A 2D Incompressible Navier-Stokes Solver Using the Finite Volume Method Implemented in C++

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Outline

Code Structure
- Overview of code structure
- Extensibility

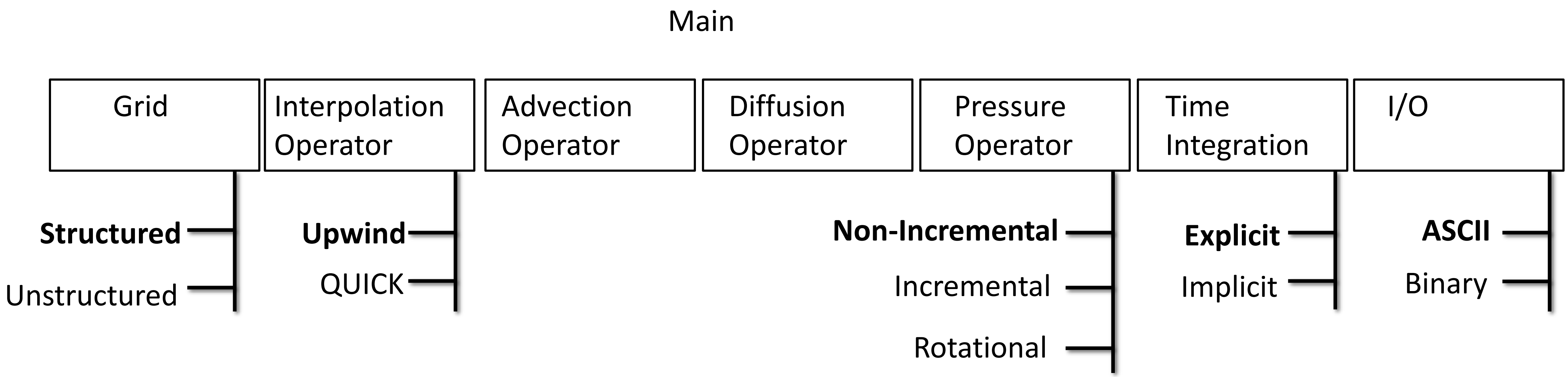
Formulation

Examples
- Burgers Equation
- Diffusion
- Poiseuille Flow
- Flow Around a Cylinder

Future Work

Code Structure

Code Diagram



Linear Algebra

Eigen is a header library with useful linear algebra data structures and functions:

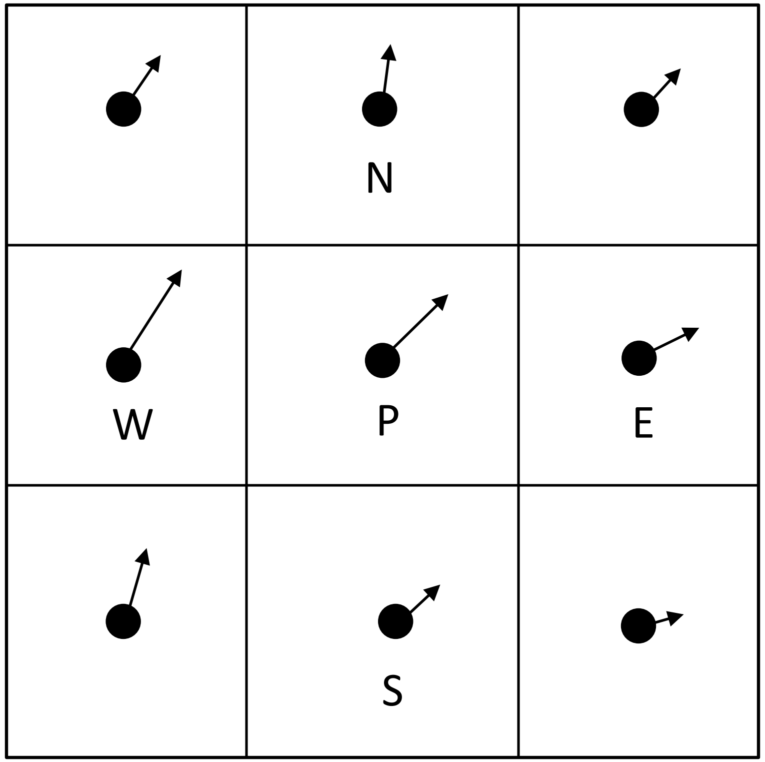
- Vector and Matrix data structures
  - Dense and sparse matrices
- Built-in direct and iterative linear solvers
  - Sparse LU used to solve Poisson Pressure Equation

http://eigen.tuxfamily.org

Formulation

Grid: Structured, Collocated

- Structured grid with all rectangular elements
- U^x -velocities, U^y -velocities and Pressures all live on cell centers
- Bookkeeping relatively easy

