Particle Methods for Solid/Fluid Mechanics Simulations
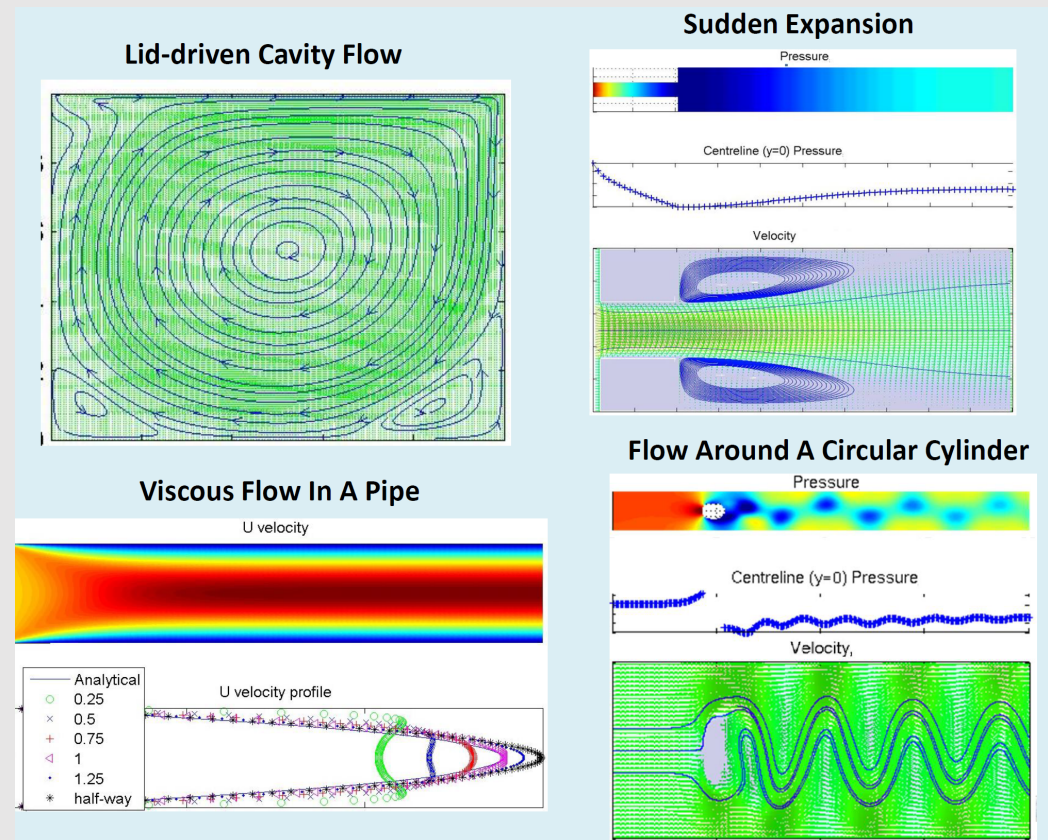
2.29 Has been all about grids

Grids are Good ✓

- Robust

- Mathematical Foundation

- Linear Algebra Ax = b

Lid-driven Cavity Flow

Sudden Expansion
Pressure
Centreline (y=0) Pressure
Velocity

Viscous Flow In A Pipe
U velocity
U velocity profile
— Analytical
○ 0.25
× 0.5
+ 0.75
◁ 1
· 1.25
✳ half-way

