Simulation of Wheel Locomotion on Bingham fluids using 2.29 code

Final Project

2.29 (Numerical Fluid Mechanics Spring 2018)

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Bingham Plastics

NS Equation: $$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}.\nabla \boldsymbol{u}\right) = -\nabla p + \nabla.\boldsymbol{\tau} + \rho \boldsymbol{b}$$

Newtonian fluid :

$$\tau = \mu \nabla \mathrm{u}$$

Bingham Fluid:

$$\dot{\gamma} = 0 \qquad \tau \le \tau_y$$
$$\tau = \left(\frac{\tau}{\dot{\gamma}} + \mu\right)\dot{\gamma} \qquad \tau \le \tau_y$$

Papanastasiou (1987) regularization :

$$\tau = \left[\frac{\tau_y}{\dot{\gamma}}\{1 - \exp(-m\dot{\gamma})\} + \mu\right]\dot{\gamma}$$

$$\textit{Where :}\ \boldsymbol{\gamma} = \boldsymbol{\nabla}\mathbf{u} + \boldsymbol{\nabla}\mathbf{u}^{\mathsf{T}} \quad \dot{\gamma} = \left[\frac{1}{2}\dot{\boldsymbol{\gamma}}:\dot{\boldsymbol{\gamma}}\right]^{1/2} \qquad \tau = \left[\frac{1}{2}\boldsymbol{\tau}:\boldsymbol{\tau}\right]^{1/2}$$

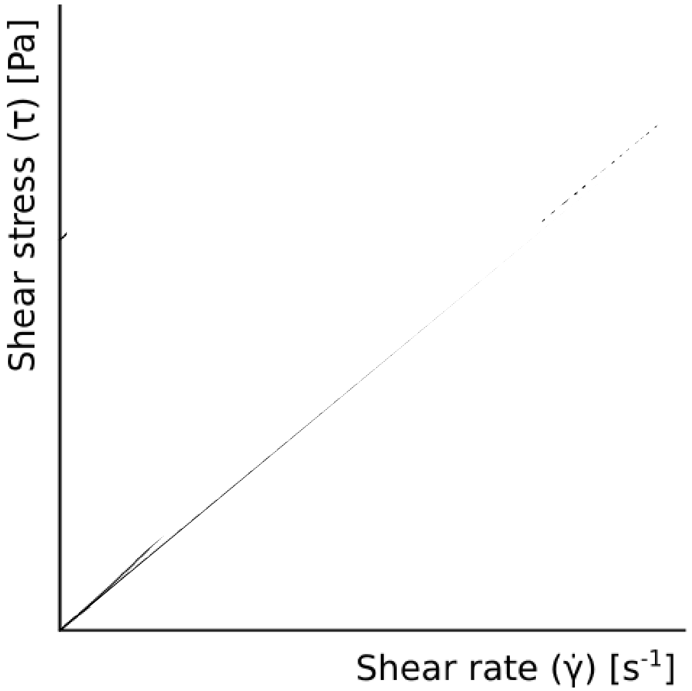
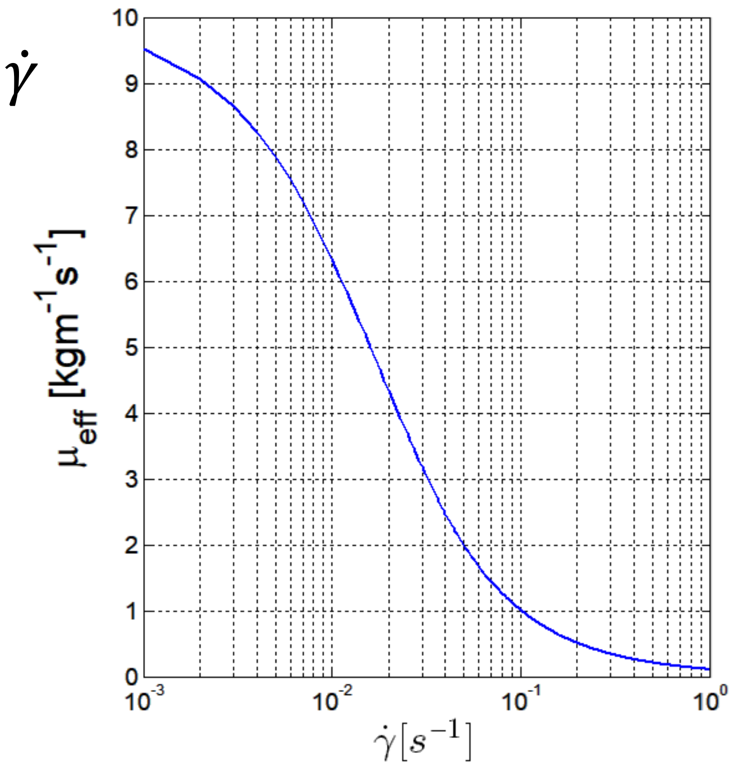


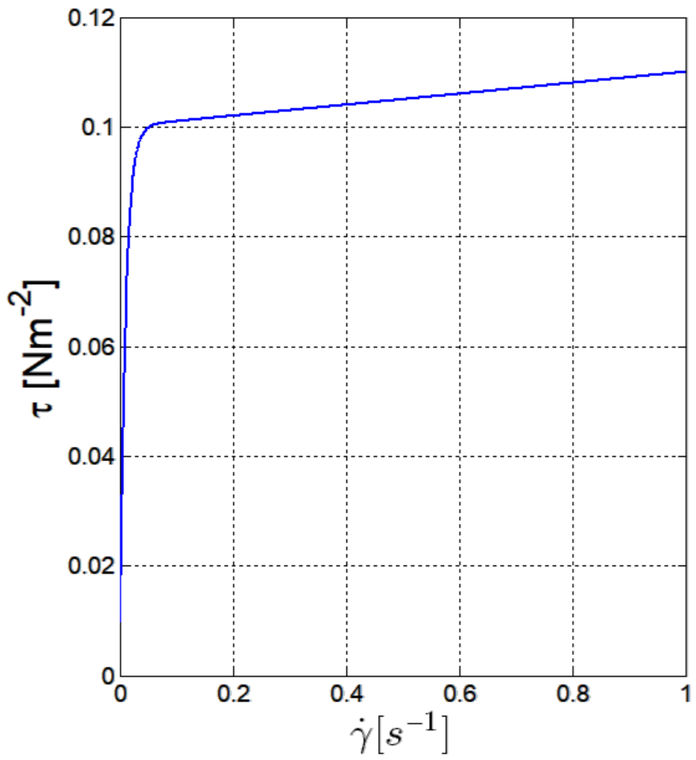
Fig: Classification of fluids with shear stress as a function of shear rate

Bingham Fluid Properties used

$$\tau = \left[\frac{\tau_y}{\dot{\gamma}} \{1 - \exp(-m\dot{\gamma})\} + \mu \right] \dot{\gamma}$$

