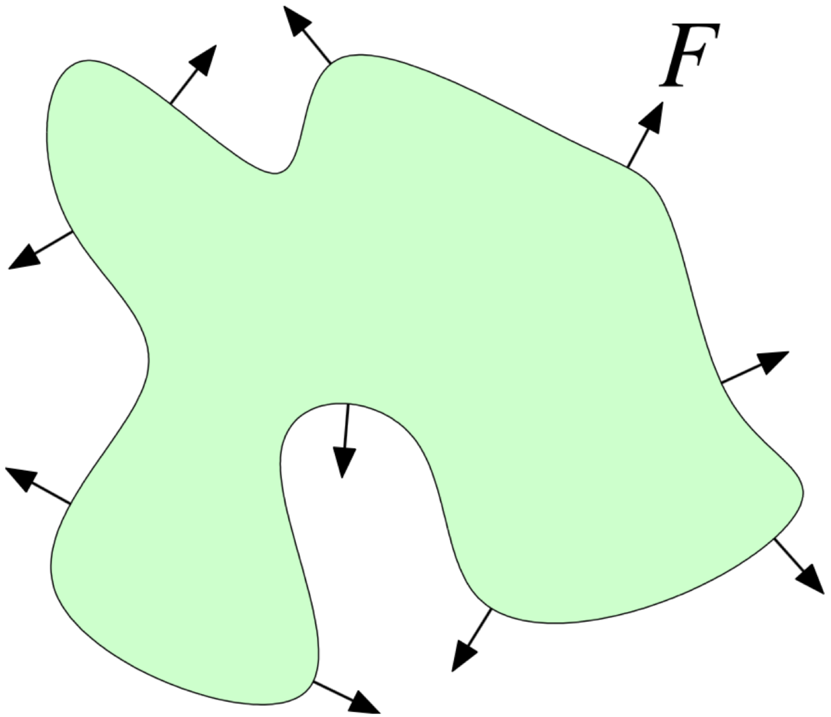
Path Planning for Sailboats in Strong Wind Fields using Level Set Methods

Project 2.29 — Fredrik Samdal Solberg

The Level Set Method

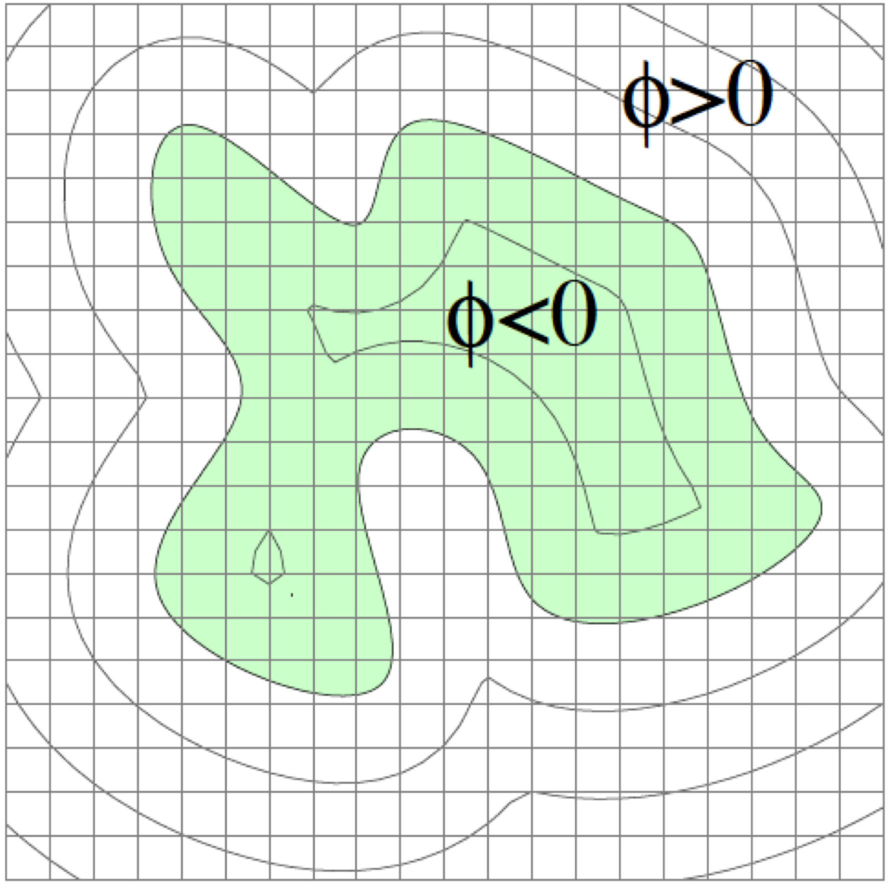
- Method for evolving curves and interfaces
- Propagating curve according to speed function F
- Surfaces in three dimension



