Introduction to the Linear Panel Method WAMIT

Emma Edwards

WaveAnalysisMIT (WAMIT)

- Created in 1987 at MIT by Dr. Chang-Ho Lee and Prof. J. Nicholas Newman
- Computes hydrodynamic properties of structures in waves
- Uses the Boundary Integral Equation Method, or the Panel Method

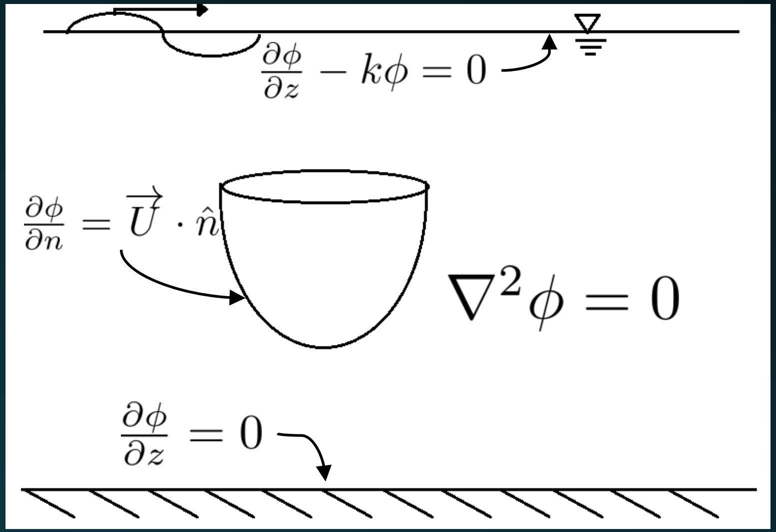
Panel Methods: problem statement

- Assumptions: Linear
  - Fluid is incompressible, inviscid
  - Flow is irrotational
  - Assume small motions relative to wavelength and body
- Laplace equation

$$\nabla^2 \Phi = 0$$

  - Linear; superposition of solutions
  - Typical solutions: source, sink, dipole
- Velocity potential

$$\Phi\left(\overrightarrow{x}, t\right) = Re\{\phi\left(\overrightarrow{x}\right) e^{i\omega t}\}$$



Panel Methods: problem statement, cont'd

- Decompose velocity potential (incident, diffraction, radiation)

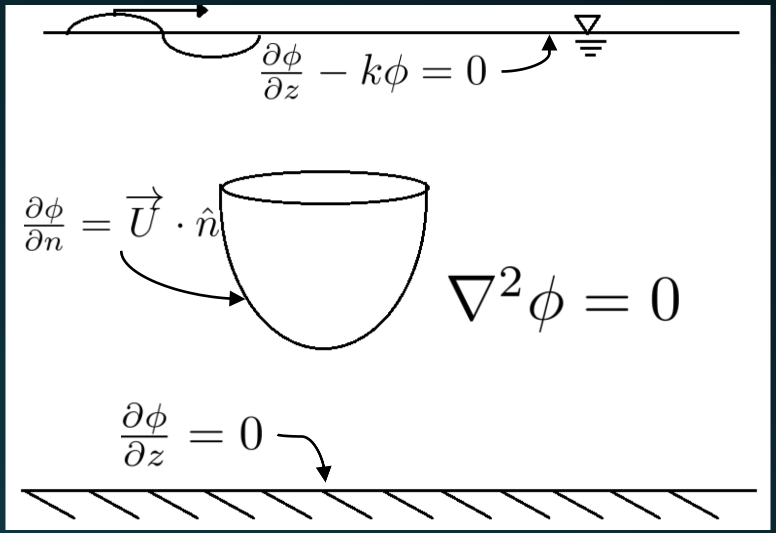
$$\phi = \phi_D + \phi_R = \phi_I + \phi_S + \phi_R$$

  - ϕ_I is the velocity potential of the incident wave
  - ϕ_S is the diffraction potential (due to disturbance of wave field in order to satisfy no-flux): $$\frac{\partial \phi_S}{\partial n} = -\frac{\partial \phi_I}{\partial n} \text{ on } S_b$$

- ϕ_R is the radiation potential (due to the body creating waves, even in absence of an incident wave)

$$\phi_R = i\omega \Sigma_{j=1}^{6} \xi_j \phi_j$$
$$\frac{\partial \phi_j}{\partial n} = n_j \text{ on } S_b$$

where ξ_j is the complex amplitude of body oscillatory motion and ϕ_j is the corresponding unit-amplitude radiation potential



Panel Methods: Overview

1. Use Green's theorem to derive integral equations for velocity potentials on the body boundary
2. Discretize the body surface by a large number N of panels
3. The sources and dipole moments are assumed constant on each panel → total of N unknowns
4. The potential is evaluated at the centroid of each panel and set equal to the normal incident potential
5. Solve system of equations
6. Compute required forces and moments