$m = 100\ [s]$
$\tau_y = 0.1\ [Pa]$
$\mu_o = 0.01\ [Pa.s]$



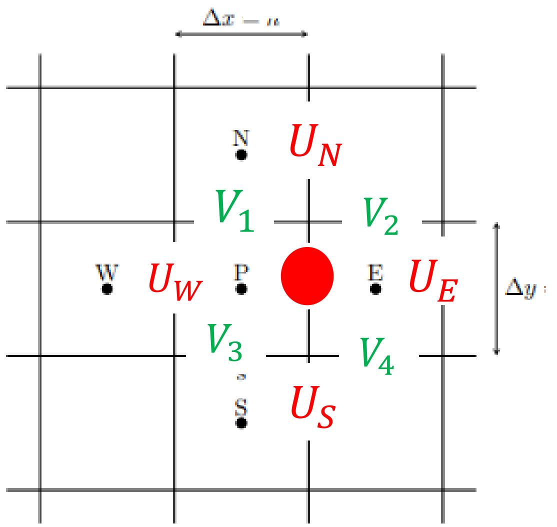


Implementation in 2.29 Code

$$\mu(\dot{\gamma}) = \left[\frac{\tau_y}{\dot{\gamma}}\{1-\exp(-m\dot{\gamma})\} + \mu\right]$$

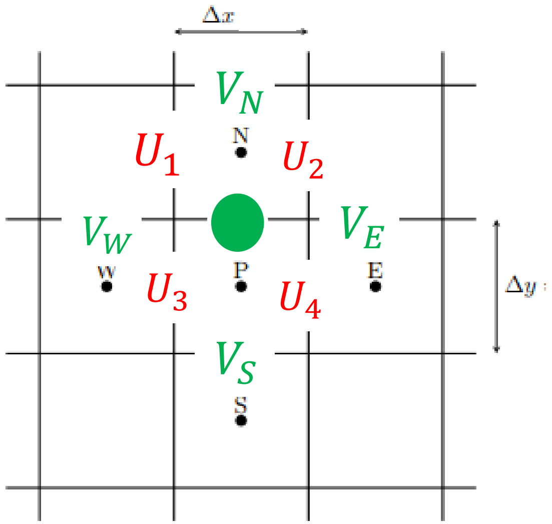
$$\mu_{n+1}\left(\dot{\gamma}_{u/v}\right) = \frac{\tau_y}{\dot{\gamma}_n|_{u/v}}\left\{1-\exp\left(-m\dot{\gamma}_n|_{u/v}\right)\right\} + \mu_o$$

$$\dot{\gamma}_u = \left[2\left(\frac{\partial u}{\partial x}\Big|_u\right)^2 + 2\left(\frac{\partial v}{\partial y}\Big|_u\right)^2 + \left(\frac{\partial v}{\partial x}\Big|_u + \frac{\partial u}{\partial y}\Big|_u\right)^2\right]^{1/2}$$



Case 1: $\frac{\partial u}{\partial x}\Big|_u = \frac{U_E - U_W}{2\Delta x}$ $\frac{\partial u}{\partial y}\Big|_u = \frac{U_N - U_S}{2\Delta y}$

$\frac{\partial v}{\partial y}\Big|_u = \frac{(V_1+V_2)-(V_3+V_4)}{2\Delta y}$ $\frac{\partial v}{\partial x}\Big|_u = \frac{(V_2+V_4)-(V_1+V_3)}{2\Delta x}$

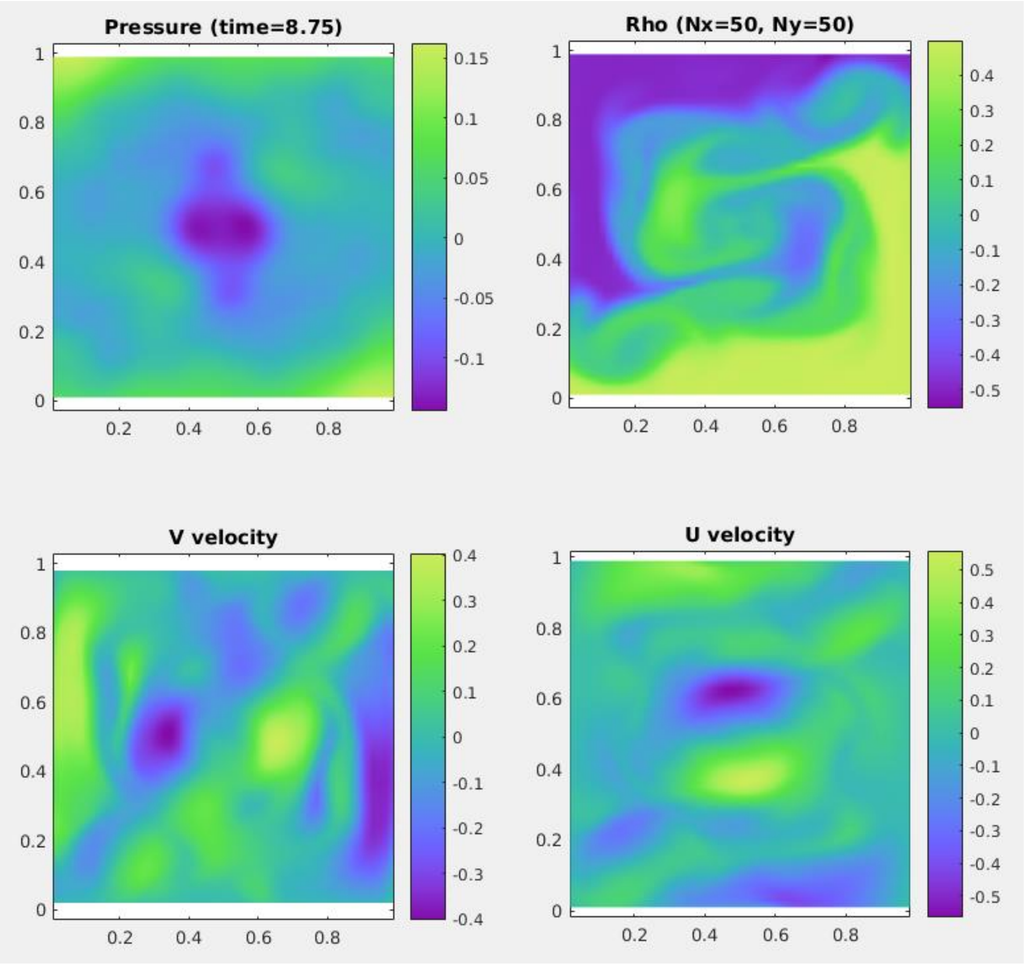


Case 2: $\frac{\partial v}{\partial x}\Big|_v = \frac{V_E - V_W}{2\Delta x}$ $\frac{\partial v}{\partial y}\Big|_v = \frac{V_N - V_S}{2\Delta y}$

