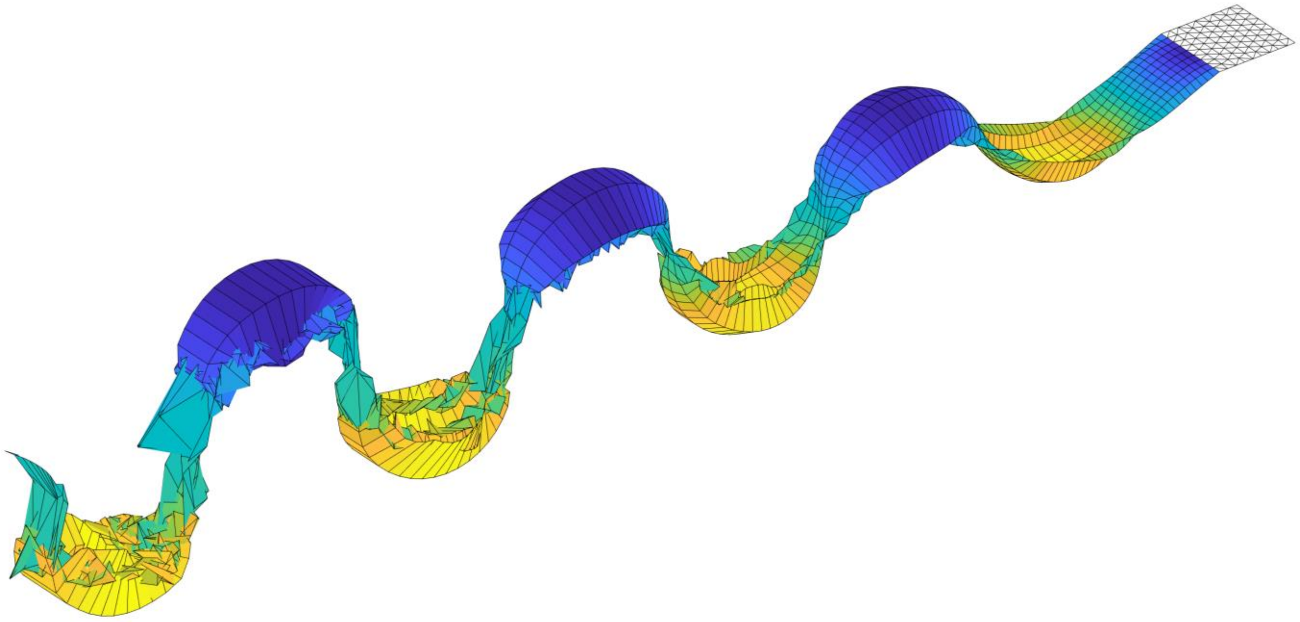
Panel Methods for Unsteady Flow over an Elastic Membrane



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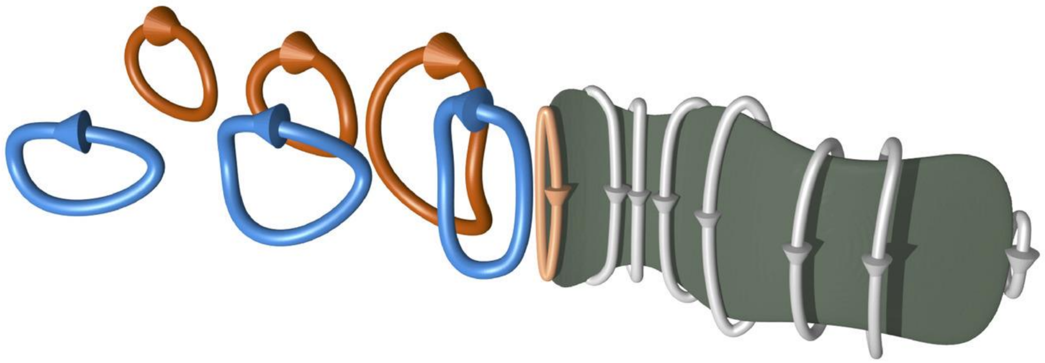
Motivation: Biological Propulsion