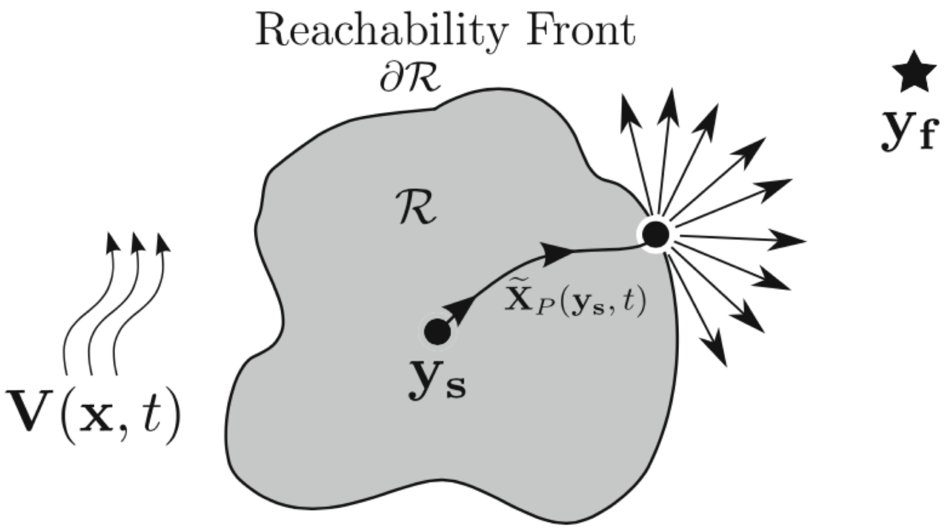
The Level Set Method

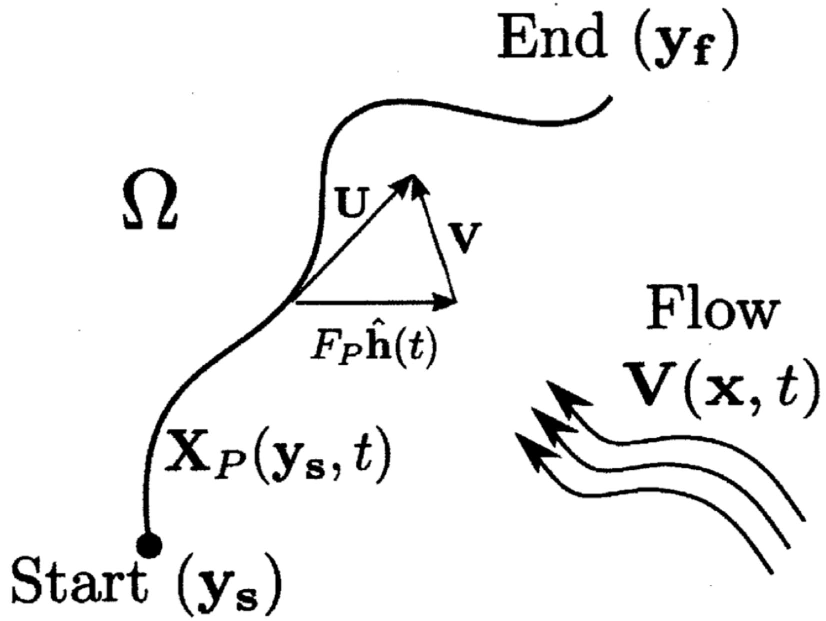
- Eulerian approch, evolve interface by solving PDEs
- Represent curve by zero level set of a function, $\phi(\mathbf{x},t) = 0$
- Solve advection equation to propagate $\phi(\mathbf{x},t)$

$$\frac{\partial \phi}{\partial t} + \Big(F_v(t)\hat{\mathbf{h}}(t) + \mathbf{V}(\mathbf{x},t) \Big) \cdot \nabla \phi = 0$$



The Level Set Method for Path Planning





$$\frac{\partial \phi}{\partial t} + \Big(F_v(t)\hat{\mathbf{h}}(t) + \mathbf{V}(\mathbf{x}, t)\Big) \cdot \nabla \phi = 0$$

Flow field

Velocity relative to the flow

The Level Set Method for Path Planning

- Forward time integration:
  Fractional step method

$$\begin{aligned} \frac{\phi^\star - \phi(\mathbf{x},t)}{\Delta t/2} &= -F|\nabla\phi(\mathbf{x},t)| \\ \frac{\phi^{\star\star} - \phi^\star}{\Delta t} &= -\mathbf{V}\left(\mathbf{x}, t + \frac{\Delta t}{2}\right)\cdot\nabla\phi^\star \\ \frac{\phi(\mathbf{x},t+\Delta t) - \phi^{\star\star}}{\Delta t/2} &= -F|\nabla\phi^{\star\star}| \end{aligned}$$

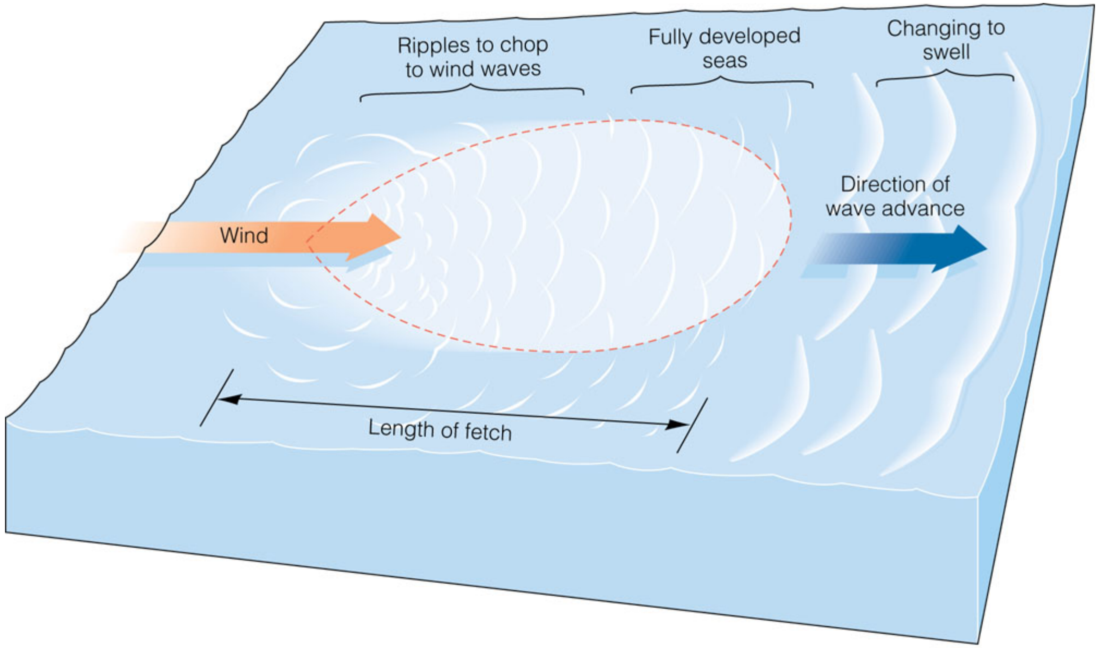
- Backtracking:
  1st order forward Euler