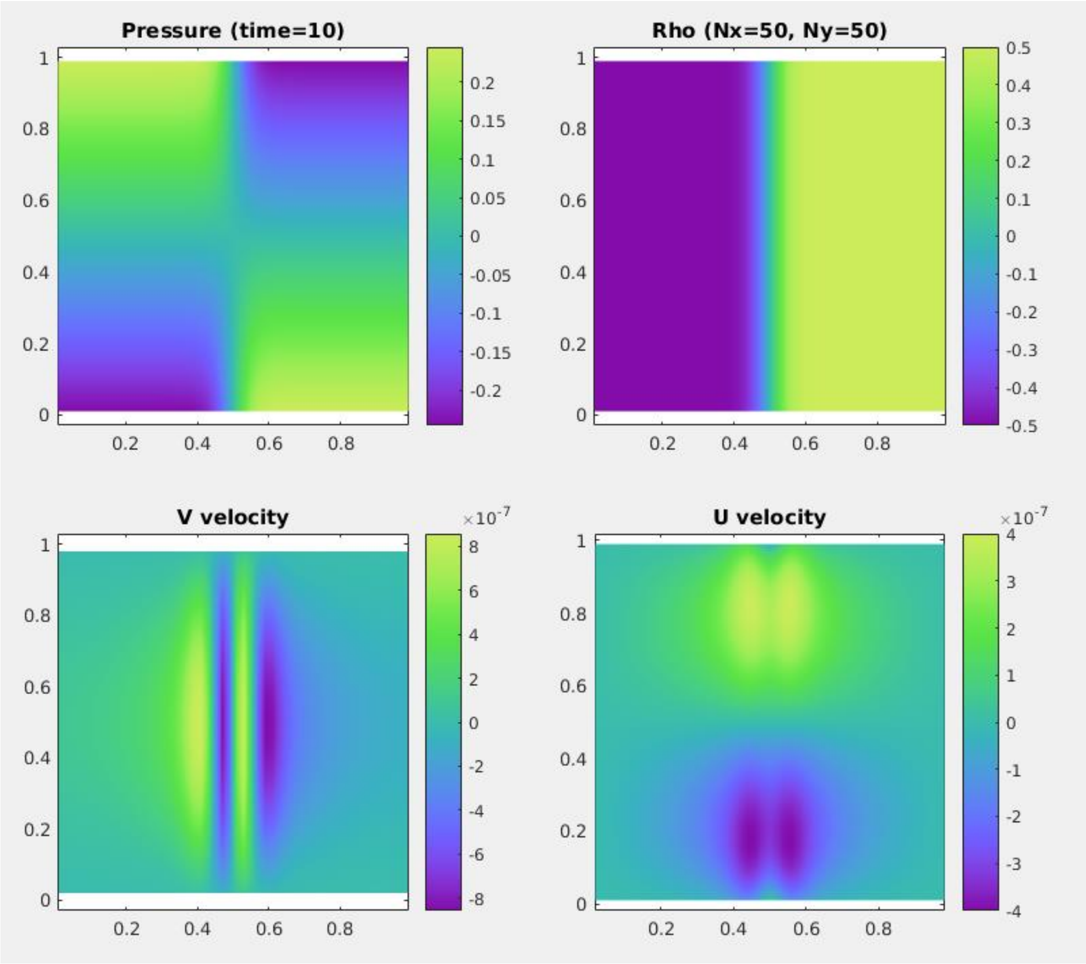
$\frac{\partial u}{\partial x}\Big|_v = \frac{(U_2+U_4)-(U_1+U_3)}{2\Delta x}$ $\frac{\partial u}{\partial y}\Big|_v = \frac{(U_1+U_2)-(U_3+U_4)}{2\Delta y}$

Litmus test 1: Lock Exchange Problem (using 2.29 code)