Step 1: using Green's theorem to derive integral equations

- $G(\vec{x};\vec{\xi})$ = the velocity potential at point $\vec{x}$ due to a periodic source with strength -4π located at point $\vec{\xi}$, satisfying free-surface boundary condition, bottom boundary condition, and radiation condition (Green function)
- Green's theorem: from Gauss' divergence theorem

$$2\pi\phi_j\left(\vec{x}\right)+\iint_{S_b}\phi_j\left(\vec{\xi}\right)\frac{\partial G\left(\vec{\xi};\vec{x}\right)}{\partial n_\xi}d\vec{\xi}=\iint_{S_b}n_jG\left(\vec{\xi};\vec{x}\right)d\vec{\xi}$$

$$2\pi\phi_D\left(\vec{x}\right)+\iint_{S_b}\phi_D\left(\vec{\xi}\right)\frac{\partial G\left(\vec{\xi};\vec{x}\right)}{\partial n_\xi}d\vec{\xi}=4\pi\phi_I\left(\vec{x}\right)$$

Step 2: discretize the body surface

- Discretize the shape into quadrilateral or triangular panels
- I use Chebyshev polynomials as basis functions to represent any shape of surface (optimization)
- Chebyshev polynomials

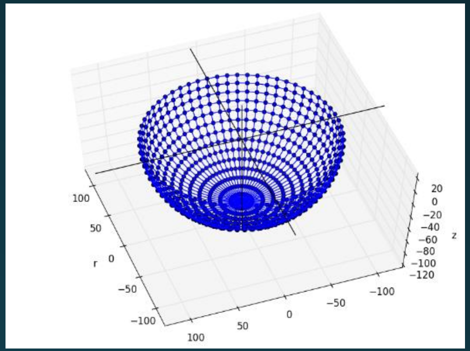
$$T_0\left(x\right)=1$$
$$T_1\left(x\right)=x$$
$$T_{n+1}\left(x\right)=2xT_n\left(x\right)-T_{n-1}\left(x\right)$$

- Represent radius r and depth z as functions of parameter s (allow for slope discontinuities), and top radius as a function of parameter t

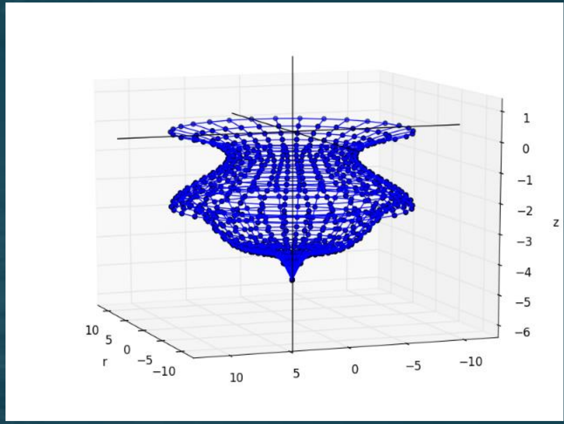
$$r_N\left(s\right)=\Sigma_{n=0}^{N}a_nT_n\left(s\right)$$
$$z_N\left(s\right)=\Sigma_{n=0}^{N}b_nT_n\left(s\right)$$
$$T_N\left(t\right)=\Sigma_{n=0}^{N}c_nT_n\left(t\right)$$

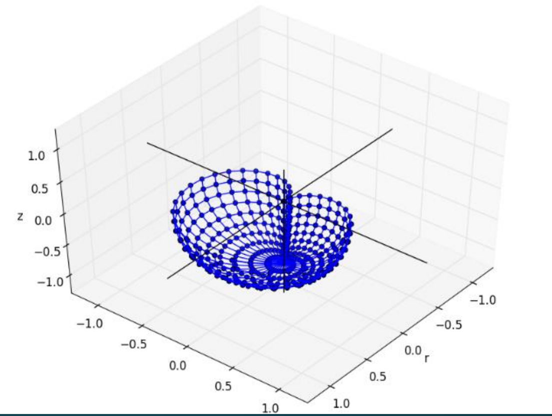
Step 2: discretize the body surface, examples

- Hemisphere: use Galerkin method to find coefficients



- General shapes (axisymmetric vs. not)





Steps 3-5

- Source is assumed constant over each panel
- The potential is evaluated at the centroid of each panel
  - Radiation potentials

$$2\pi\phi_i + \Sigma_{k=1}^{N} D_{ij}\phi_k = \Sigma_{k=1}^{N} S_{ik}\left(\frac{\partial\phi}{\partial n}\right)_k$$