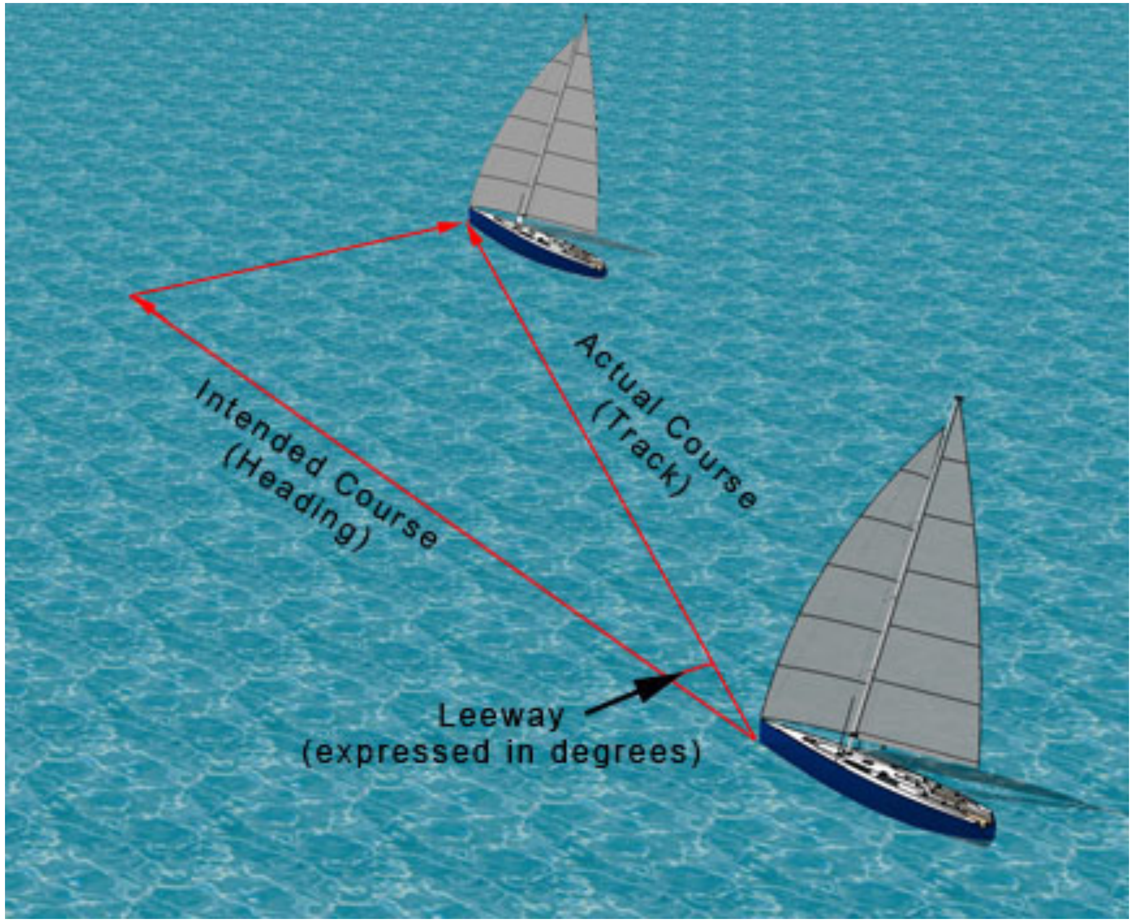
$$\frac{\mathbf{x}(t-\Delta t) - \mathbf{x}(t)}{\Delta t} = -\mathbf{V}(\mathbf{x},t) - F\underbrace{\frac{\nabla\phi(\mathbf{x},t)}{|\nabla\phi(\mathbf{x},t)|}}_{\hat{\mathbf{n}}_w(\mathbf{x}',t)}$$