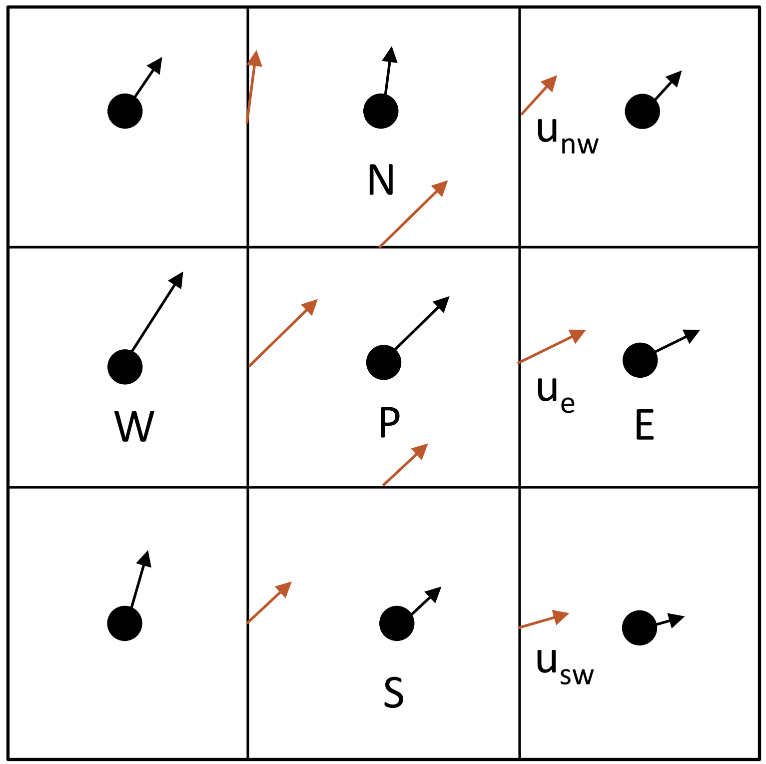


Velocity Interpolation: Upwinding

Upwinding used for velocity interpolation

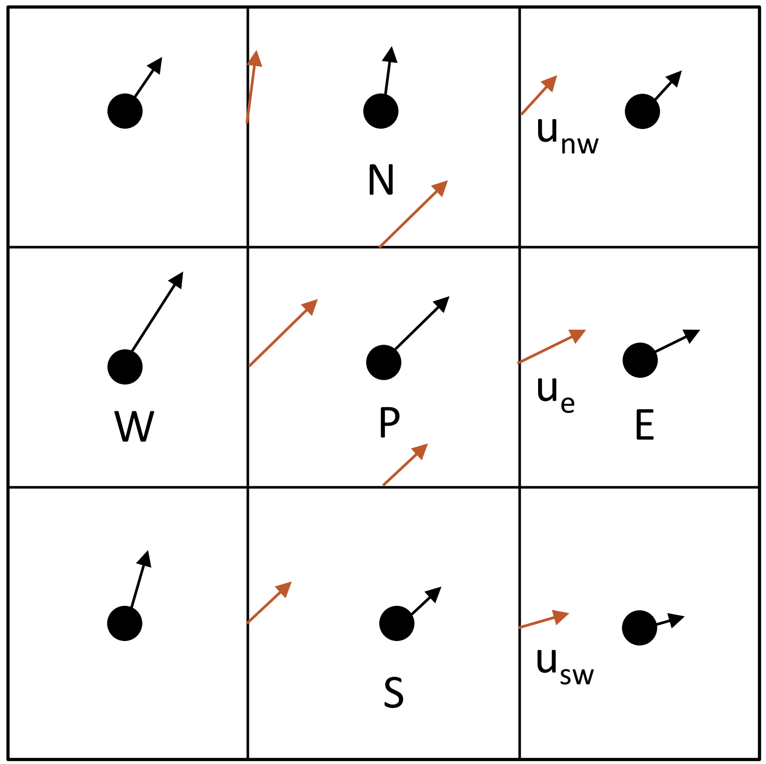
- $\boldsymbol{u_e} = \begin{cases} \boldsymbol{U_P} & \boldsymbol{if\ U_E^x > 0} \\ \boldsymbol{U_E} & \boldsymbol{if\ U_E^x < 0} \end{cases}$

- $\boldsymbol{u_n} = \begin{cases} \boldsymbol{U_P} & \boldsymbol{if\ U_N^y > 0} \\ \boldsymbol{U_N} & \boldsymbol{if\ U_N^y < 0} \end{cases}$



Velocity Gradient Interpolation

- Gradients in direction of normal to surfaces are straightforward
- Gradients in direction parallel to surface normal require special treatment
  - Use interpolated face-centered values
- Ex:
  - $\nabla \boldsymbol{u}_e = \begin{pmatrix} \frac{\partial u_e^x}{\partial x} & \frac{\partial u_e^x}{\partial y} \\ \frac{\partial u_e^y}{\partial x} & \frac{\partial u_e^y}{\partial y} \end{pmatrix}$
  - $\frac{\partial u_e^y}{\partial y} = \frac{u_{nw}^y - u_{sw}^y}{2 * \Delta y}$



Examples

Burgers Equation: Mesh, Initial Conditions, and Boundary Conditions

Initial Condition:

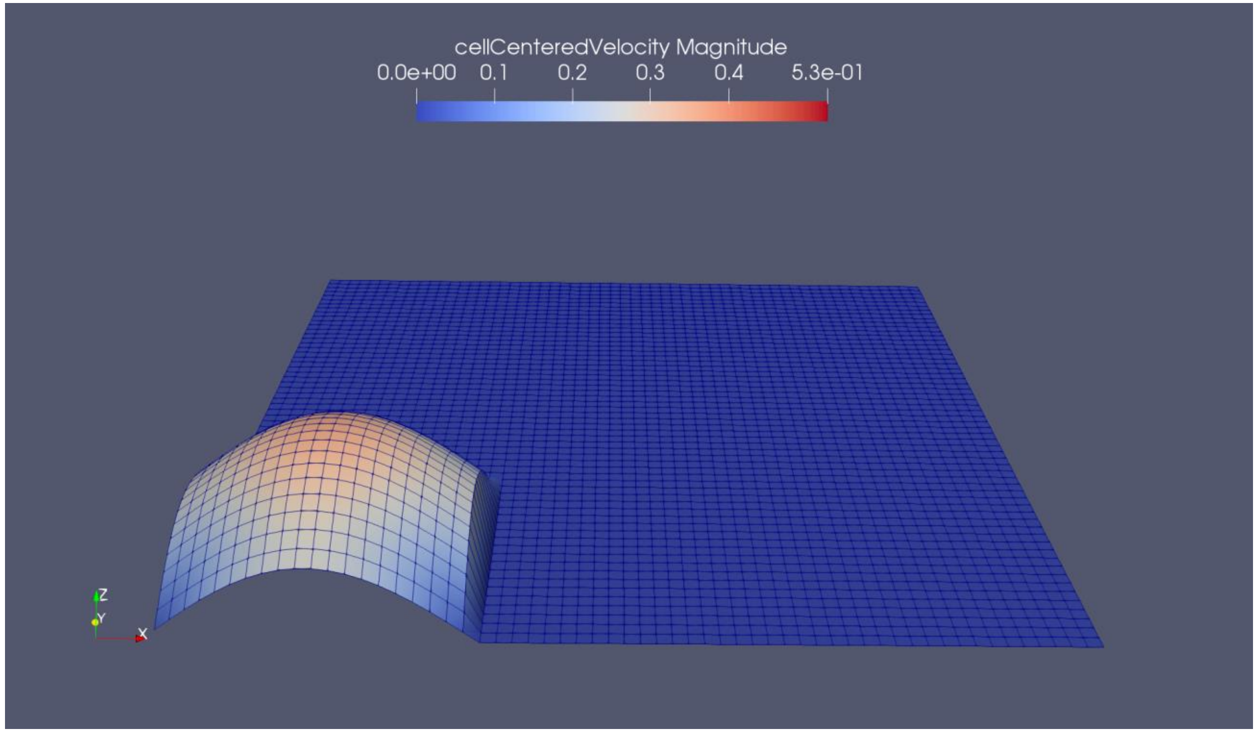
- $U^x = 0.25\sin(\pi x)\,,\ x \in [0.0,1.0]$
- $U^y = 0.25\sin(\pi y)\,,\ y \in [0.0,1.0]$
- $U^x = U^y = 0\ elsewhere$

Dimensions:

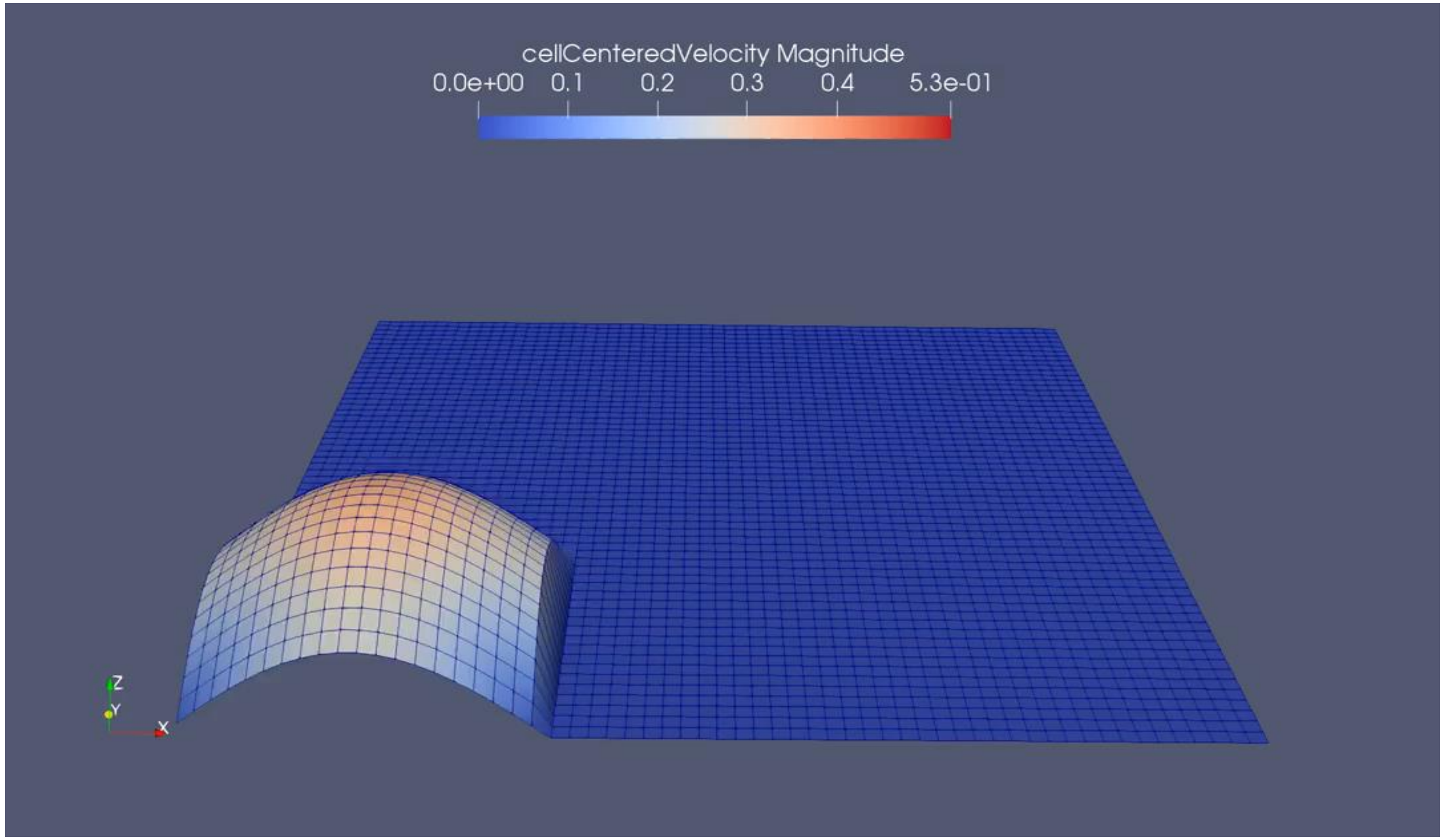
- Length = 2
- Height = 2
- $\Delta x = \Delta y = 0.05$

Material Parameters:

- ρ = 1



Burgers Equation: Result



Diffusion: Mesh, Initial Conditions, and Boundary Conditions

Initial Condition:

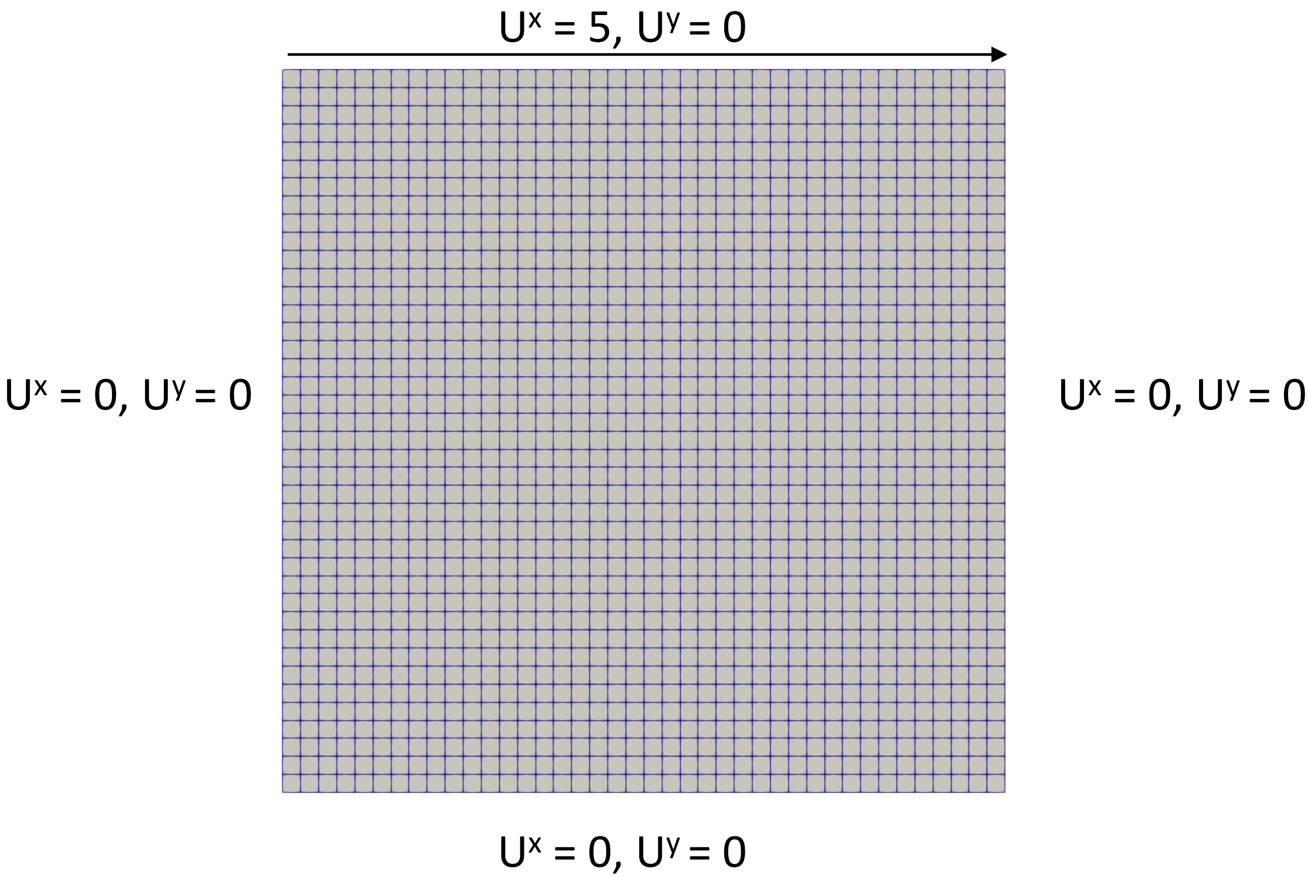
- $U^x = 0$, $U^y = 0$

Dimensions:

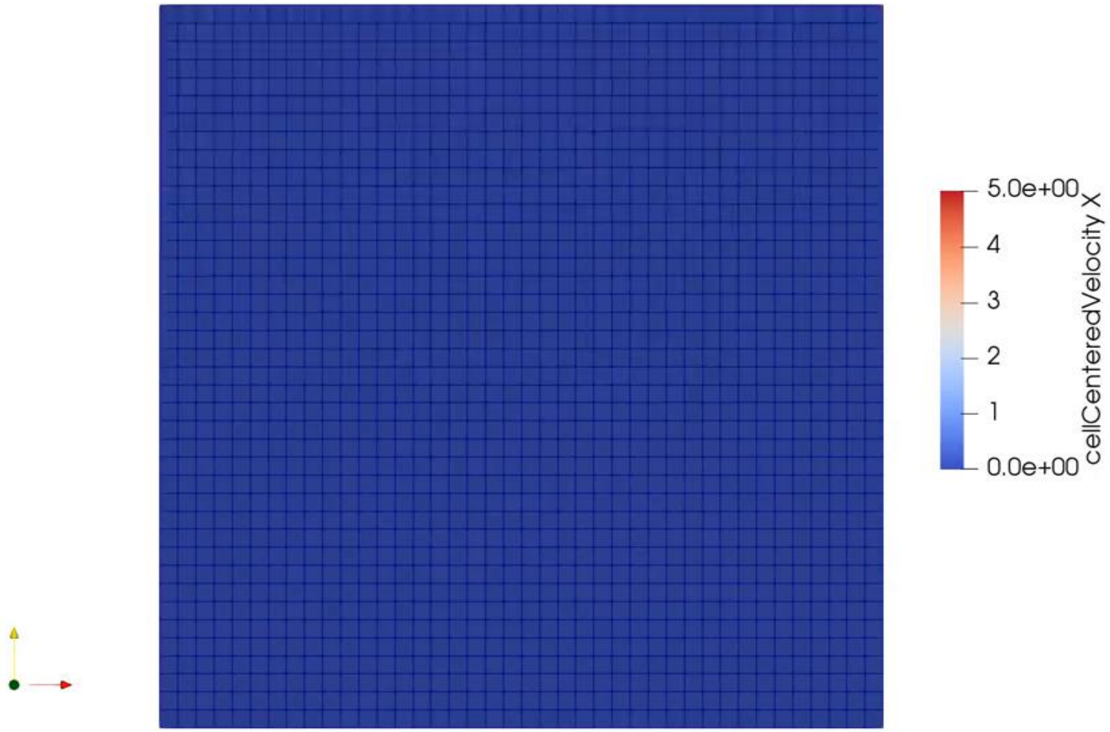
- Length = 2
- Height = 2
- $\Delta x = \Delta y = 0.05$

Material Parameters:

- μ = 1



Diffusion: Result



Poiseuille Flow: Mesh, Initial Conditions, and Boundary Conditions

Initial Condition:

- $U^x = 0$, $U^y = 0$

Dimensions:

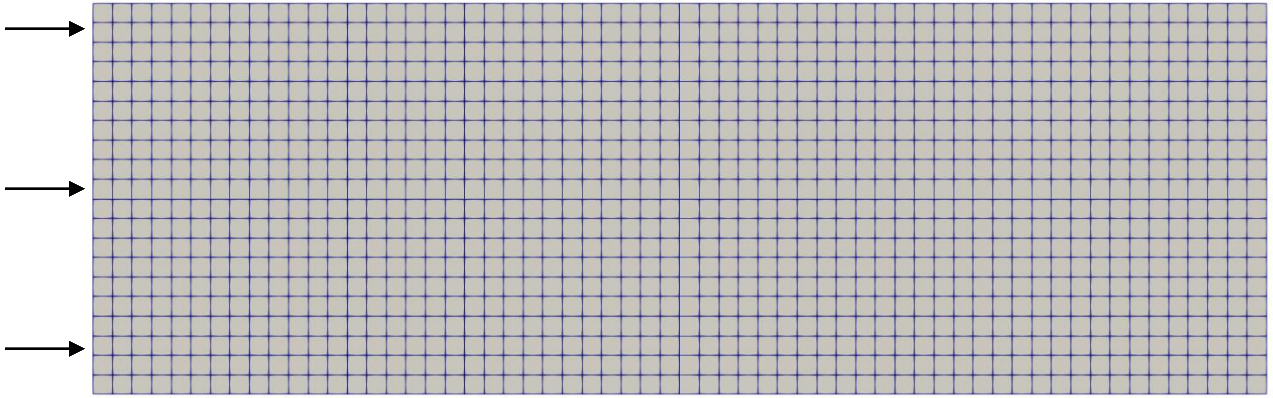
- Length = 3
- Height = 1
- $\Delta x = \Delta y = 0.05$