- Most animals that fly or swim use thin, flexible structures that deform in a non-negligible way
- Meshing a deforming computational domain is expensive
- Meshing a deforming surface is much easier (Animals undergo nearly-isometric deformation)
- For propulsion, surface forces are often the goal, flow fields are secondary



Panel Methods and Potential Flow

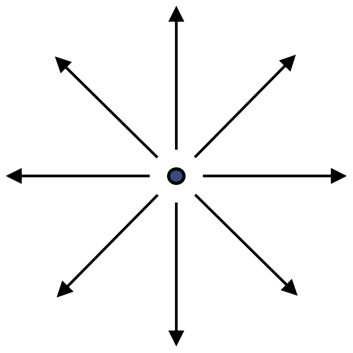
Pros:

- Reduces the dimension of the problem (faster computations)
- Reduced number of computational elements require less memory
- Designed for complex geometries
- Naturally handle open BC's and unbounded exterior flows

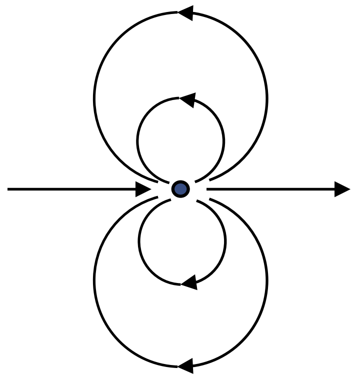
Cons:

- Only appropriate for attached high Reynold's number flows
- Limited Physics (no viscous effects or turbulence)
- Interactions are nonlocal: dense matrices, cost grows as $O(N^2)$ without special treatment
- Higher order methods require complex surface integrals or specialized quadrature algorithms

Two Fundamental Singularities



Singular Source



Singular Doublet

- The Green's Function for the Laplace Operator is a function satisfying

$$\nabla^2 G = \delta(r)$$

- For potential flow, this Green's function is a source: a singular point that radiates fluid equally in all directions. In 3D,

$$\phi_S = G(r) = \frac{1}{4\pi|r|}$$

- The directional derivative of a source flow gives a doublet flow, with potential given by

$$\phi_D = \hat{n} \cdot \nabla G(r) = \frac{\hat{n} \cdot r}{4\pi|r|^3}$$

- This acts like a jet, accelerating fluid to infinite velocity at a single point

Green’s Identity and Layer Potentials

- Functions satisfying $\nabla^2\phi = 0$ also satisfy Green’s Identity,

$$\phi(r) = -\int_S \underbrace{\frac{\partial\phi}{\partial n}G(r-s)}_{\text{Sources (Single Layer)}}ds + \int_S \underbrace{\phi n \cdot \nabla G(r-s)ds}_{\text{Doublets (Double Layer)}}$$

Sources
(Single Layer)

Doublets
(Double Layer)

- All of the information needed to construct ϕ exists on the boundary
- Every such ϕ can be built up from sources and doublets on the boundary

Green's Identity and Layer Potentials

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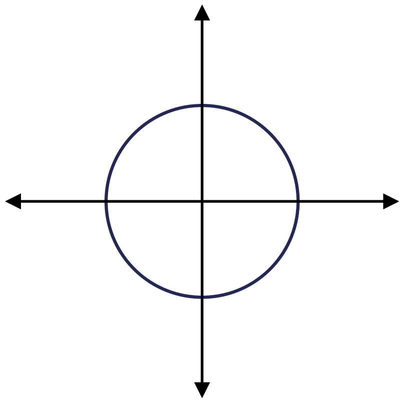
Sources (Single Layer) — Doublets (Double Layer)

- All of the information needed to construct ϕ exists on the boundary
- Every such ϕ can be built up from sources and doublets on the boundary

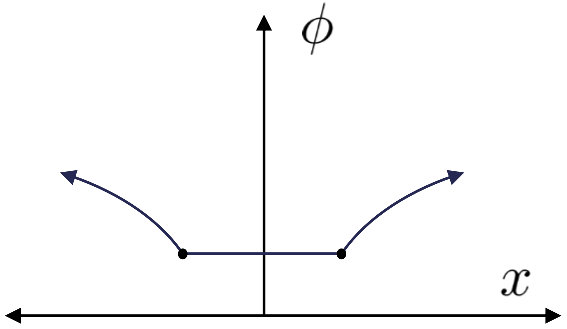
Instead of solving for ϕ, why not solve for two boundary functions?