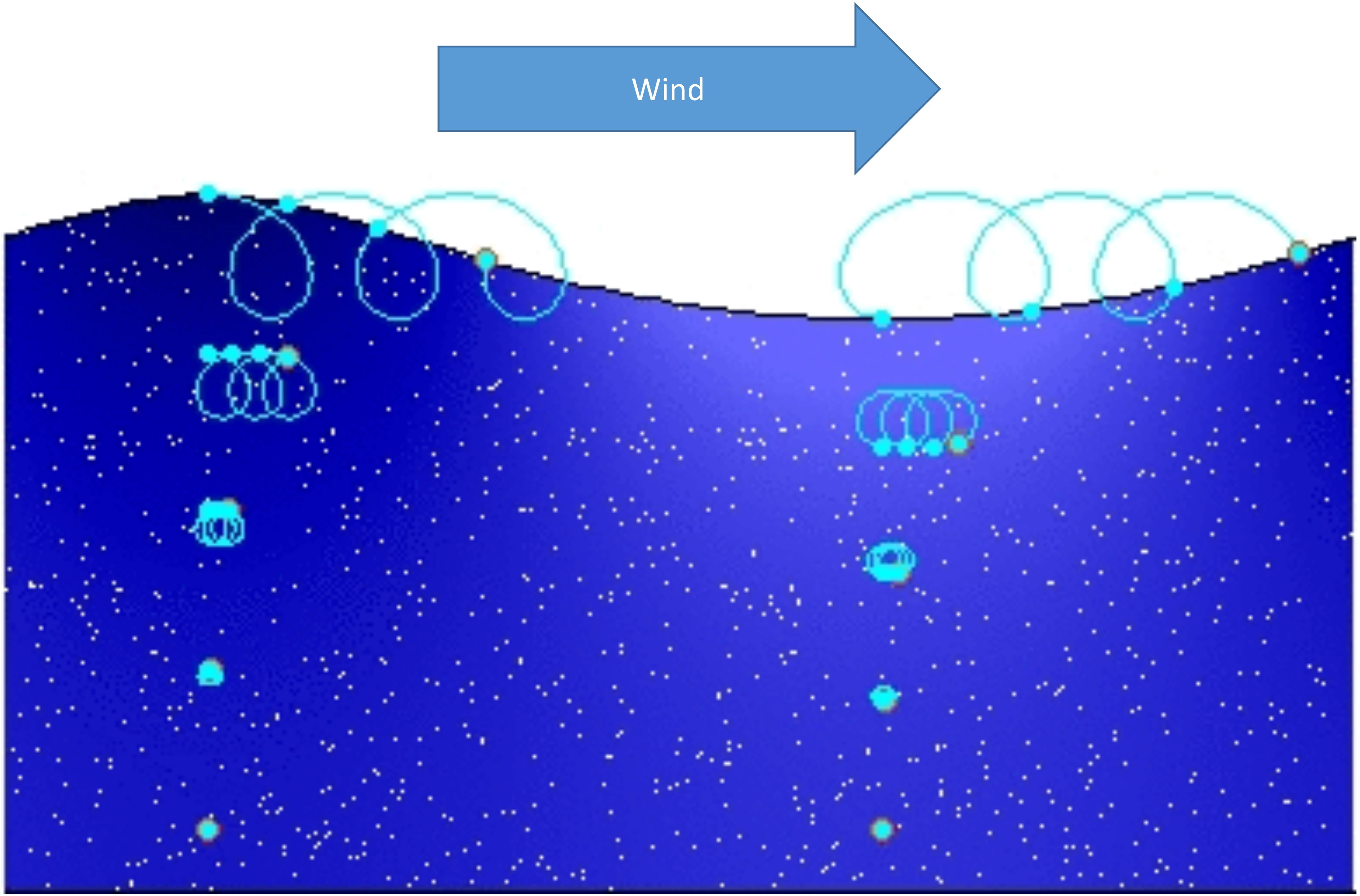
Path Planning for Sailboats in Strong Wind Fields

”The motion of the object induced by wind and waves relative to the ambient current”

Allen and Plourde






Wind

Modeling of Leeway Drift

- Following U.S. Coast Guard Research and Development Center, Report No. CG-D-06-99, ”Modeling of Leeway Drift

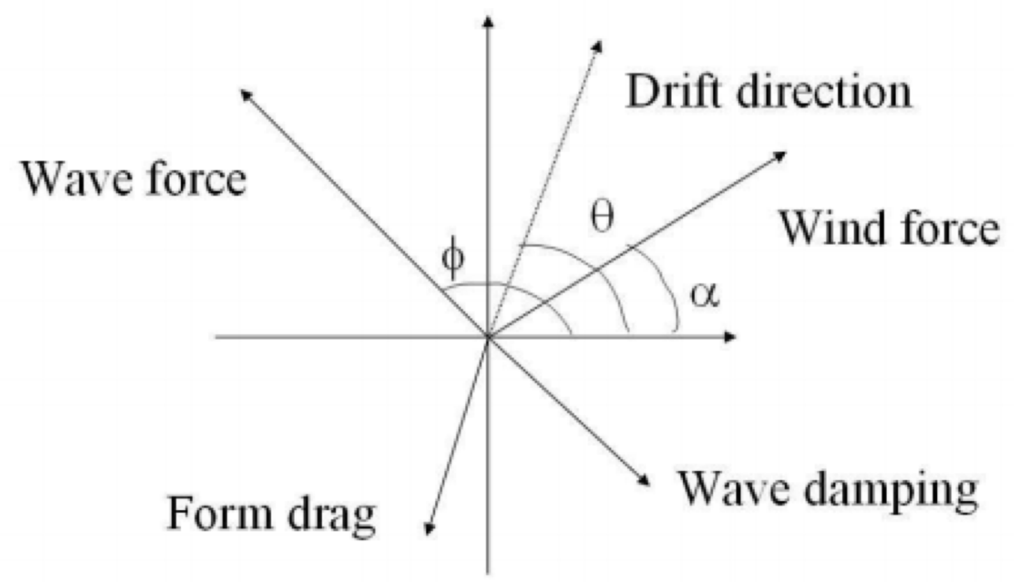
$$M' \, du_b/dt = F_a + F_w + F_c$$

where:

F_a = air force
F_w = wave force
F_c = water force
$M' \, du_b/dt$ = inertial force
M’ is total mass (Lamb, 1932.)

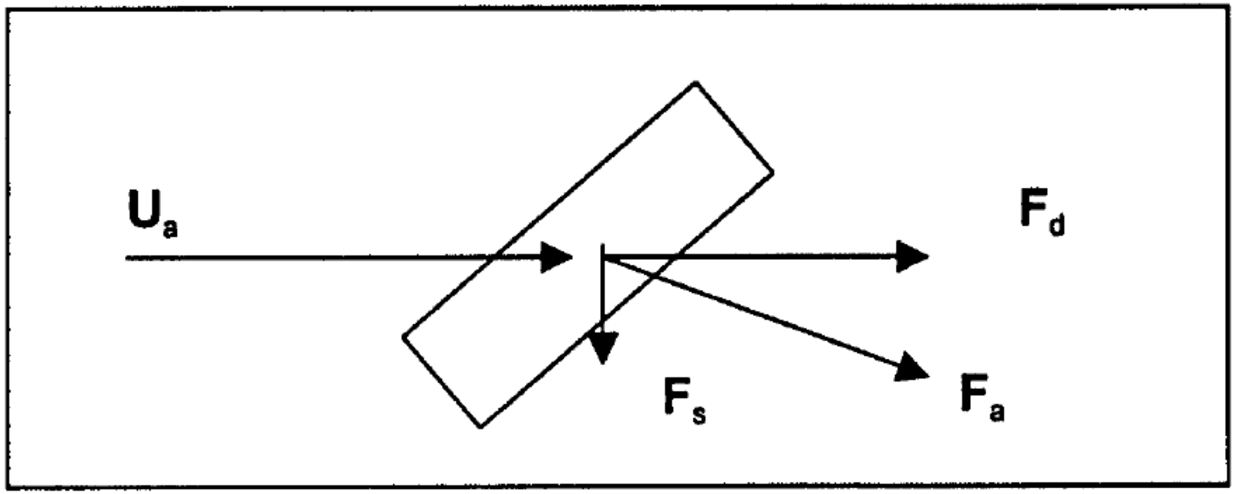
$M' = m + km'$

where:
m is the body mass,
km’ is added mass,
m’ is the mass of fluid displaced by the floating body.