Newtonian Fluid

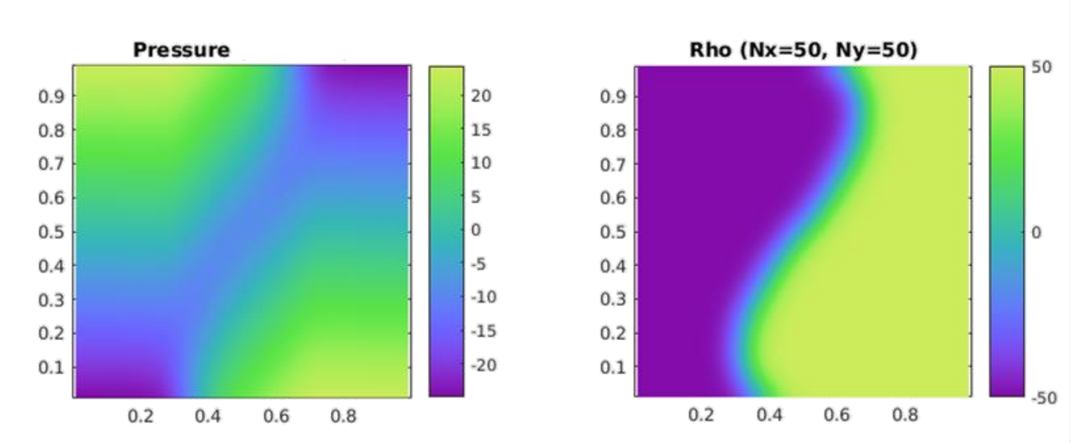


Bingham Plastic: Same density

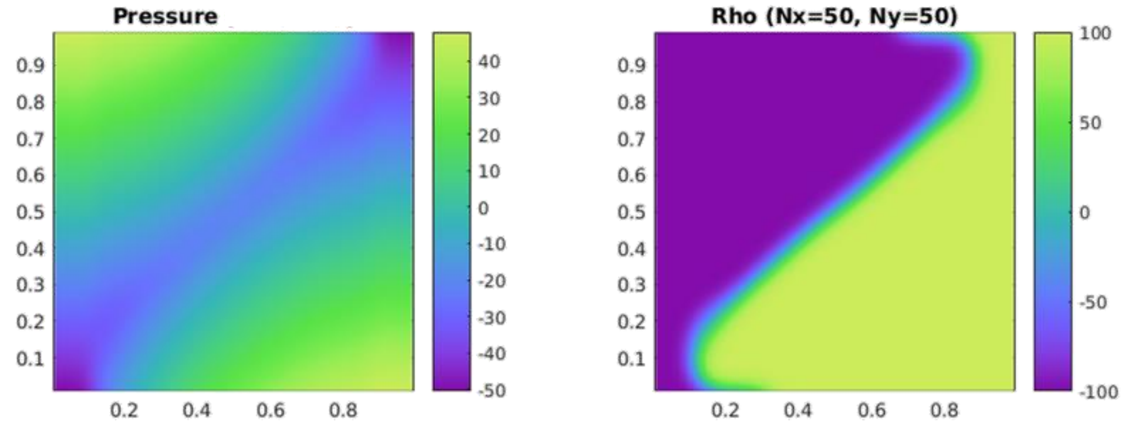


Litmus test 1: Lock Exchange Problem

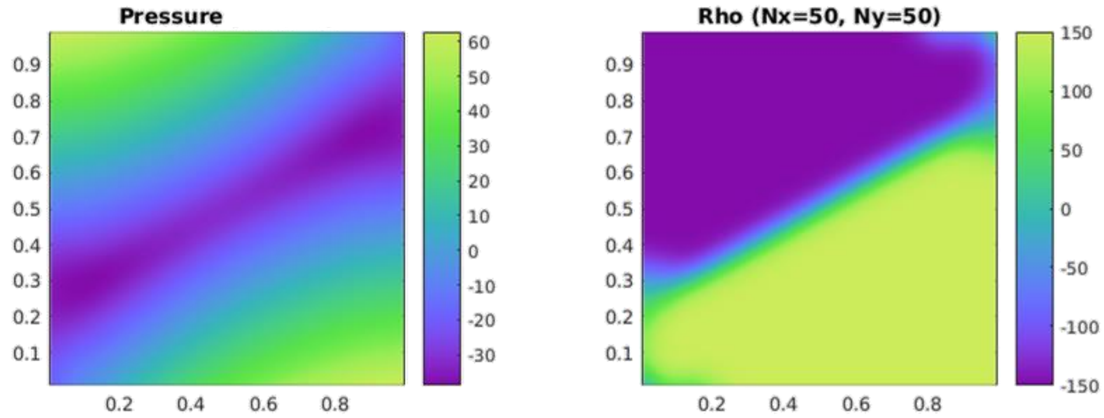
Bingham Fluid: density x100



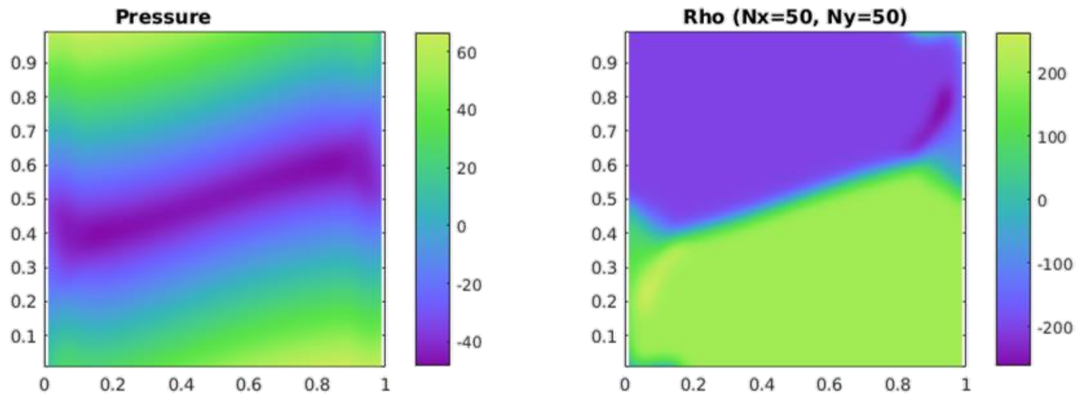
Bingham Fluid: density x200



Bingham Fluid: density x300

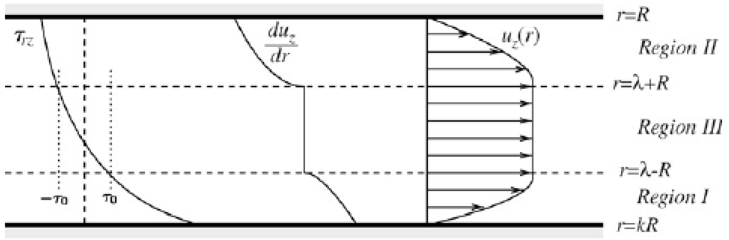


Bingham Fluid: density x400

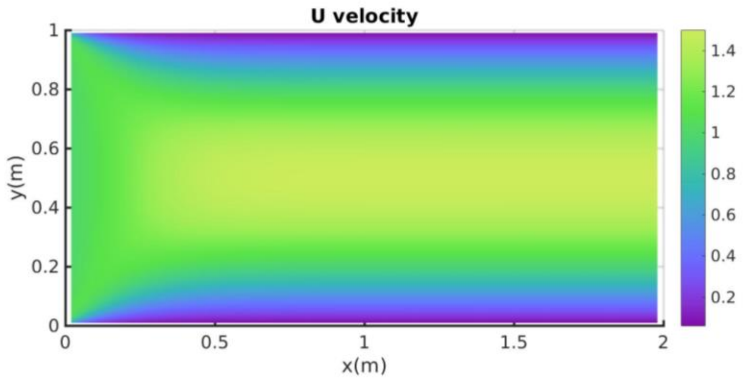


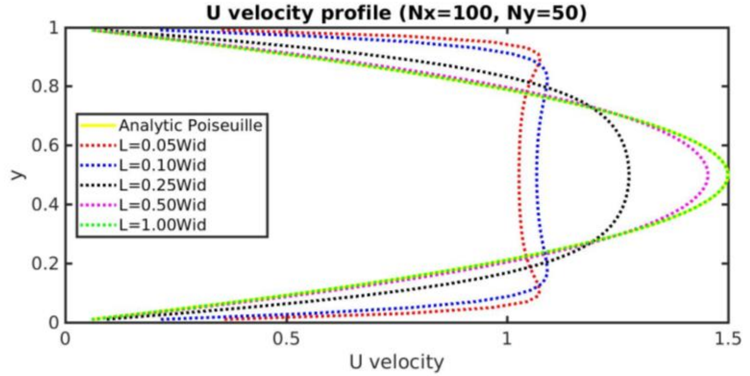
Litmus test 2 : Poiseuille flow

Expected Profile at equilibrium state

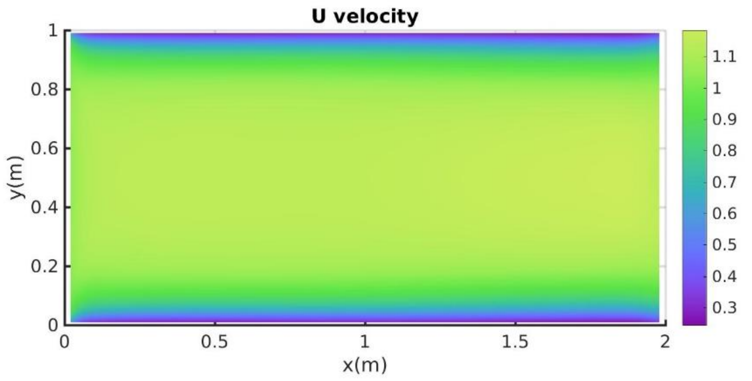


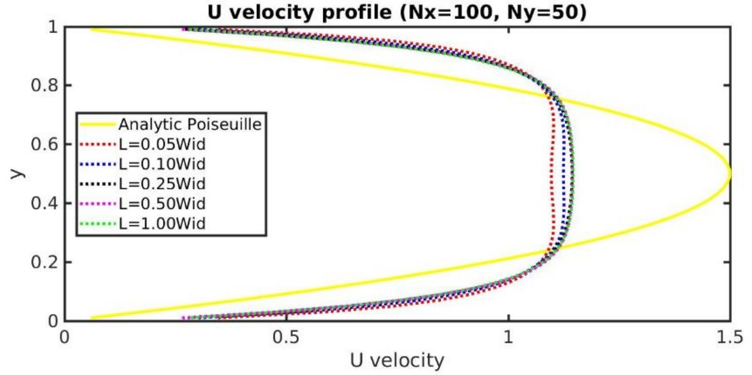
Geometry (As provided in 2.29 sample code):
Length= 20m
Radius = 3m