Flow Around A Circular Cylinder
Pressure
Centreline (y=0) Pressure
Velocity,

Grids are worth a lot

Several Companies Based on FVM/FEM

- Market size: > $20B

Sometimes though, mesh methods don't perform well

➢ High Distortion

➢ Nonlinear Physics

➢ Surface Tracking

➢ Fluid-Solid Interaction

Sometimes though, mesh methods don't perform well

➢ High Distortion

➢ Nonlinear Physics
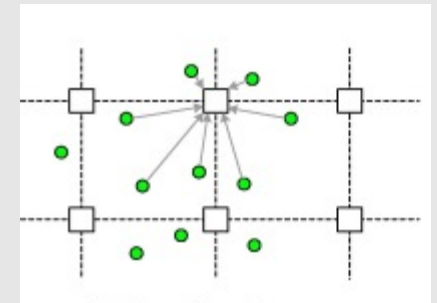
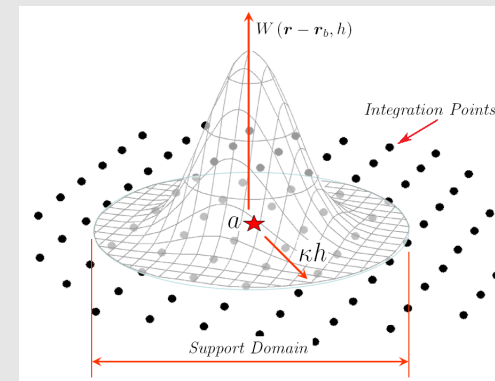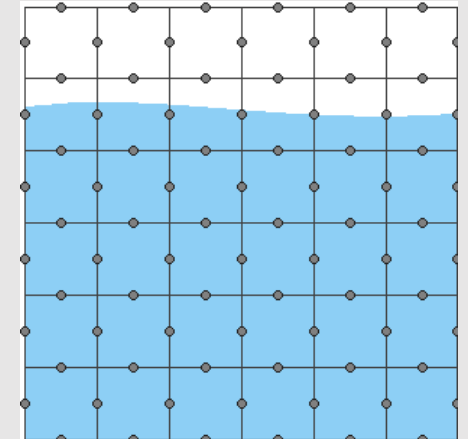➢ Surface Tracking

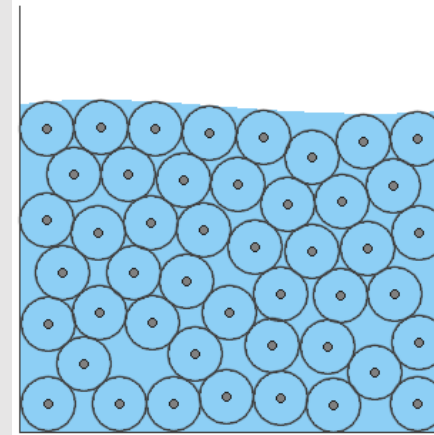➢ Fluid-Solid Interaction

Is there another way?

Particle Methods

How do they work?

- Nodes are free to move relative to one another

- Many ways to get information from neighbors

- Lagrangian Mechanics
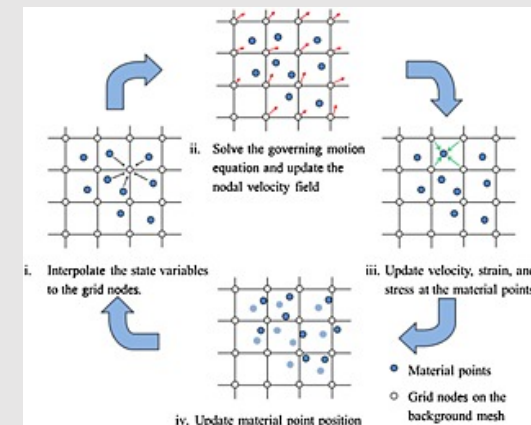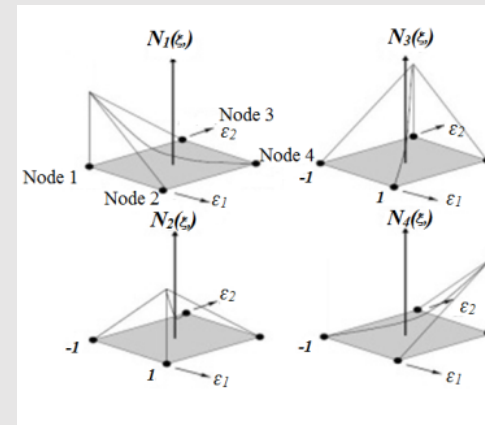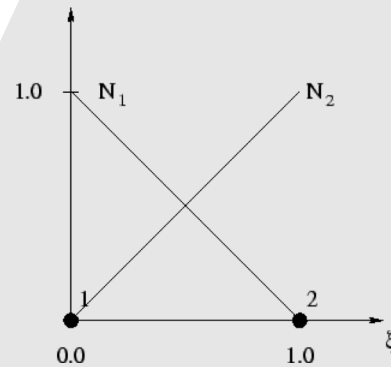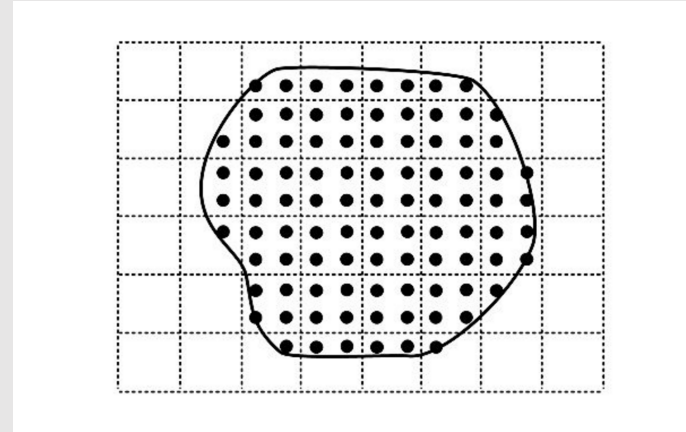
Adoption in academia has been slow