Modeling of Leeway Drift

Wind Drag Force



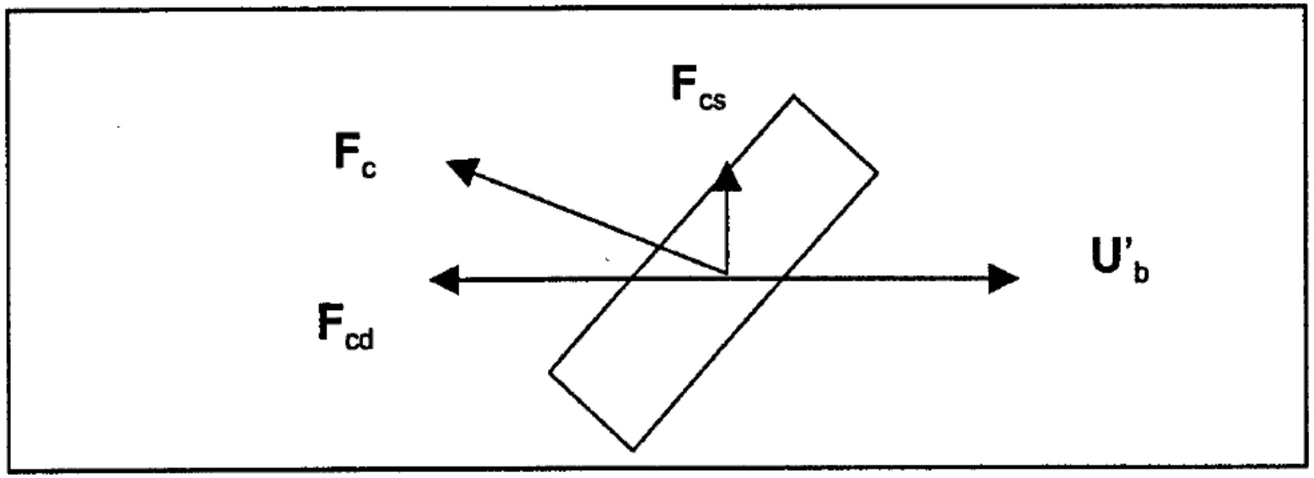
$$F_d = \tfrac{1}{2}\, C_d\, \rho_a\, A_a\, U_a^2$$

where: F_d is the magnitude of the drag force,
C_d is the air drag coefficient for the particular body shape,
ρ_a is the density of air, and
A_a is the projected frontal area of the floating body above the water's surface.

Modeling of Leeway Drift

Water Drag Force

U_b' = body velocity relative to water (leeway)
F_c= water force
F_{cd}= water drag force (opposite direction of body velocity)
F_{cs} = water lift force (normal to direction of body velocity)



$$F_{cd} = \frac{1}{2} C_{cd} \rho_w A_w U_b'^2$$

where ρ_w is water density
C_{cd} is an empirical drag coefficient, and
A_w is the projected area of the floating body under the water.

Modeling of Leeway

Wave Force

$$F_w = \frac{1}{2} C_{iw}\, g\, \rho_w\, L_t\, A^2$$

where: A is wave amplitude,
C_{iw} is the incident wave reflection coefficient,
g is gravitational acceleration,
ρ_w is the density of water,
L_t is the body length scale.

Modeling of Leeway

$$\mathbf{F}_a + \mathbf{F}_w + \mathbf{F}_c = 0$$

$$F_d = \tfrac{1}{2} C_d \rho_a A_a U_a^2$$

$$F_{cd} = \tfrac{1}{2} C_{cd} \rho_w A_w U_b'^2$$

$$F_w = \tfrac{1}{2} C_{iw}\, g\, \rho_w L_t A^2$$

$$C_{iw}\, g\, \rho_w L_t A^2 \mathbf{C}_g / |\mathbf{C}_g| + C_d \rho_a A_a \mathbf{U}_a' |\mathbf{U}_a'| = C_{cd} \rho_w A_w \mathbf{U}_b' |\mathbf{U}_b'|$$

- Terms on LHS known
- Daniel et al. (2001) simplifies further and estimate the leeway ratio for small objects as

$$\frac{u_{\text{leeway}}}{u_{\text{wind}}} \sim \sqrt{\frac{\rho_{\text{air}} C_D A_{\text{air}}}{\rho_{\text{water}} C_W A_{\text{water}}}}.$$