What do Layer Potentials do?

When singularities are spread out over a surface, they lead to non-smooth potentials. For a uniform distribution on the unit circle,

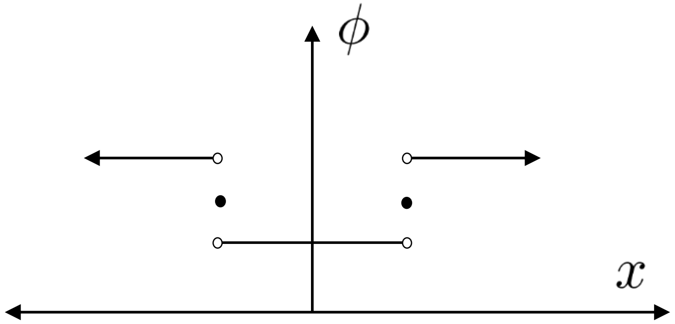


Constant Source Strength
=
Jump in Normal Derivative



$$\phi(x) = \int_S G(x-s)ds$$

Constant Doublet Strength
=
Jump in Potential



$$\phi(x) = \int_S n \cdot \nabla G(x-s)ds$$

Panel Methods

- For thin panels, normal velocity is zero on both sides. No jump in normal velocity means no source distribution.

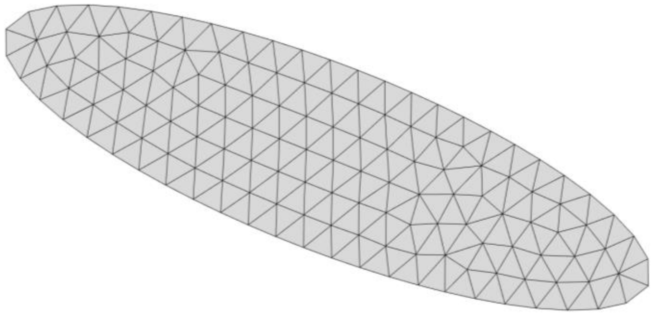
- The potential flow solution will be a double layer potential

- The goal is to find a function $\mu(s)$ on the surface so that

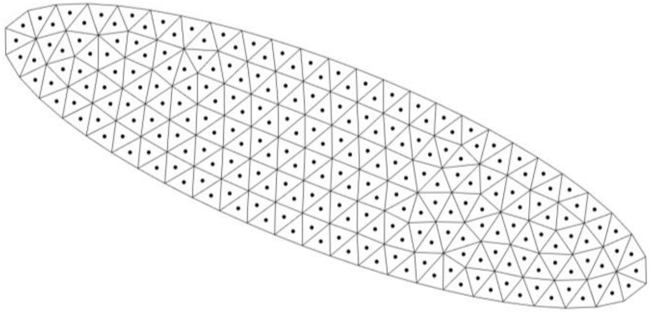
$$\phi(r) = \int_S \mu(s) \frac{\partial}{\partial n} G(r - s) ds$$

$$\frac{\partial \phi}{\partial n} + U_s \cdot n = 0 \text{ on } S$$

- In practice, assume that $\mu(s) = \sum \mu_i \phi_i(s)$, integrate over each individual panel, and solve a linear system to find values of μ_i