➢ Error Estimation

➢ "fudge" factors

➢ Rigorous Testing

The Material Point Method

Background grid + particles

Particles carry the state variables
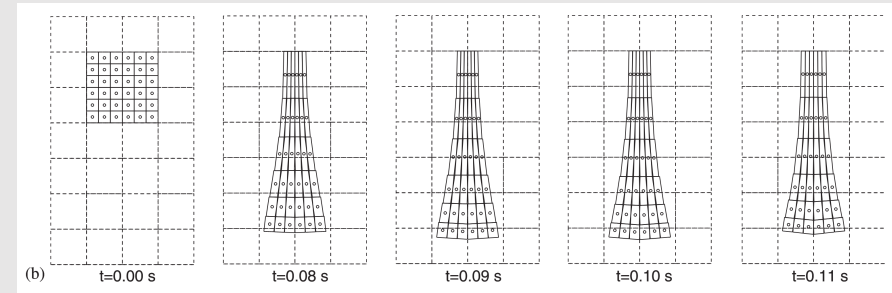
1) Information is sent to the grid

2) Rates of change are found on the nodes

3) Gradients are sent to the particles

4) Particles are updated, grid is reset

Advantages of MPM

- Exact Advection

- Large Deformations

- Fast Neighbor Search

- Multiphase/physics

$$m\frac{Dv}{Dt} = F_{int} + F_{ext}$$

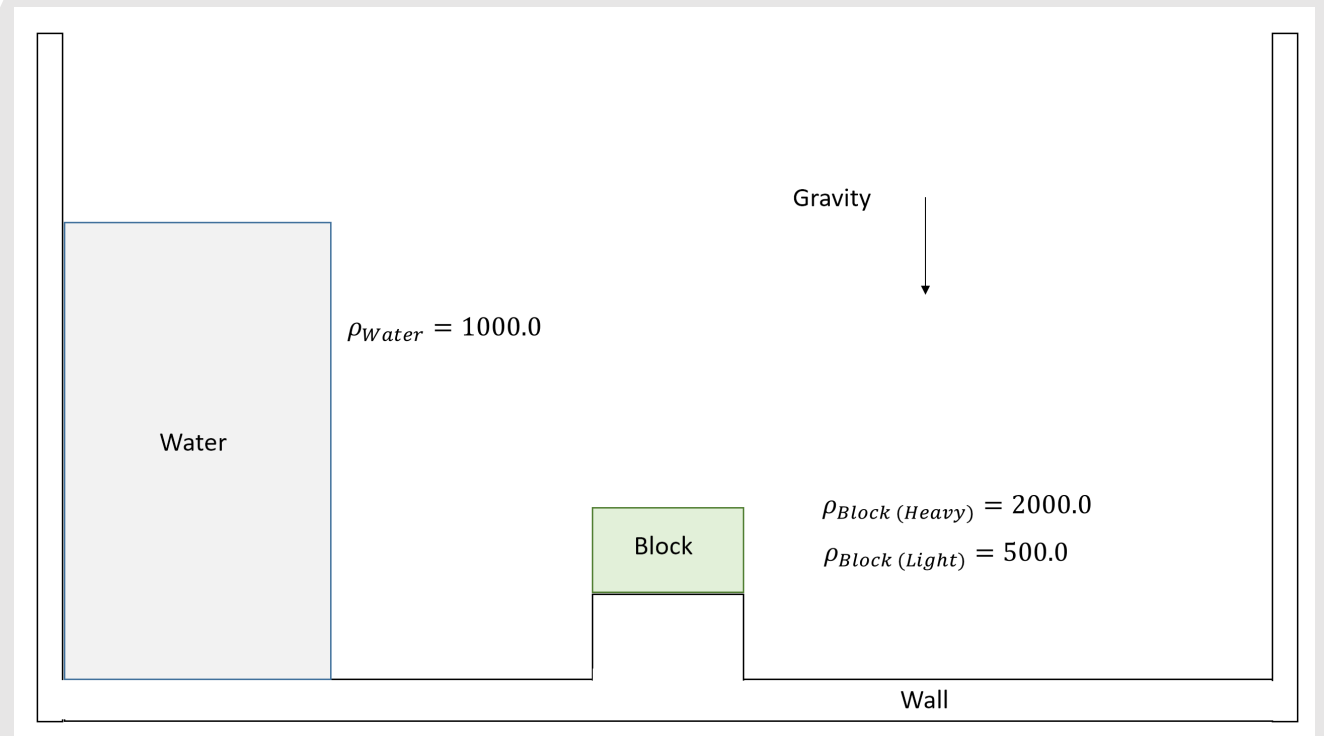(b) t=0.00 s t=0.08 s t=0.09 s t=0.10 s t=0.11 s

Particle Update:

$$V_P^{n+1} = V_P^n + \Delta t \sum N_{GP} a_{grid}$$

$$x_P^{n+1} = x_P^n + V_P^{n+1} \Delta t$$

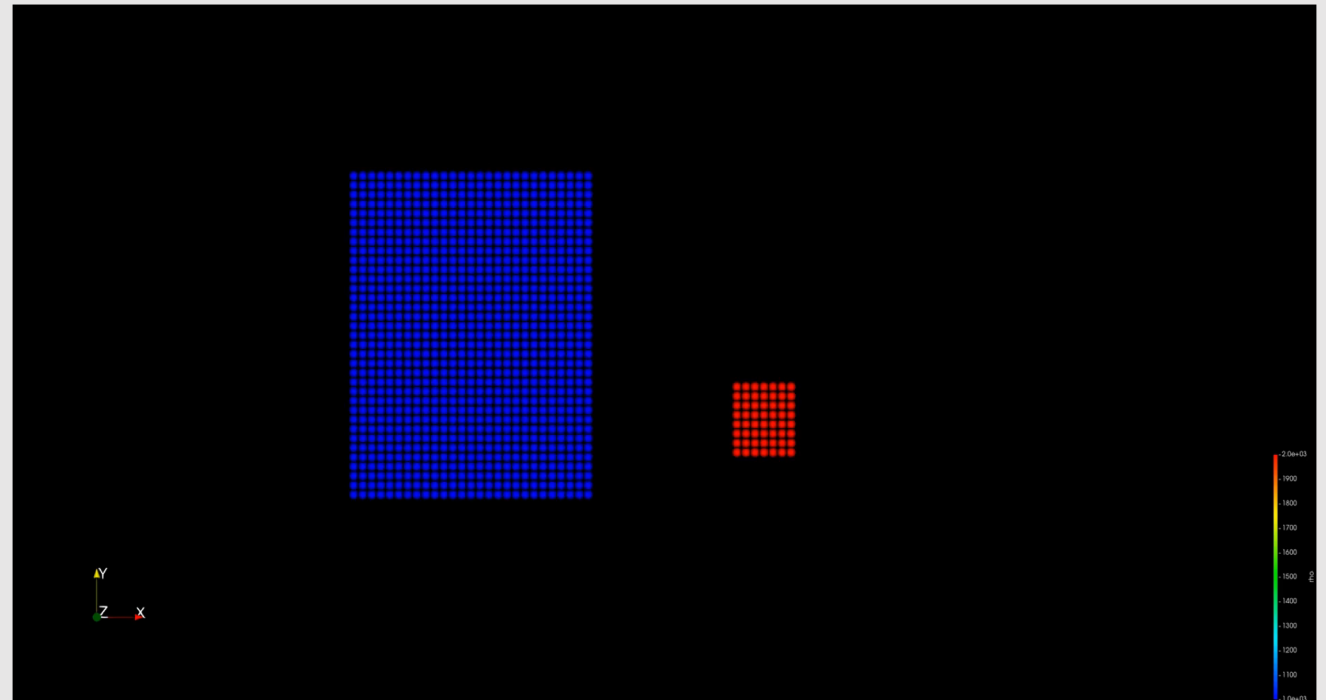$$\sigma_P^{n+1} = f(\sigma_P^n, \dot{\varepsilon}_P^{n+1}, \ldots)$$

