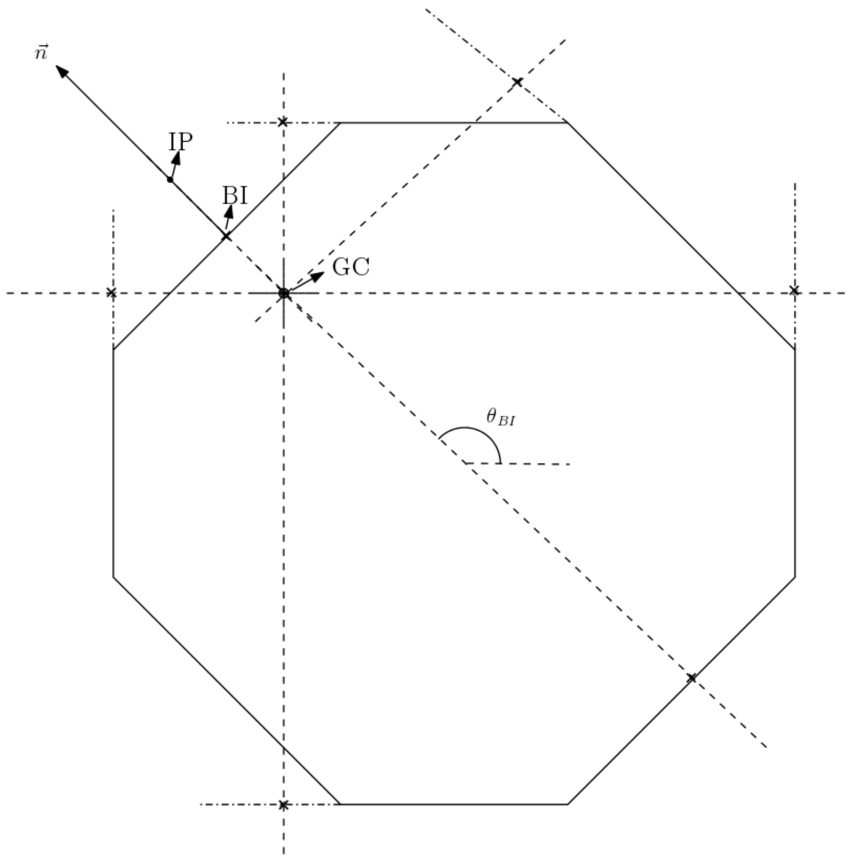
$$\phi_{IP} = \begin{bmatrix} 1 & x_{IP} & y_{IP} & x_{IP}y_{IP} \end{bmatrix} \begin{bmatrix} 1 & \cos(\theta_{BI}) & \sin(\theta_{BI}) & x_{BI}\sin(\theta_{BI}) + y_{BI}\sin(\theta_{BI}) \\ 1 & x_1 & y_1 & x_1y_1 \\ 1 & x_2 & y_2 & x_2y_2 \\ 1 & x_3 & y_3 & x_3y_3 \end{bmatrix}^{-1} \begin{bmatrix} \frac{d\phi_{BI}}{d\vec{n}} \\ \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}$$