Boundary Panels



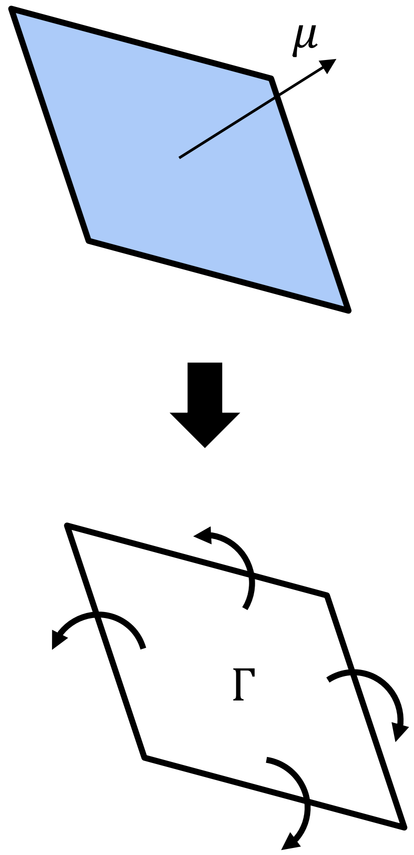
Collocation Points

Doublet / Vortex Equivalence

- High order basis functions lead to difficult surface integrals, which can be expensive to evaluate (analytically or through quadrature).
- For the special case of constant-strength doublet panels, these integrals reduce to line integrals around the boundary:

A doublet panel of constant strength μ is equivalent to a vortex ring with circulation $\Gamma = \mu$.

- An array of doublet panels can be replaced with a grid of singular vortices. Methods that take advantage of this are collectively known as Vortex Lattice Methods.



Forces acting on Each Panel

- In compressible flow, surfaces do not experience shear force.

- Pressure forces can be recovered from the momentum equation

$$p + \frac{1}{2}\rho v^2 + \rho\frac{\partial\phi}{\partial t} = const.$$

- After some juggling of material derivatives and limiting processes, pressure differences can be written in terms of dipole strength

For a vortex lattice, v is continuous across the surface except at the singular vortex lines. The $\rho v^2/2$ term becomes

$$F_\Gamma = \rho\Gamma(\vec{v} \times \vec{e})$$

The time-dependent term relates to added-mass effects, and can be calculated from changes in doublet strength:

$$F_t = \rho\frac{d\mu}{dt}A_{panel}$$

Wake Modeling

- Wake dynamics are an important part of real flows. To treat wakes in a potential flow model, a vortex sheet is created at the trailing edges of thin bodies.

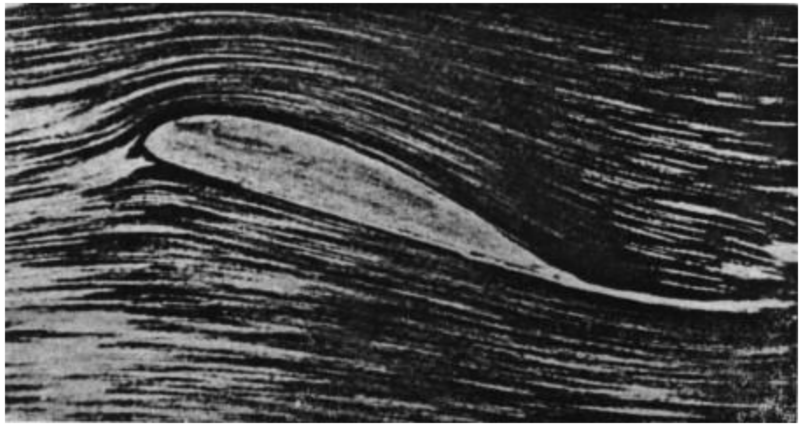
- Vortex sheet is used to satisfy the Kutta condition (flow leaves the trailing edge smoothly). For Vortex Lattice, this means

$$\Gamma_{TE} = 0$$

- Once it enters the flow, vorticity satisfies

$$\frac{D\omega}{Dt} = 0$$

- It can be tracked using Lagrangian particles or panels.