  - Diffraction potential

$$2\pi\phi_i + \Sigma_{k=1}^{N} D_{ij}\phi_k = 4\pi\left(\phi_I\right)_i$$

  - where

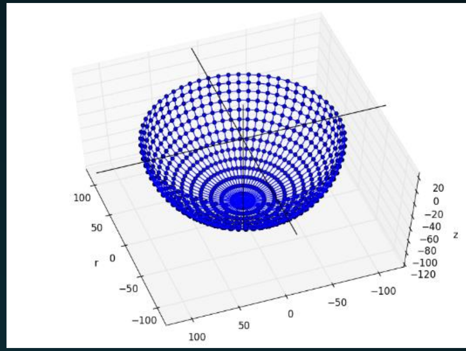
$$D_{ik} = \iint_{s_k} \frac{\partial G\left(\overrightarrow{\xi}; \overrightarrow{x_i}\right)}{\partial n_\xi} d\overrightarrow{\xi} \qquad S_{ik} = \iint_{s_k} G\left(\overrightarrow{\xi}; \overrightarrow{x_i}\right) d\overrightarrow{\xi}$$

- System of N equations is solved, and $\phi = \Sigma_{i=1}^{N}\phi_i$

WAMIT: properties calculated (for range of frequencies)

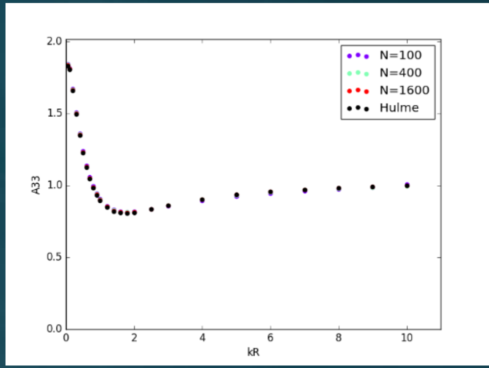
- Added-mass and damping coefficients
- Exciting forces
- Body motions in waves
- Hydrodynamic pressure
- Free-surface elevation

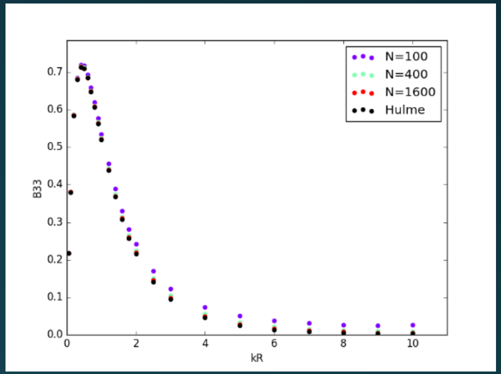
Example: Convergence of hemisphere properties

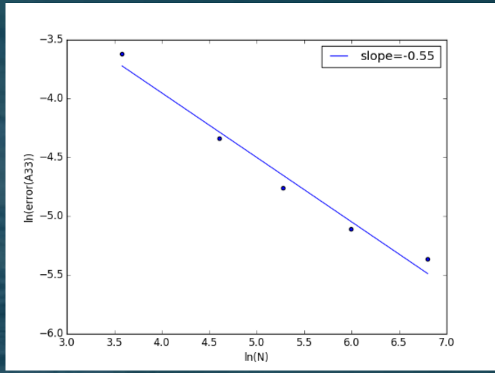


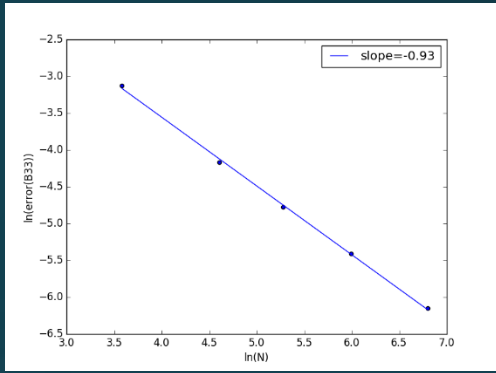
- Added mass and damping coefficients

$$A_{ij} - \frac{i}{\omega} B_{ij} = \rho \iint_{S_b} n_i \phi_j dS$$









Example: hemisphere properties

- Exciting force

$$X_i = -i\omega\rho \iint_{S_b} \left(n_i \phi_I - \phi_i \frac{\partial \phi_I}{\partial n} \right) dS$$

- Hydrodynamic pressure

$$p = -\rho \frac{\partial \phi}{\partial t}$$
$$\phi = \phi_D + i\omega \Sigma_{j=1}^{6} \xi_j \phi_j$$

- Free-surface elevation

$$\eta = -\frac{1}{g} \left(\frac{\partial \phi}{\partial t} \right)_{z=0}$$

Example: hemisphere properties

- Velocity field

$$\overrightarrow{V} = \nabla \phi$$

- Quiver plot in MATLAB to show the flow field