Examples

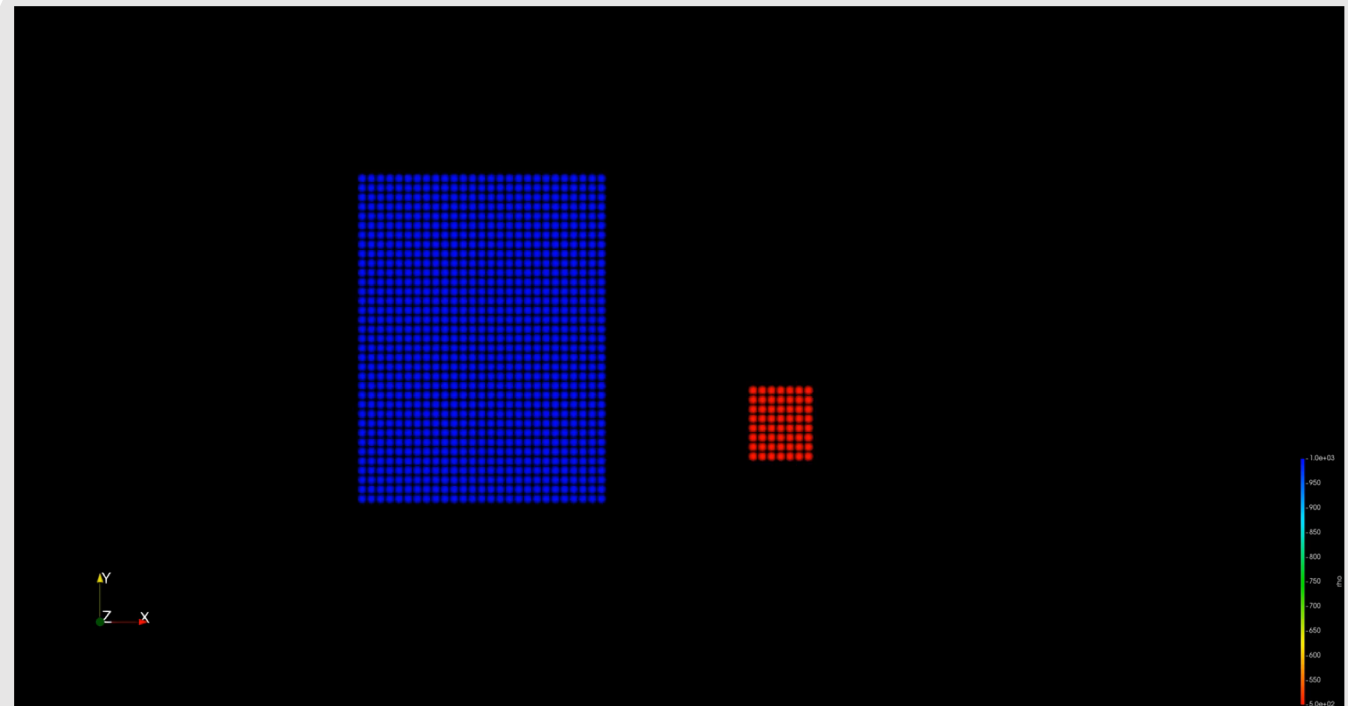- Dam Break – Heavy Object

- Dam Break – Light Object

Gravity

$\rho_{Water} = 1000.0$

Water

$\rho_{Block\ (Heavy)} = 2000.0$

Block

$\rho_{Block\ (Light)} = 500.0$

Wall

Examples

- **Dam Break – Heavy Object**

- Dam Break – Light Object

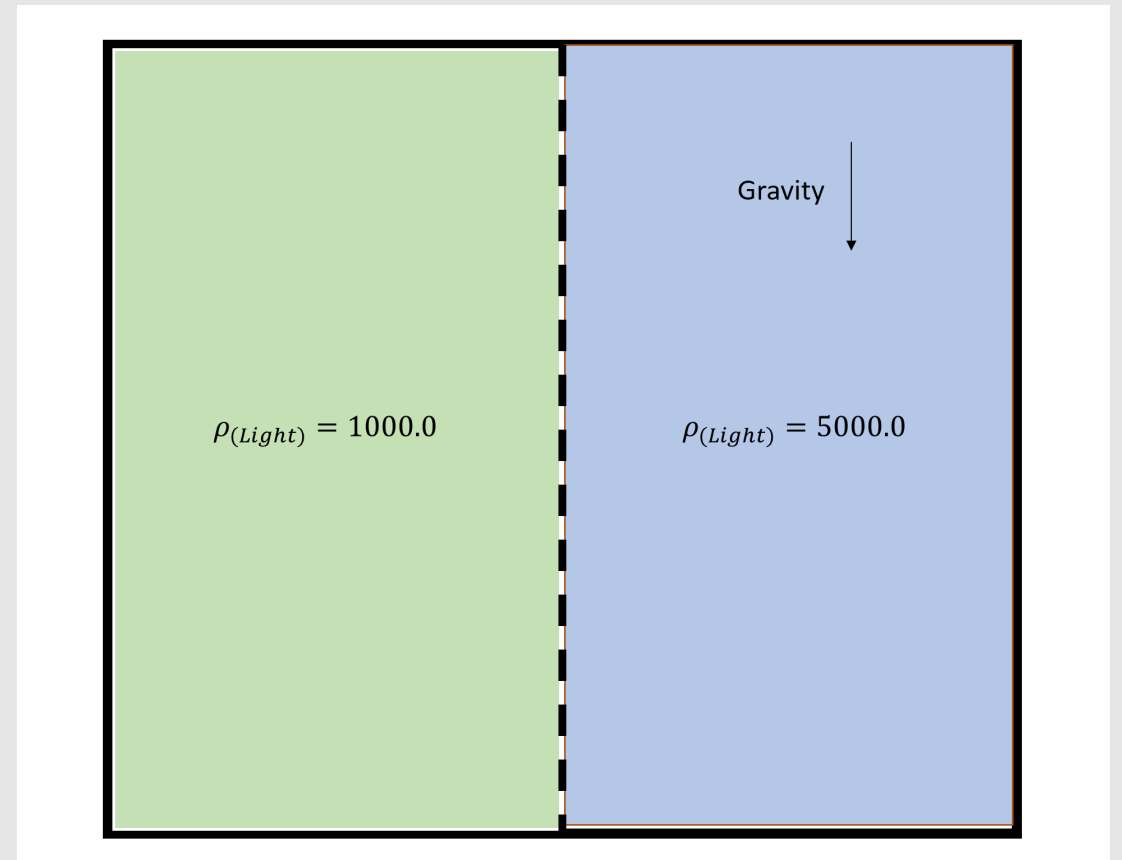# Examples

- Dam Break – Heavy Object

- **Dam Break – Light Object**