Properties:
$\mu = 0.01\ Pa.s$



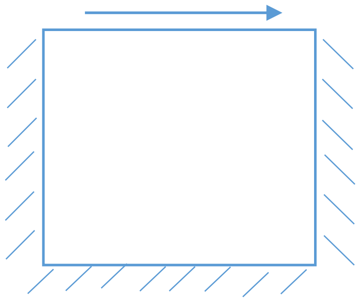


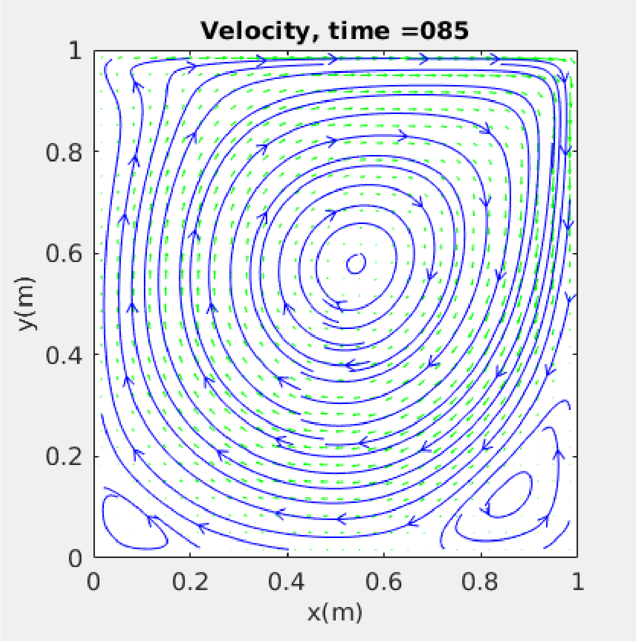
Properties:
$\mu_{eq} = 0.01\ Pa.s \quad m = 100$
$\tau = 50\ Pa$

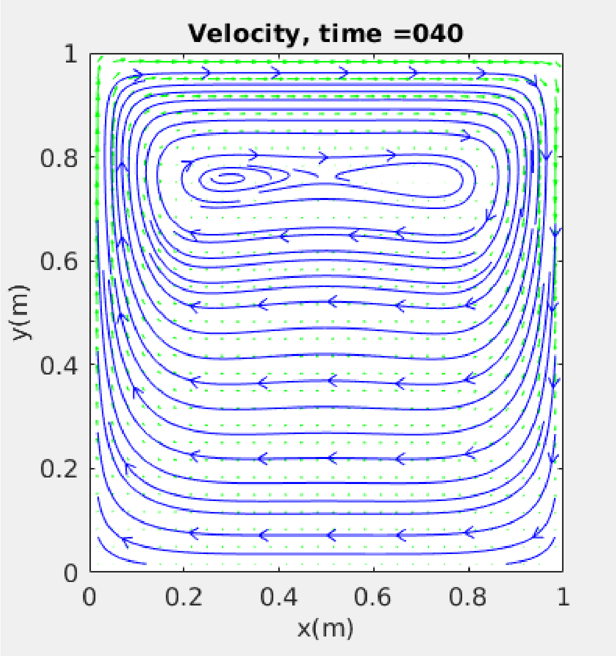
Litmus test 3: Lid Driven Cavity Flow

Geometry (As provided in 2.29 sample code):
Dim= 1x1 (m^2)

Properties: Generic







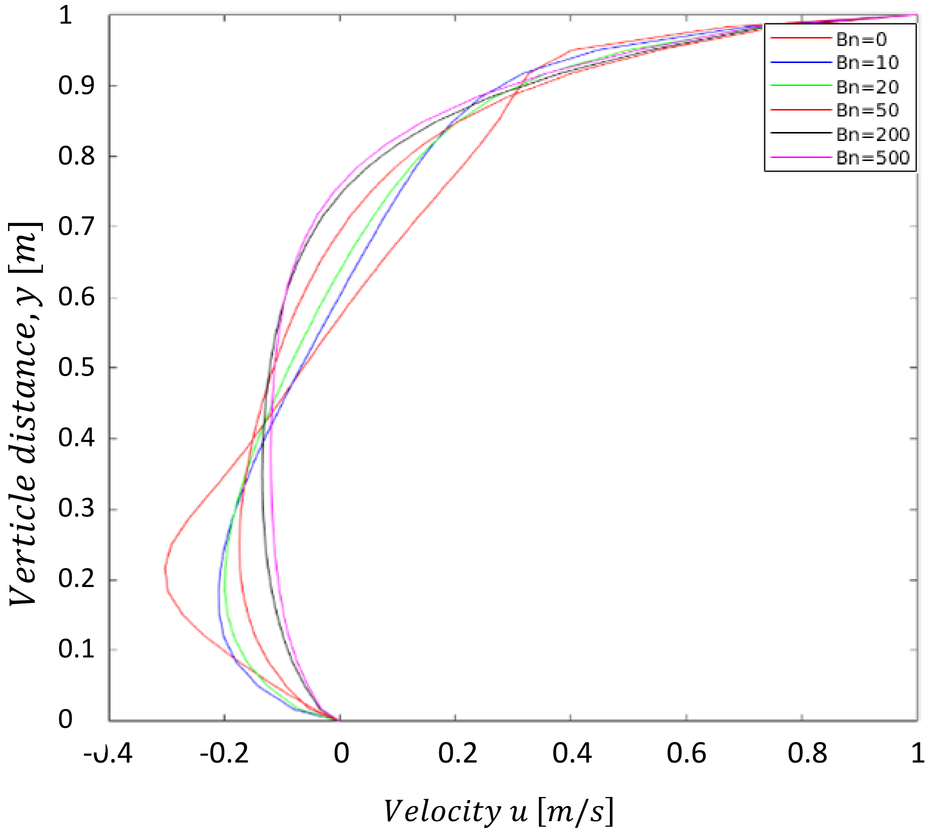
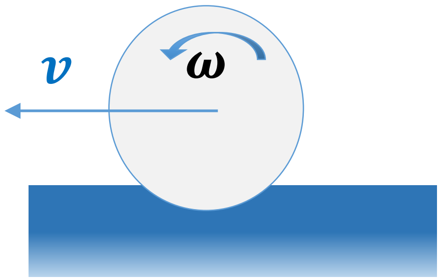