$$\phi_{IP} = 2\phi_{BI} - \phi_{GC}$$

$$\phi_{IP} = 2L\frac{d\phi_{BI}}{d\vec{n}} + \phi_{GC}$$

Implementation

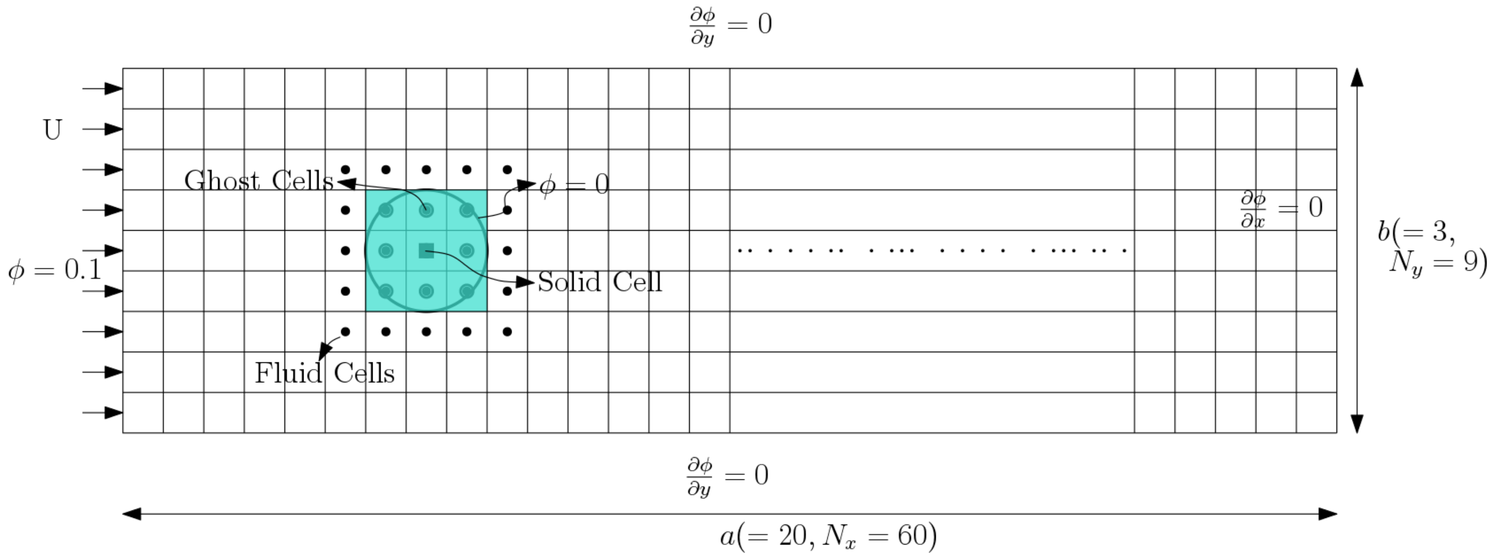
- Find location of BI, IP for each GC
- Find weighing coefficients corresponding to each GC
- Solve for Fluid cells and Boundary cells implicitly as a single linear system



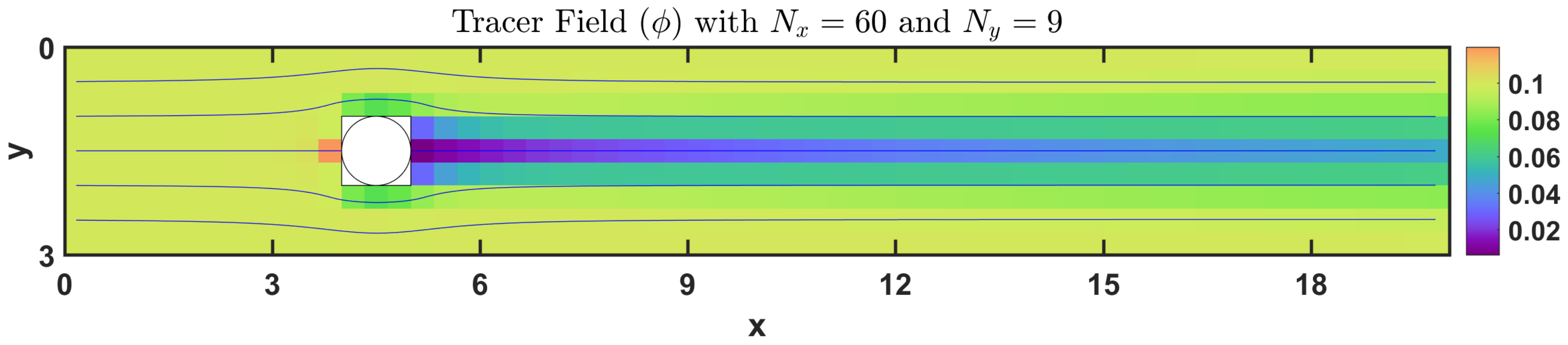
Results

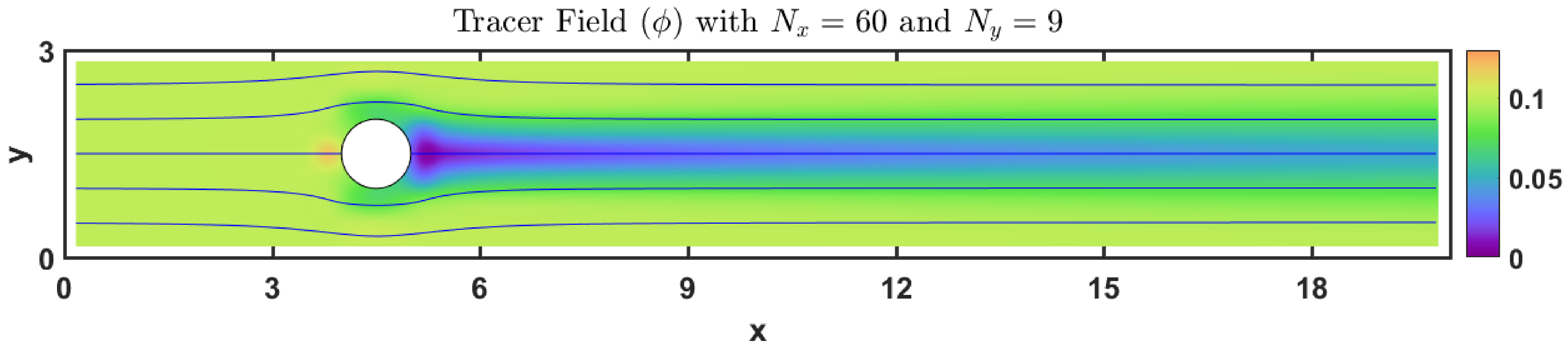
- Currently implemented the GCIBM only for tracer field.
- Test case:
  - Potential velocity field past a circular cylinder
  - Solved Advection-Diffusion-Reaction equation for tracer