Examples

- Dam Break – Heavy Object

- Dam Break – Light Object

- Lock Exchange

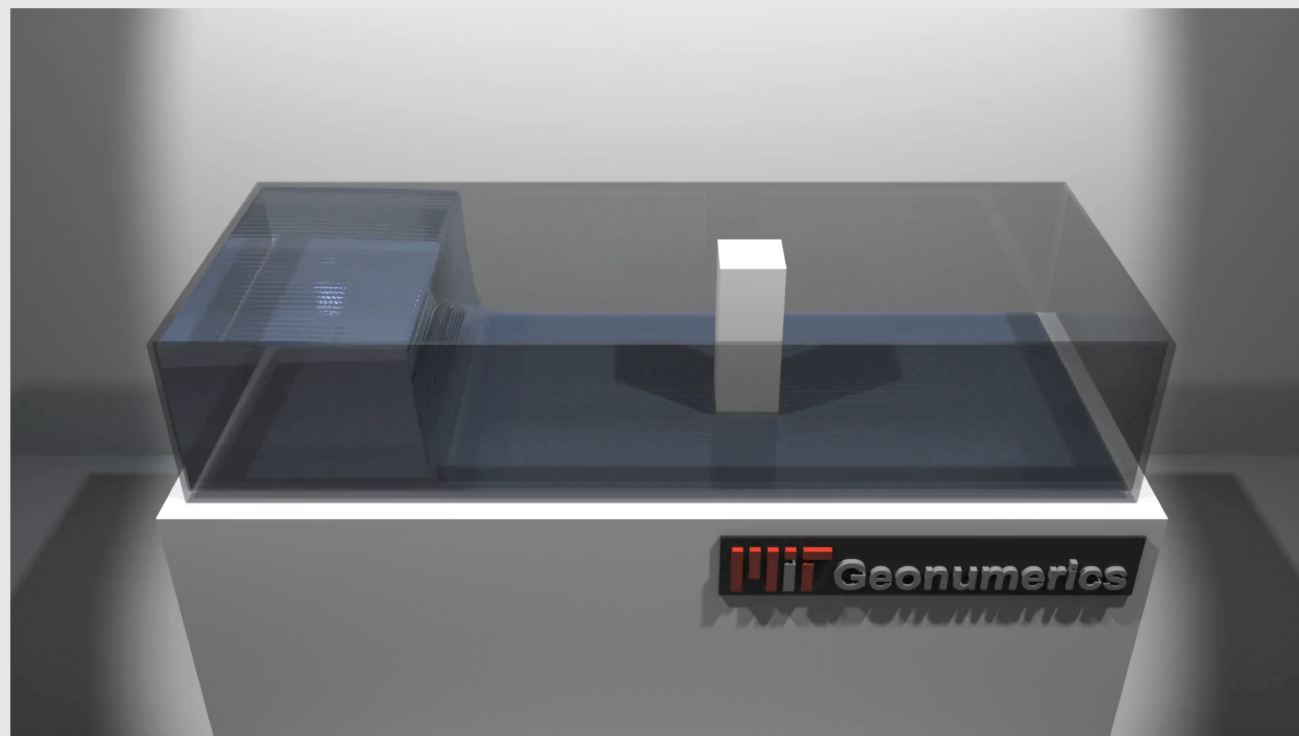$\rho_{(Light)} = 1000.0$ Gravity $\rho_{(Light)} = 5000.0$

Examples

- Dam Break – Heavy Object

- Dam Break – Light Object

- **Lock Exchange**

Physics Animations

Particle methods are ideal for animation software

Physics Animations

Disney is using MPM for more realistic animations in their movies:

Frozen, Big Hero 6, etc.

Effect of Young's Modulus

$E_0 = 4.8 \times 10^4$
$\theta_c = 2.5 \times 10^{-2}$
$\theta_s = 7.5 \times 10^{-3}$
$\xi = 10$

$E_0 = 1.4 \times 10^5$
$\theta_c = 2.5 \times 10^{-2}$
$\theta_s = 7.5 \times 10^{-3}$
$\xi = 10$

©Disney

Physics Animations

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Thank You!