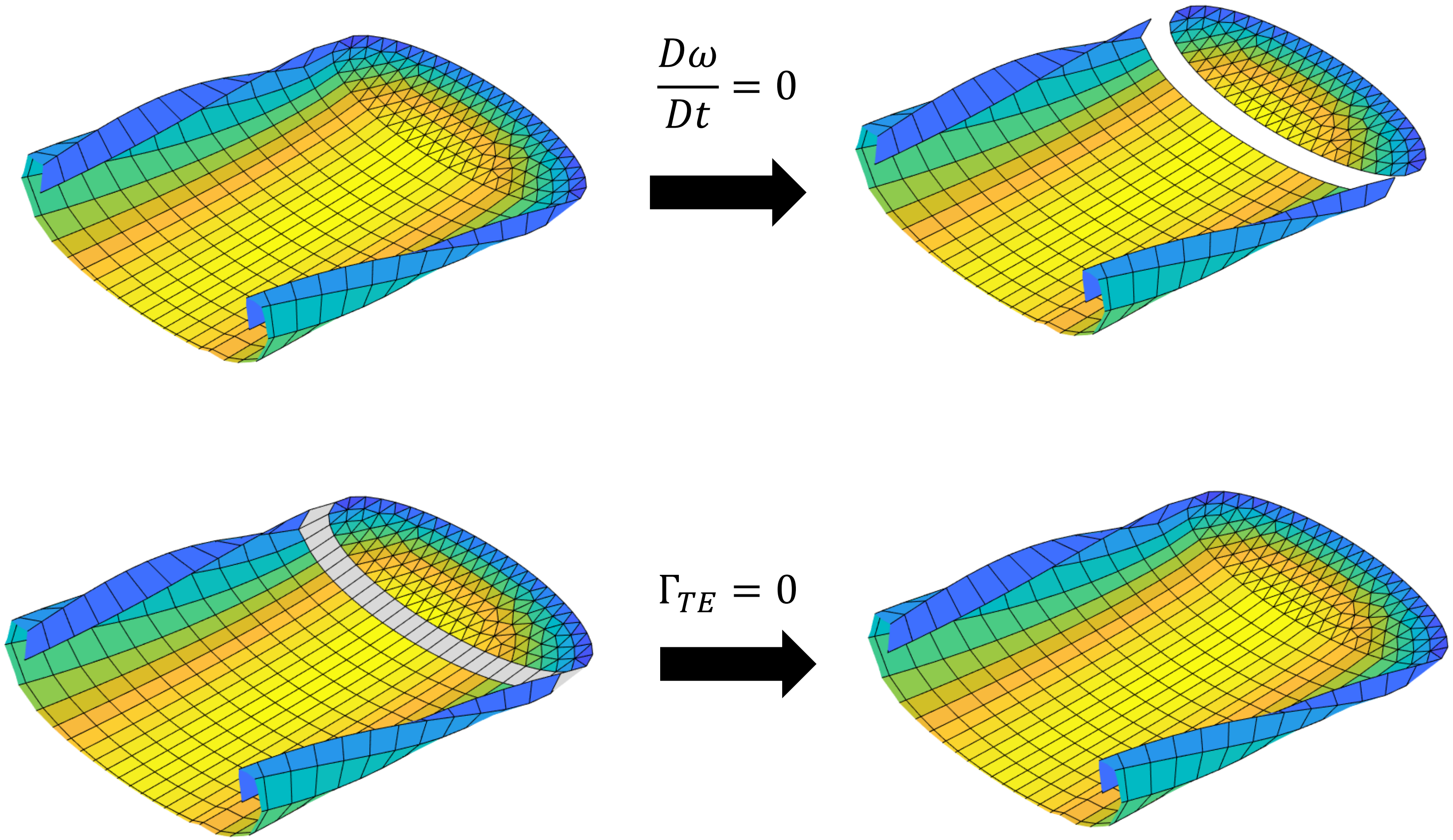


sciencedirect.com/topics/engineering/aerodynamic-characteristic