Material Parameters:

- $\mu = 1$
- $\rho = 1$

$U^x = 0$, $U^y = 0$, $\frac{\partial P}{\partial \boldsymbol{n}} = 0$

$U^x = 1$, $U^y = 0$, $\frac{\partial P}{\partial \boldsymbol{n}} = 0$

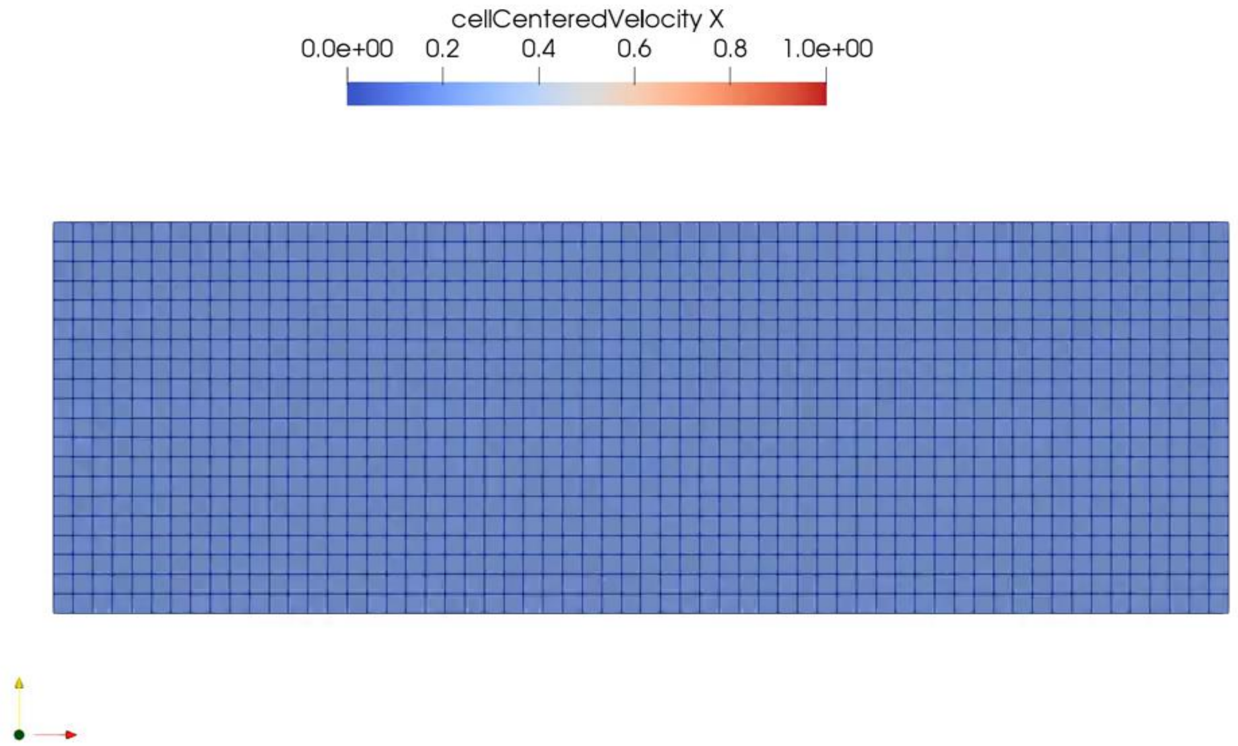


$\frac{\partial U^x}{\partial x} = 0$, $\frac{\partial U^y}{\partial x} =$

$P = 0$

$U^x = 0$, $U^y = 0$, $\frac{\partial P}{\partial \boldsymbol{n}} = 0$

Poiseuille Flow: Result



Flow Around a Cylinder: Mesh, Initial Conditions, and Boundary Conditions

Initial Condition:

- $U^x = 0$, $U^y = 0$

Dimensions:

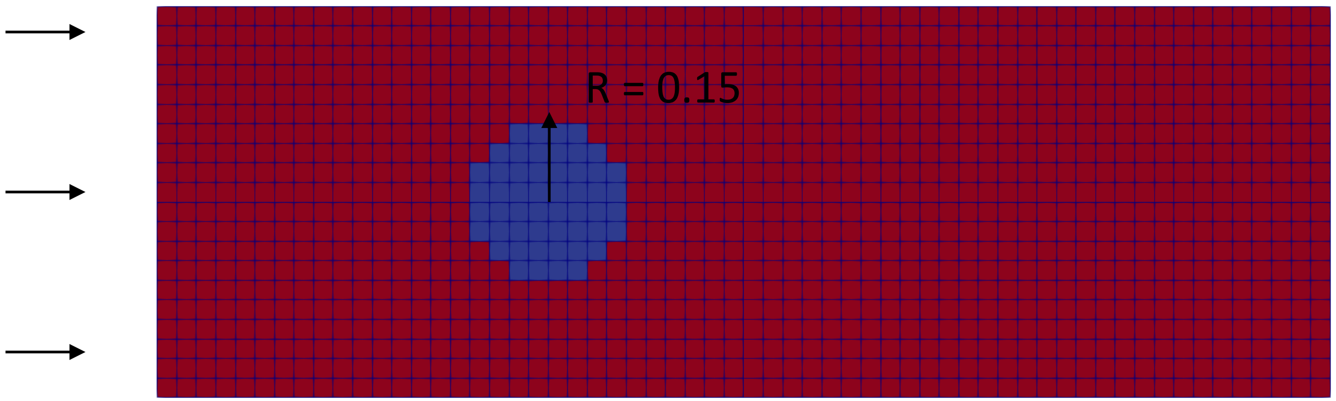
- Length = 3
- Height = 1
- $\Delta x = \Delta y = 0.05$

Material Parameters:

- $\mu = 1$
- $\rho = 1$

$U^x = 0$, $U^y = 0$, $\frac{\partial P}{\partial \boldsymbol{n}} = 0$

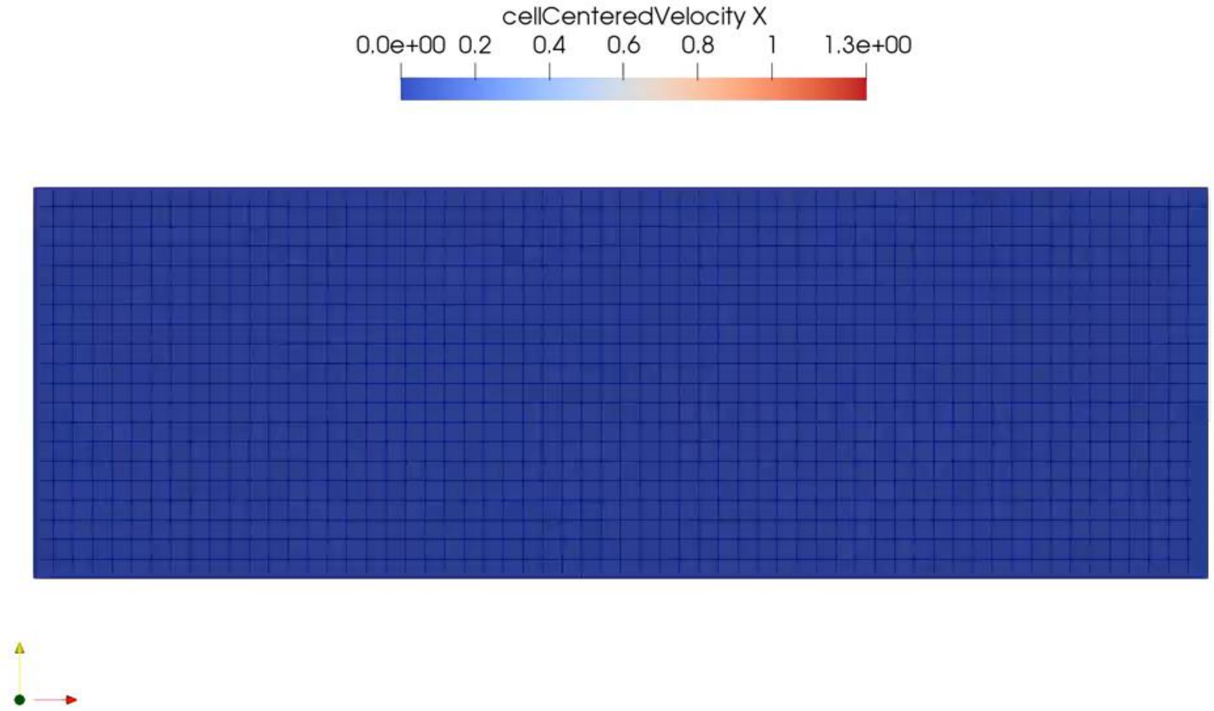
$U^x = 1$, $U^y = 0$, $\frac{\partial P}{\partial \boldsymbol{n}} = 0$