Modeling of Leeway

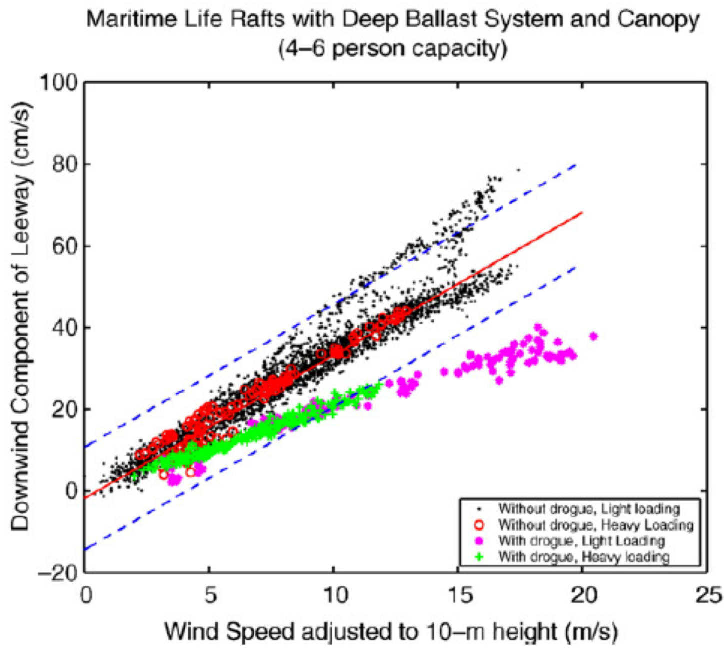
- Field studies indicate an almost linear relationship between the wind speed and the leeway of small objects (0.1 - 25 m) (Breivik et al. 2008)

$$L_d = a_d W_{10} + b_d + \varepsilon_d, \quad \text{Downwind leeway speed (DWL)}$$

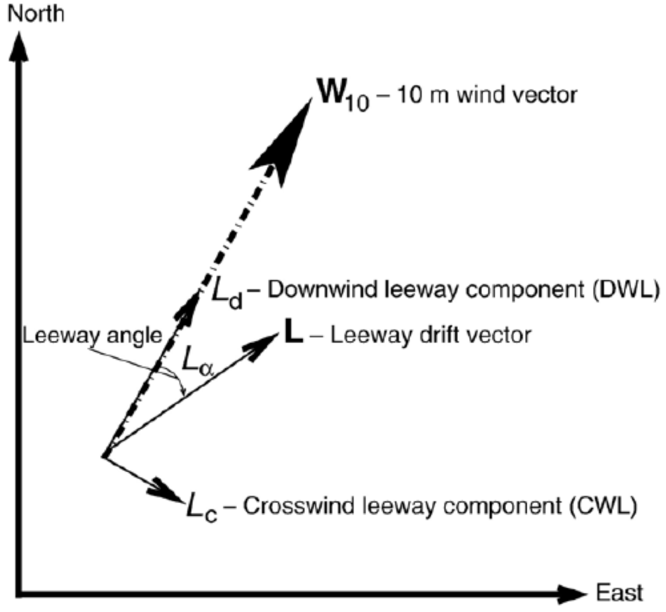
$$L_{c+} = a_{c+} W_{10} + b_{c+} + \varepsilon_{c+}, \quad \text{Right crosswind speed (CWL+)}$$

$$L_{c-} = a_{c-} W_{10} + b_{c-} + \varepsilon_{c-}, \quad \text{Left crosswind speed (CWL−).}$$

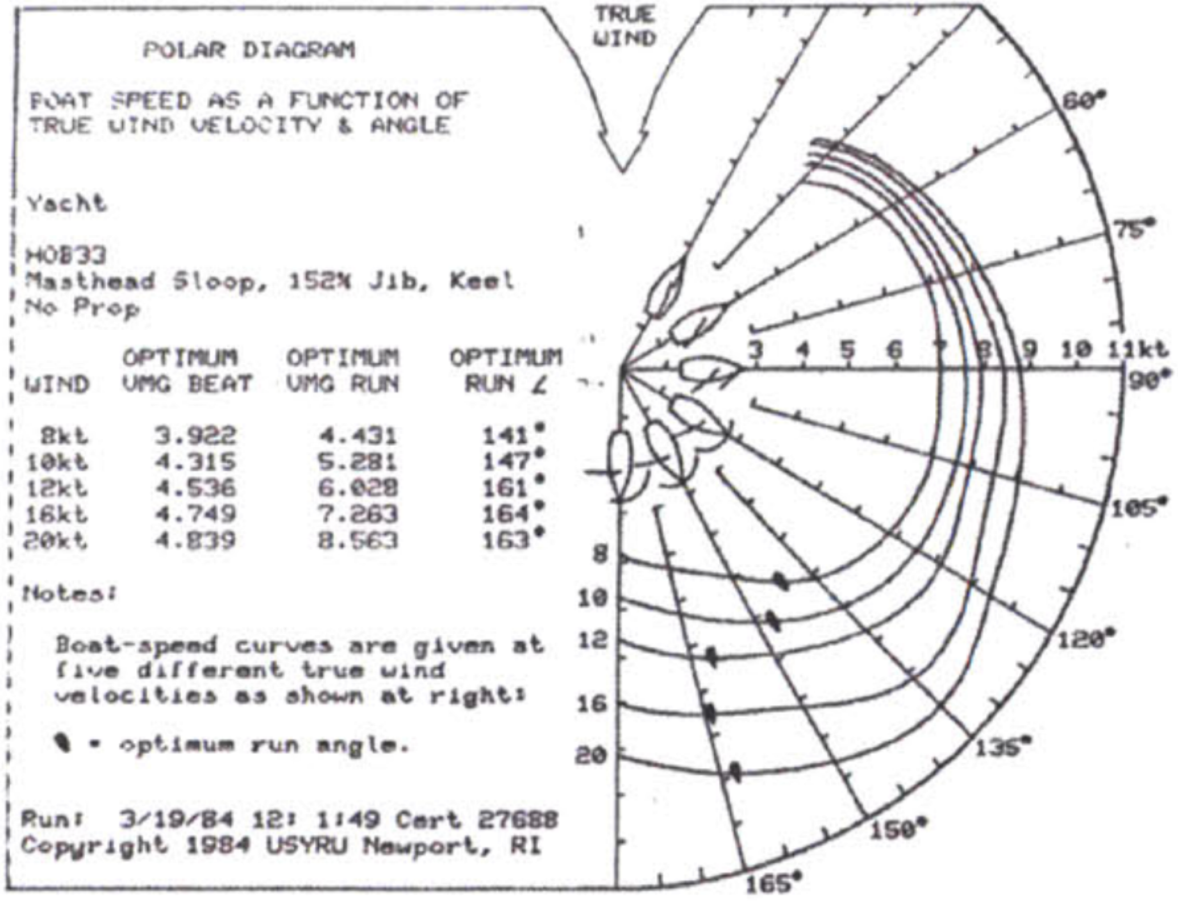
- Allen and Plourde have estimated coefficients for 63 different objects, including sailboats