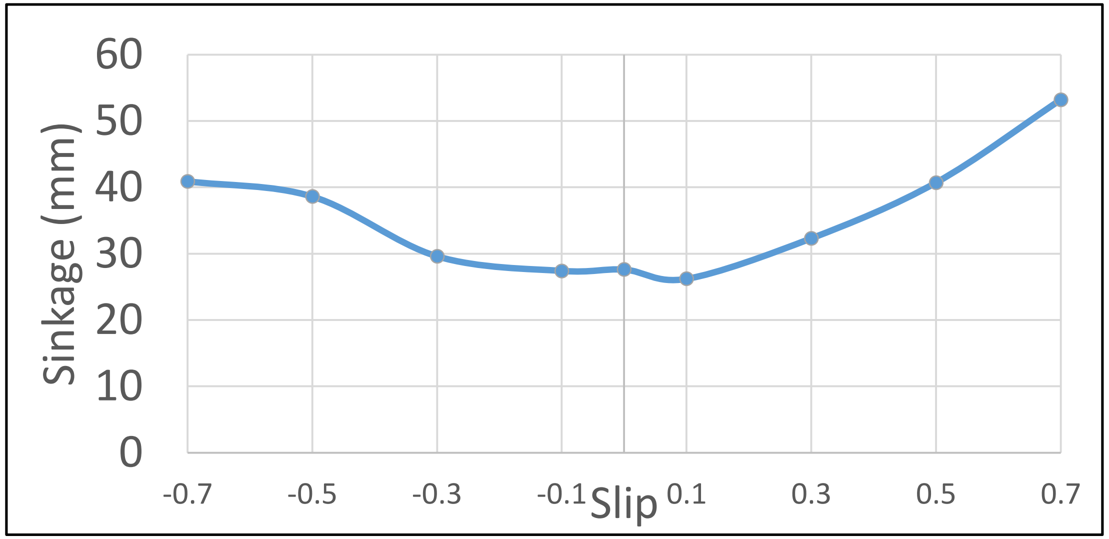
Fig: Velocity variation along center line using Bingham fluid on 2.29 code at various Bingham numbers

Drag Lift Variation on Wheel locomotion on Bingham fluid

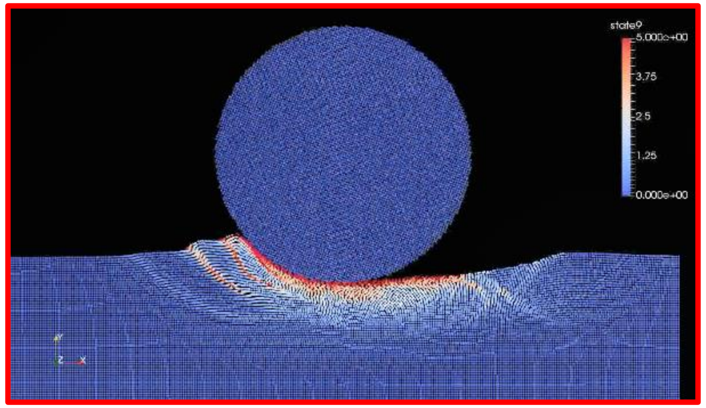


$$Slip = 1 - \frac{v}{r\omega}$$

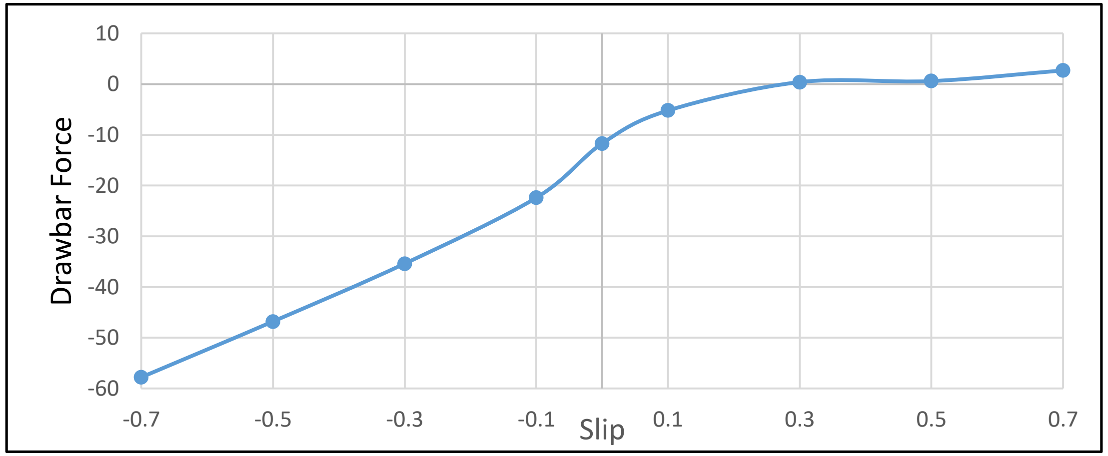
Sinkage of wheel in forced slip conditions



For Granular media:

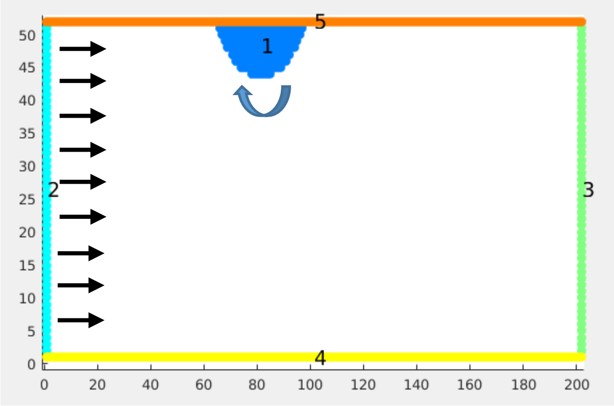


Additional Drag force on wheel in forced slip conditions

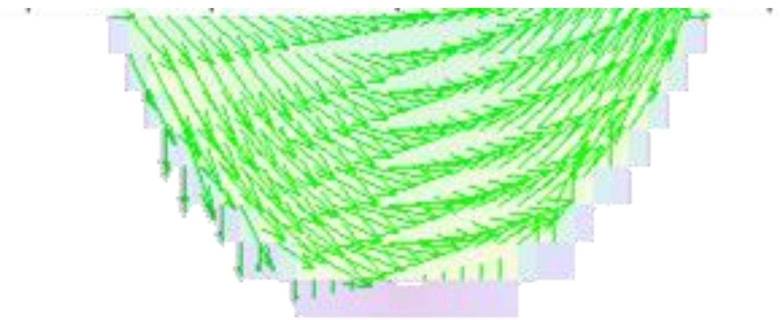


Drag Lift Variation on Wheel locomotion on Bingham fluid

Implementation



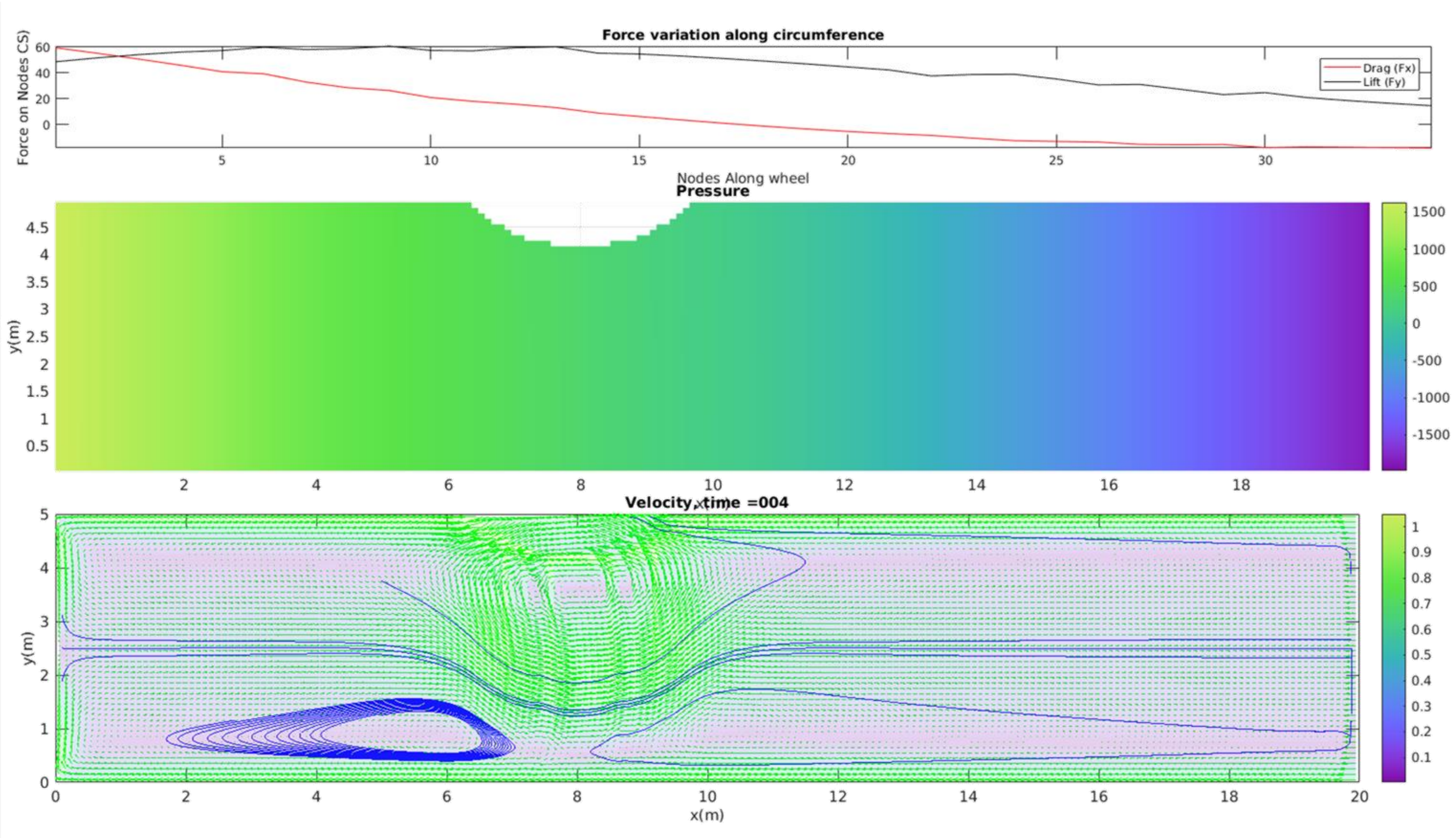
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
95	105	115	125	135	145	2	2	2	2	2	2	2	2	209	219	229	239	249	259	269	279	289
96	106	116	126	136	146	2	2	2	2	2	2	2	2	210	220	230	240	250	260	270	280	290
97	107	117	127	137	147	155	2	2	2	2	2	2	201	211	221	231	241	251	261	271	281	291
98	108	118	128	138	148	156	163	2	2	2	2	194	202	212	222	232	242	252	262	272	282	292
99	109	119	129	139	149	157	164	170	176	182	188	195	203	213	223	233	243	253	263	273	283	293
100	110	120	130	140	150	158	165	171	177	183	189	196	204	214	224	234	244	254	264	274	284	294
101	111	121	131	141	151	159	166	172	178	184	190	197	205	215	225	235	245	255	265	275	285	295
102	112	122	132	142	152	160	167	173	179	185	191	198	206	216	226	236	246	256	266	276	286	296
103	113	123	133	143	153	161	168	174	180	186	192	199	207	217	227	237	247	257	267	277	287	297
104	114	124	134	144	154	162	169	175	181	187	193	200	208	218	228	238	248	258	268	278	288	298
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4

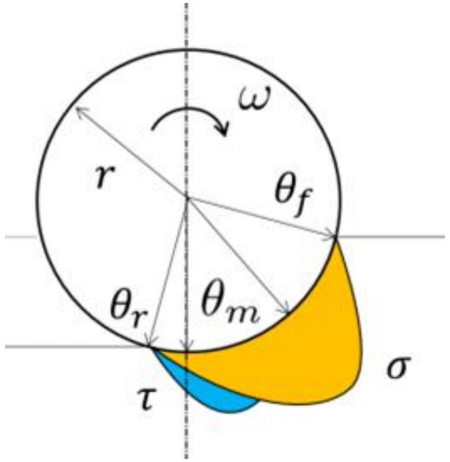


Node	x	y	theta	Area
145	-2.2500	0.7500	-1.2490	0.5000
146	-2.2500	1.2500	-1.0637	0.5000
155	-1.7500	1.7500	-0.7854	0.5000
163	-1.2500	2.2500	-0.5071	0.5000
170	-0.7500	2.7500	-0.2663	0.5000
176	-0.2500	2.7500	-0.0907	0.5000
182	0.2500	2.7500	0.0907	0.5000
188	0.7500	2.7500	0.2663	0.5000
194	1.2500	2.2500	0.5071	0.5000
201	1.7500	1.7500	0.7854	0.5000
210	2.2500	1.2500	1.0637	0.5000
209	2.2500	0.7500	1.2490	0.5000

$$Velocity_{bc|Dmv} = (r - r_o)\text{*omega}$$

Sample Results:





Convergence study for grid size verification

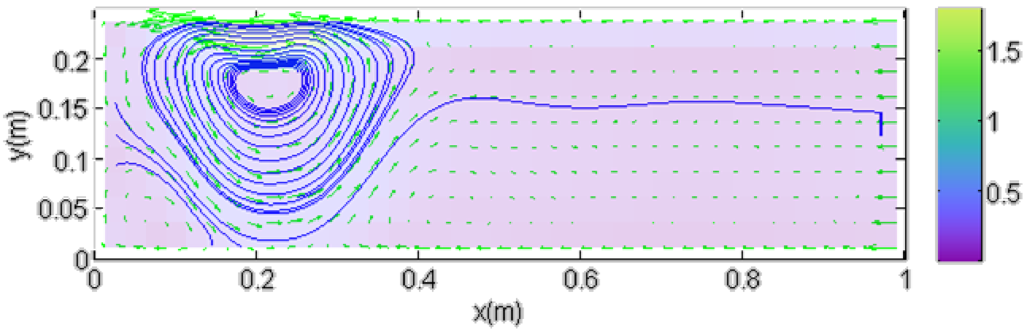
Lift and Drag Force variation with grid size:

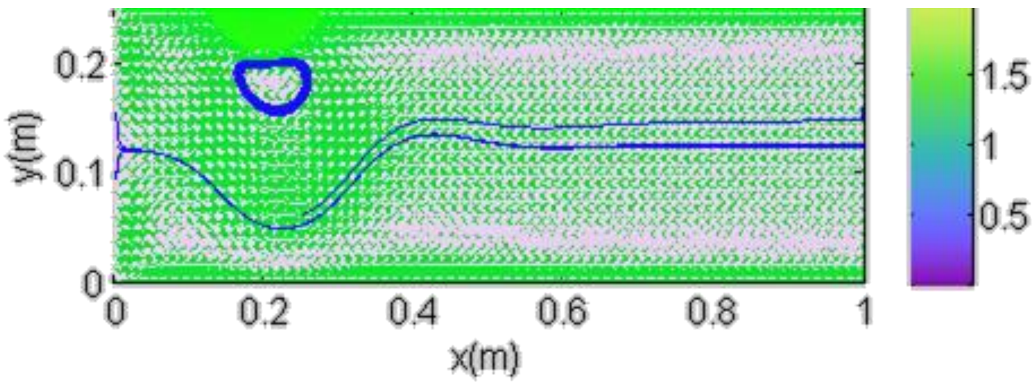
Domain size: 1m x 0.25m

Wheel Radius: 10 cm

Sinkage : 4 cm

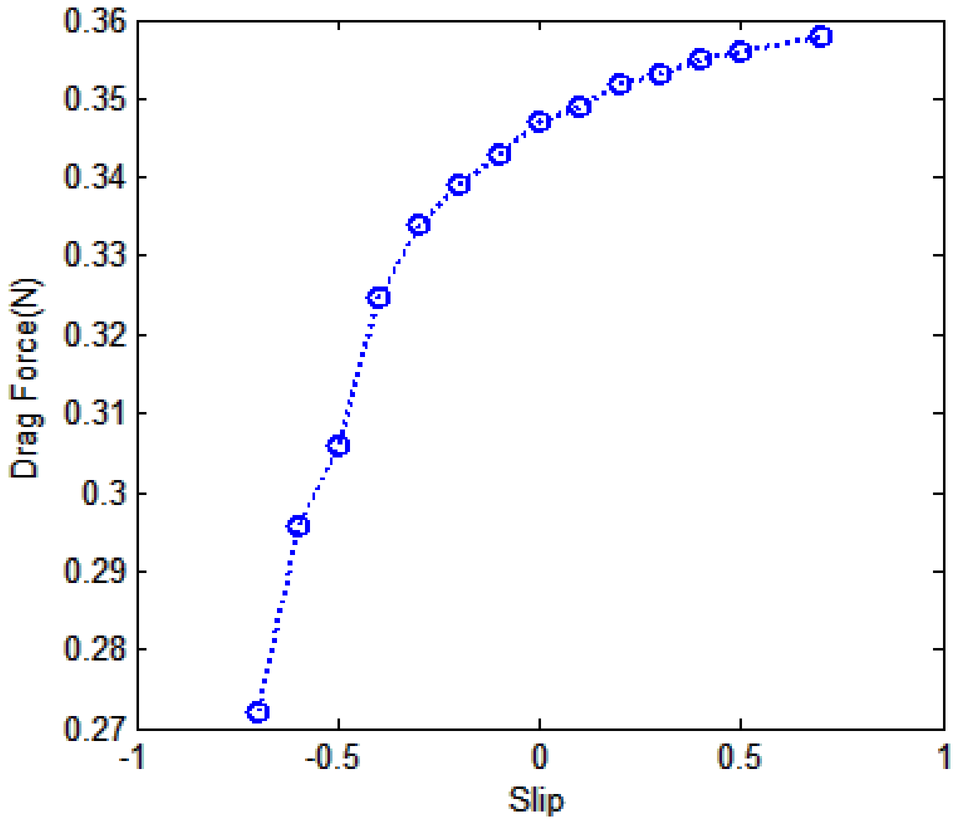
Grid size	Drag Force (N)	Lift Force (N)
40x10	0.420	2.798
100x25	0.388	2.838
200x50	0.347	2.701
400x100	0.330	2.705

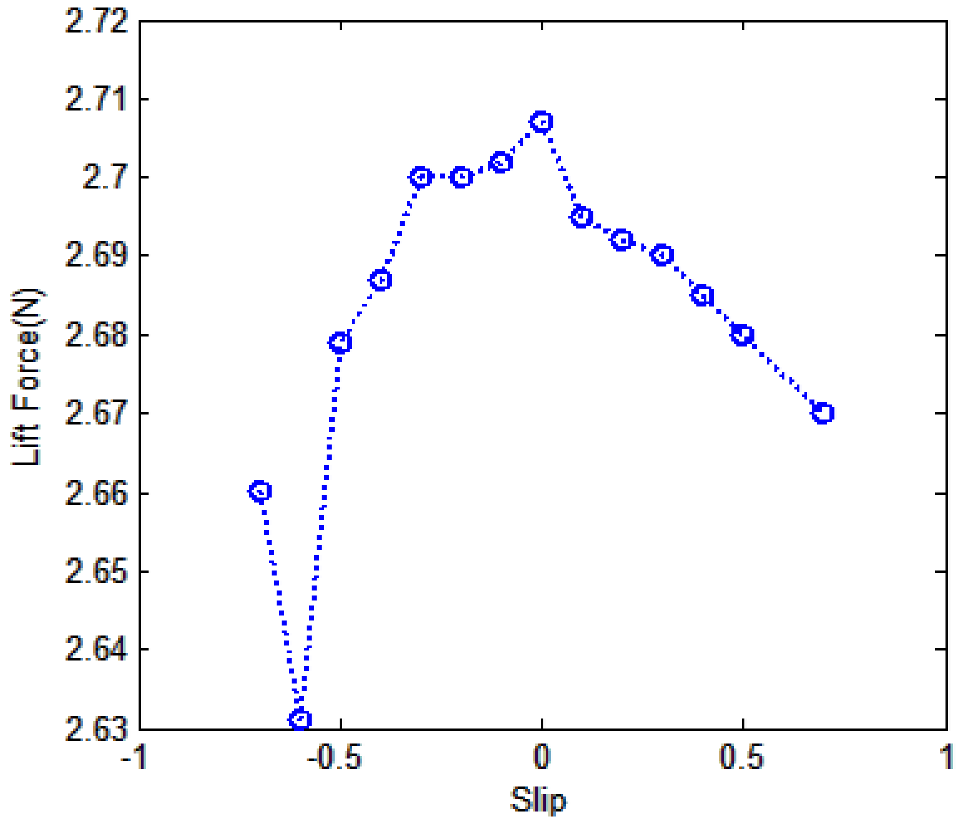




Results:

Variation of Lift and Drag Force on Locomotion of cylindrical wheel on Bhingham Fluid





Future work

- Make viscosity Pressure sensitive for simulating granular media

- **Constitutive Model for Non-cohesive Granular media:**

$$\bar{\tau} = \mu_s P \quad if\ \ \dot{\gamma} > 0$$

$$\boldsymbol{\sigma} = \mathbf{0} \qquad if\ \rho < \rho_{critical}$$