$$\frac{\partial \phi}{\partial t} + \vec{u}.\nabla\phi - \frac{1}{PE}\nabla^2\phi = 0$$

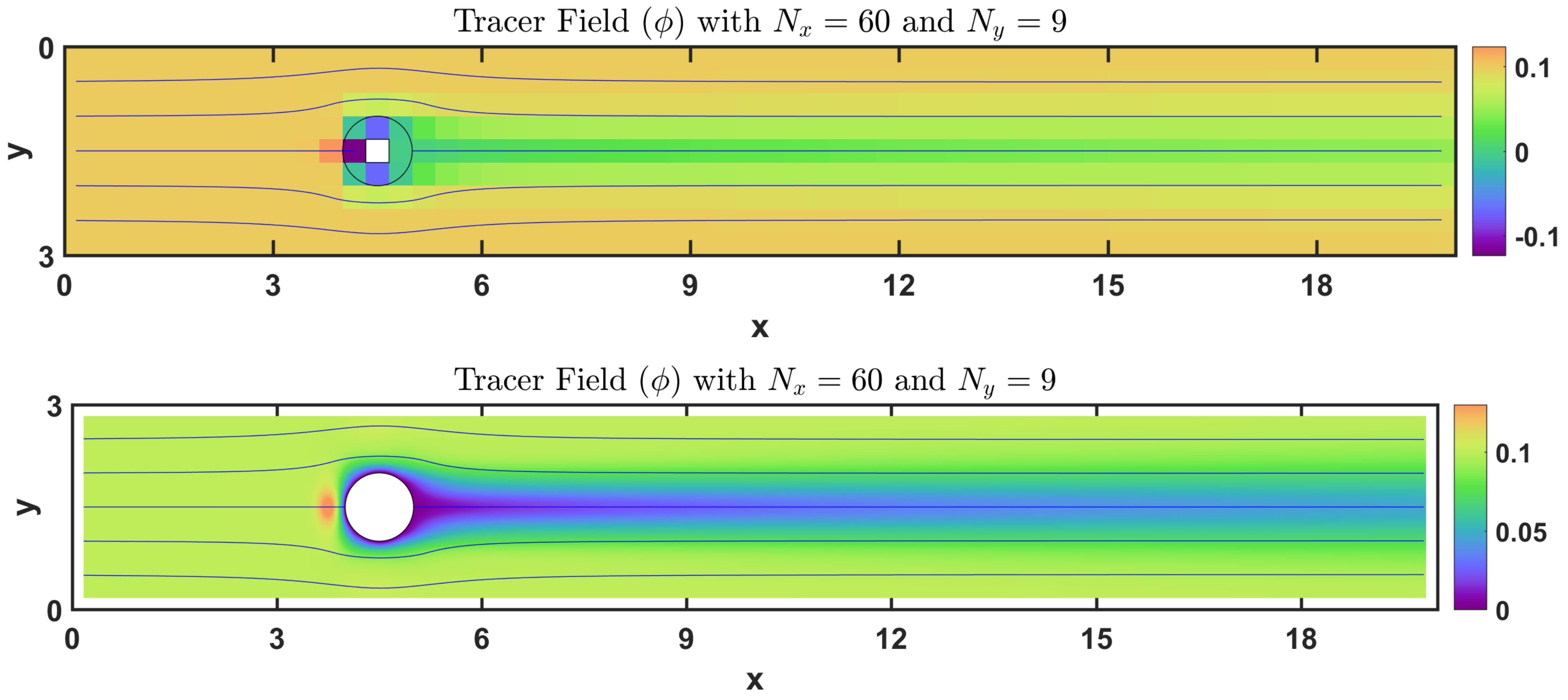


Result: Staircase Approach





Result: GCIBM



Conclusions

- This approach should also be extended to the momentum equations
- Can resolve complex geometries without increasing the grid resolution

Thank You!

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