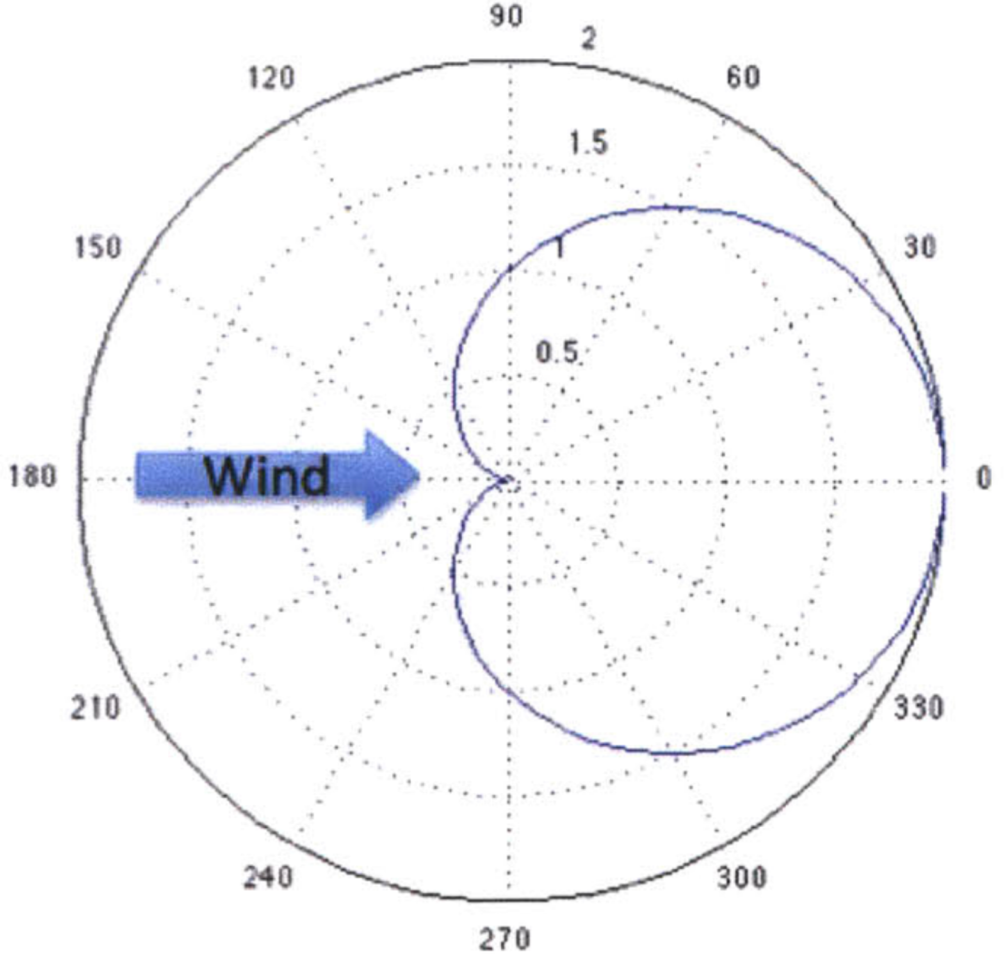


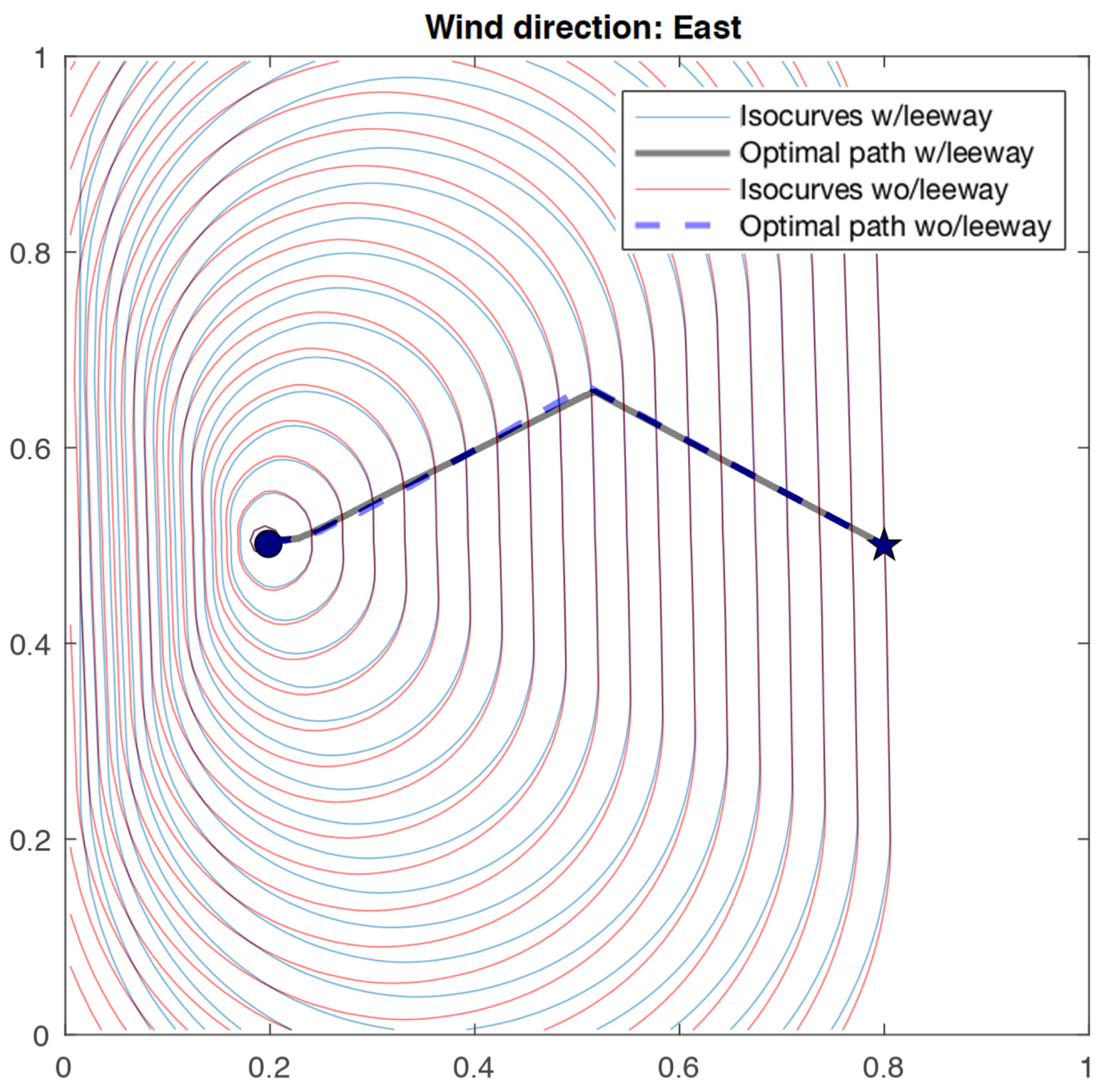
(Breivik et al. 2011)