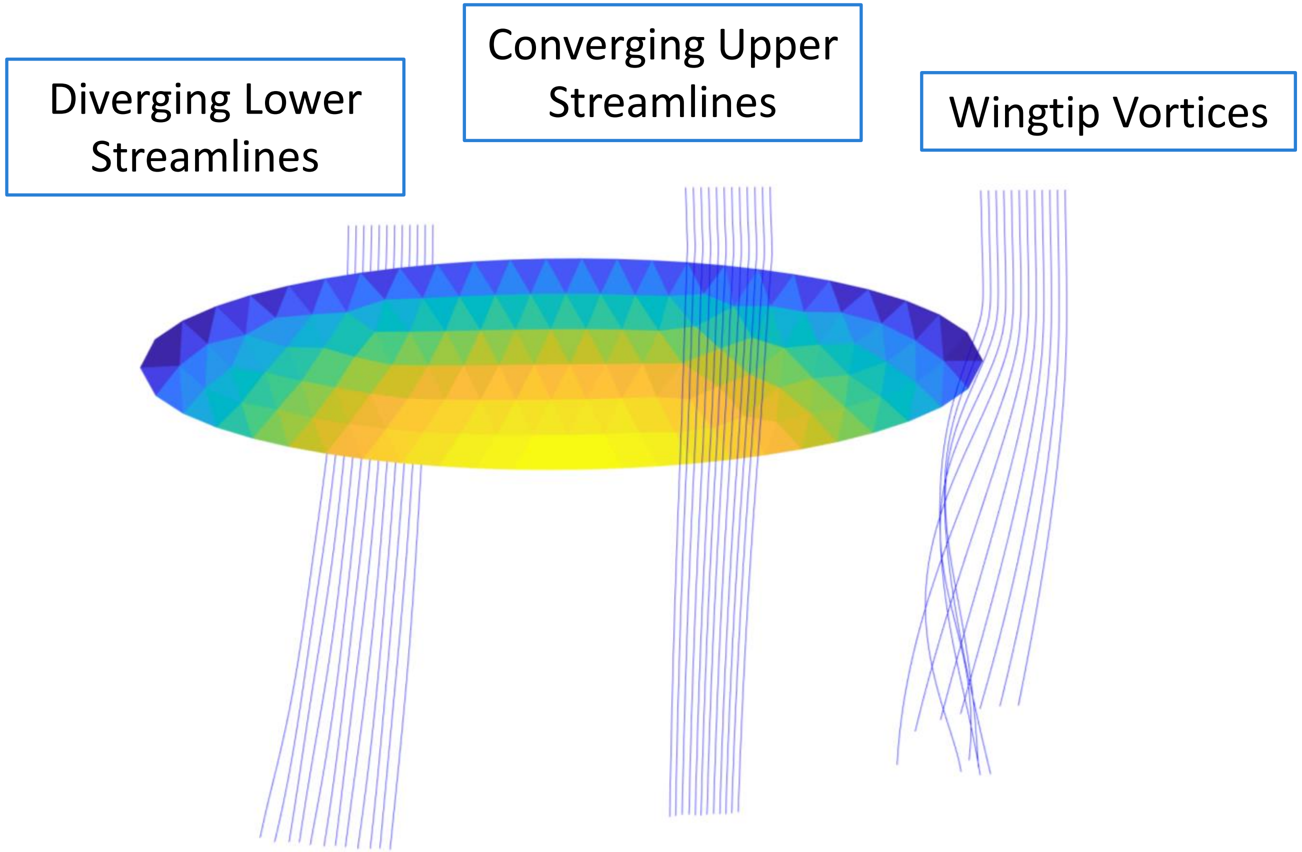
commons.wikimedia.org/wiki/File:VFW_614_ATTAS_Wingtip_vortices_at_start.jpg