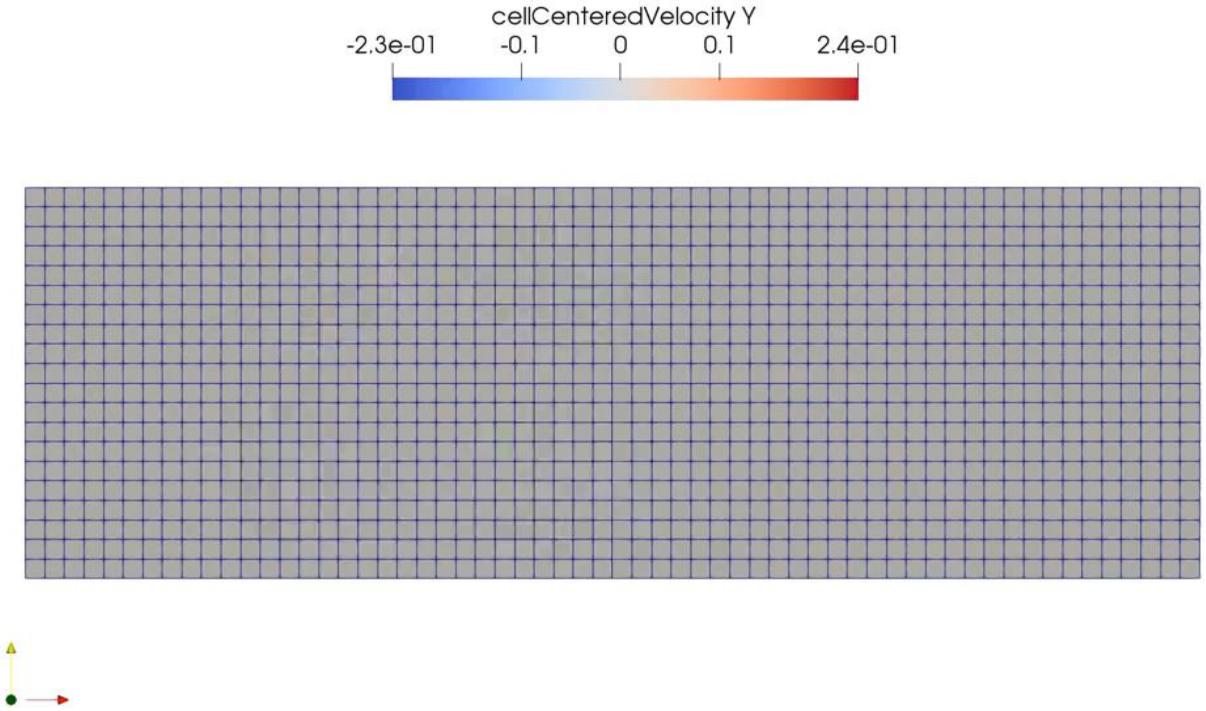
$\frac{\partial U^x}{\partial x} = 0$, $\frac{\partial U^y}{\partial x} = 0$, $P = 0$

$U^x = 0$, $U^y = 0$, $\frac{\partial P}{\partial \boldsymbol{n}} = 0$

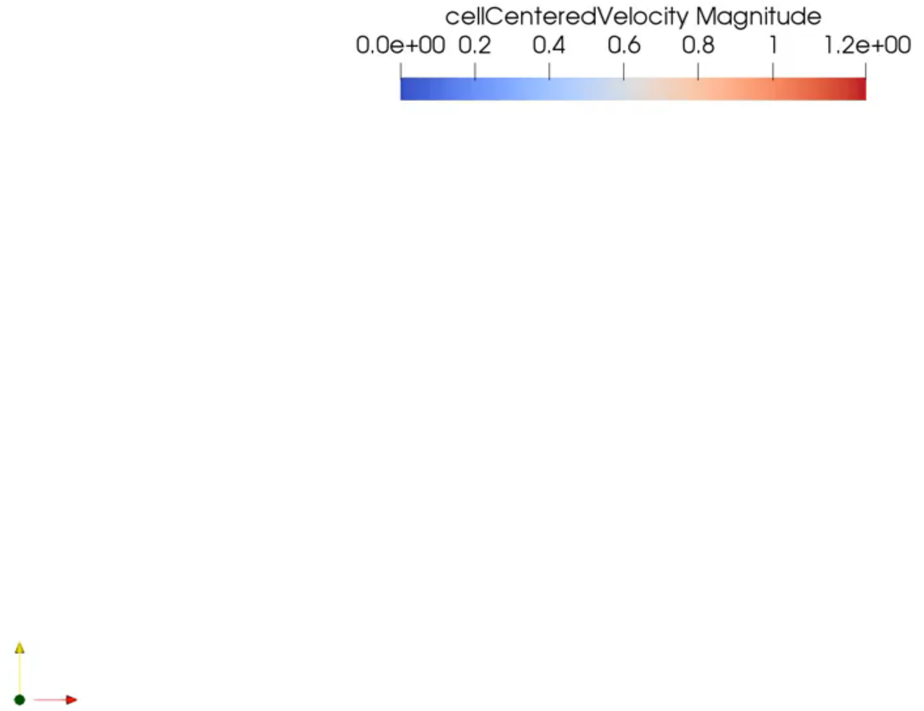
Flow Around a Cylinder: U-Velocity



Flow Around a Cylinder: V-Velocity



Flow Around a Cylinder: Velocity Vector Field



Future Work

Code Extensions

- Add different velocity interpolation functions
- Nonuniform grids

Extend to 3D

- More complex bookkeeping but code structure remains same