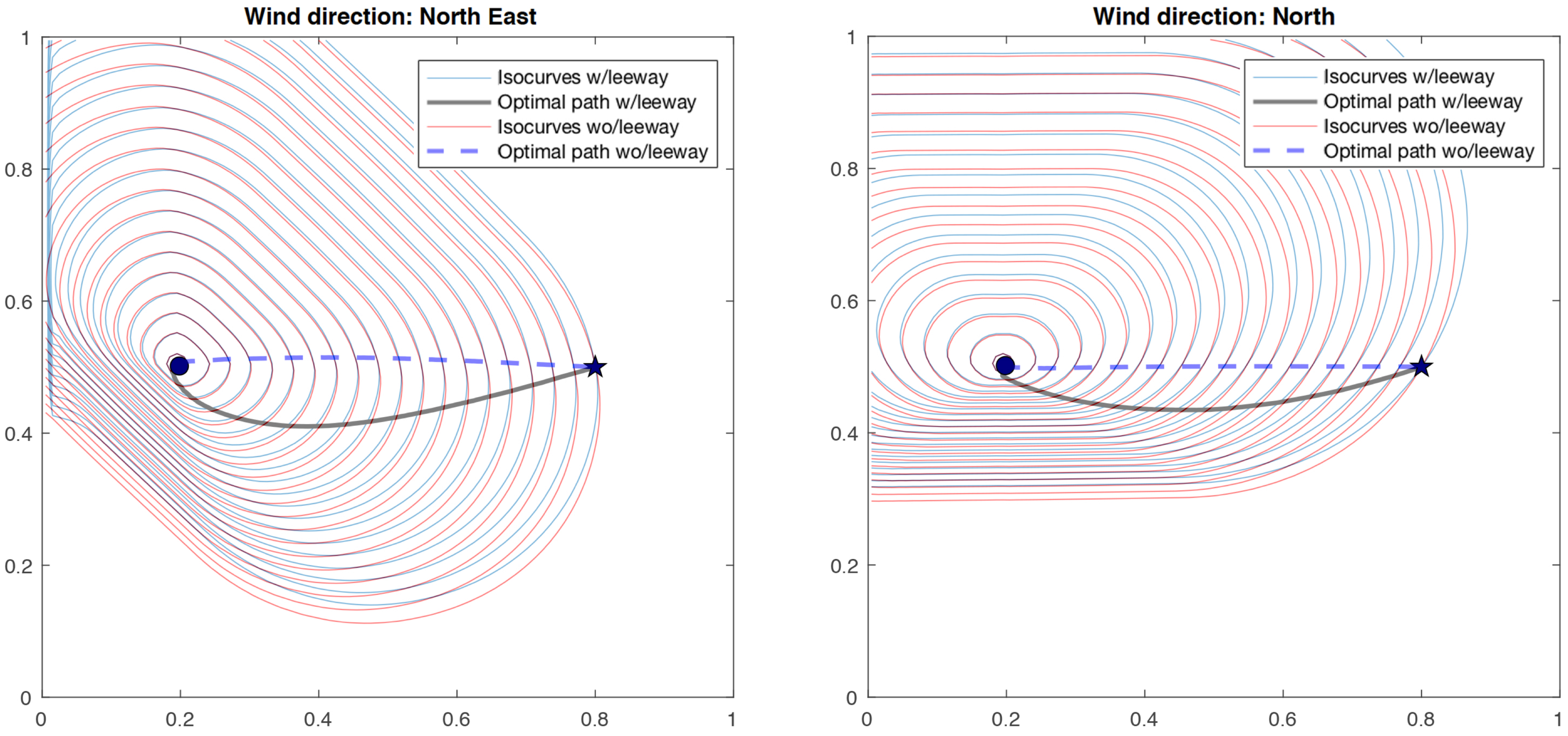
Polar Speed Diagram







Wind direction: East
Isocurves w/leeway
Optimal path w/leeway
Isocurves wo/leeway
Optimal path wo/leeway
0
0.2
0.4
0.6
0.8
1


Wind direction: North East
Isocurves w/leeway
Optimal path w/leeway
Isocurves wo/leeway
Optimal path wo/leeway
0
0.2
0.4
0.6
0.8
1
Wind direction: North
Isocurves w/leeway
Optimal path w/leeway
Isocurves wo/leeway
Optimal path wo/leeway
0
0.2
0.4
0.6
0.8
1