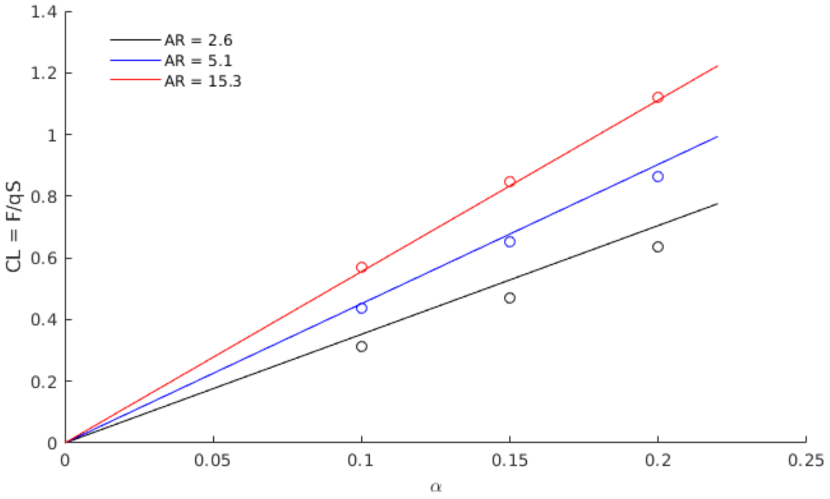
Explicit Kutta Condition

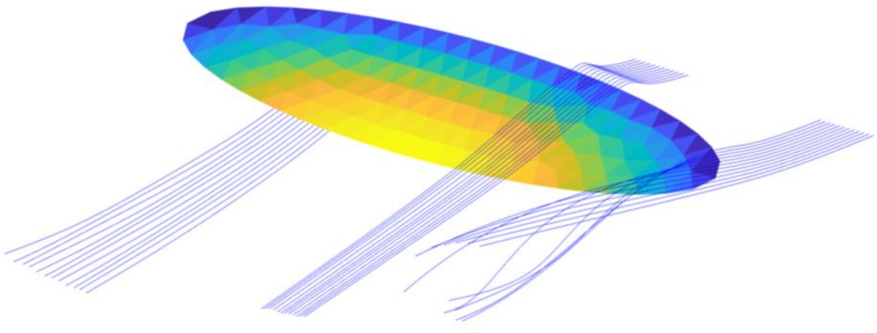
1. Solve
2. Advect
3. Connect
4. Assign Strength



Steady Results: Finite Wings

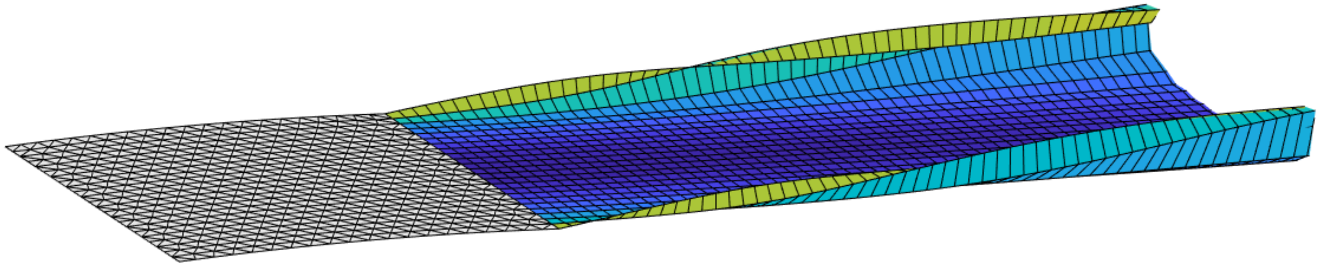




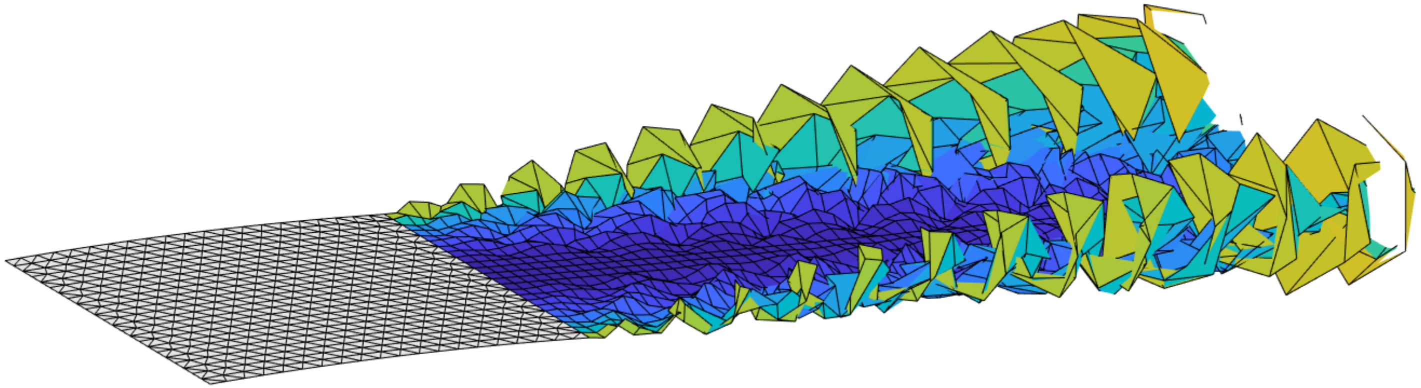


Wingtip Vortices and Wake Advection

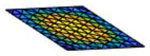
Wake Advection with Forward Euler



Wake Advection with Low Storage RK3

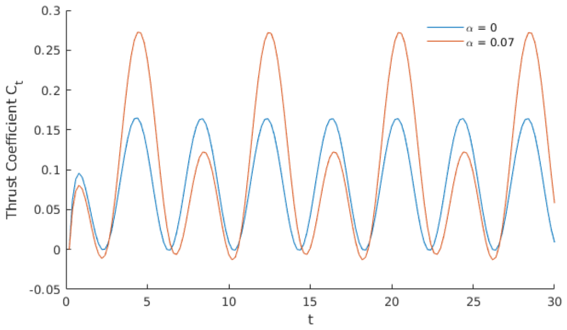


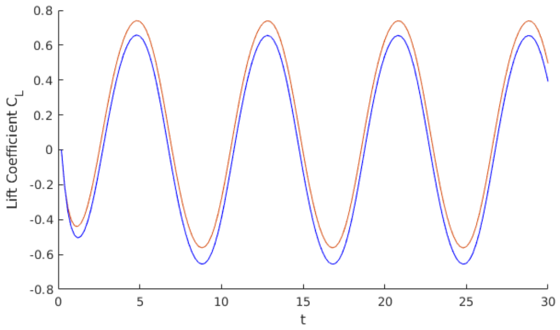
Flapping Foils



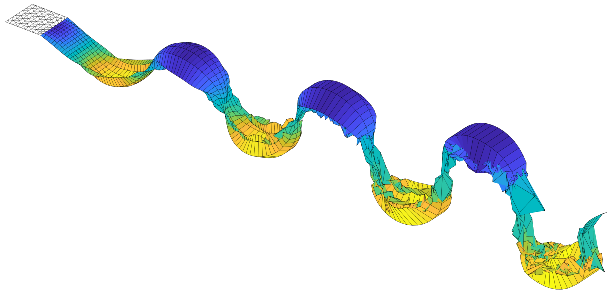
Flapping Foils

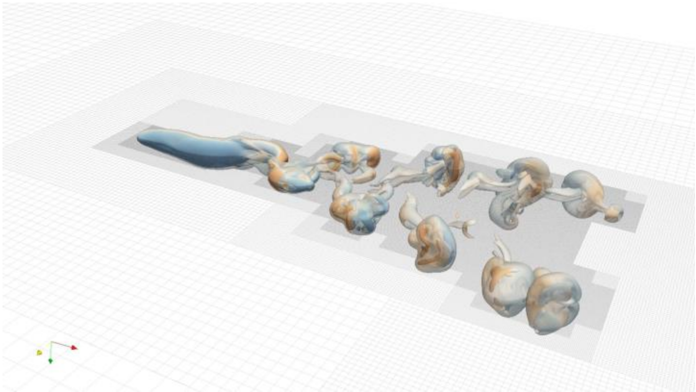
Qualitatively correct Lift/Thrust Patterns



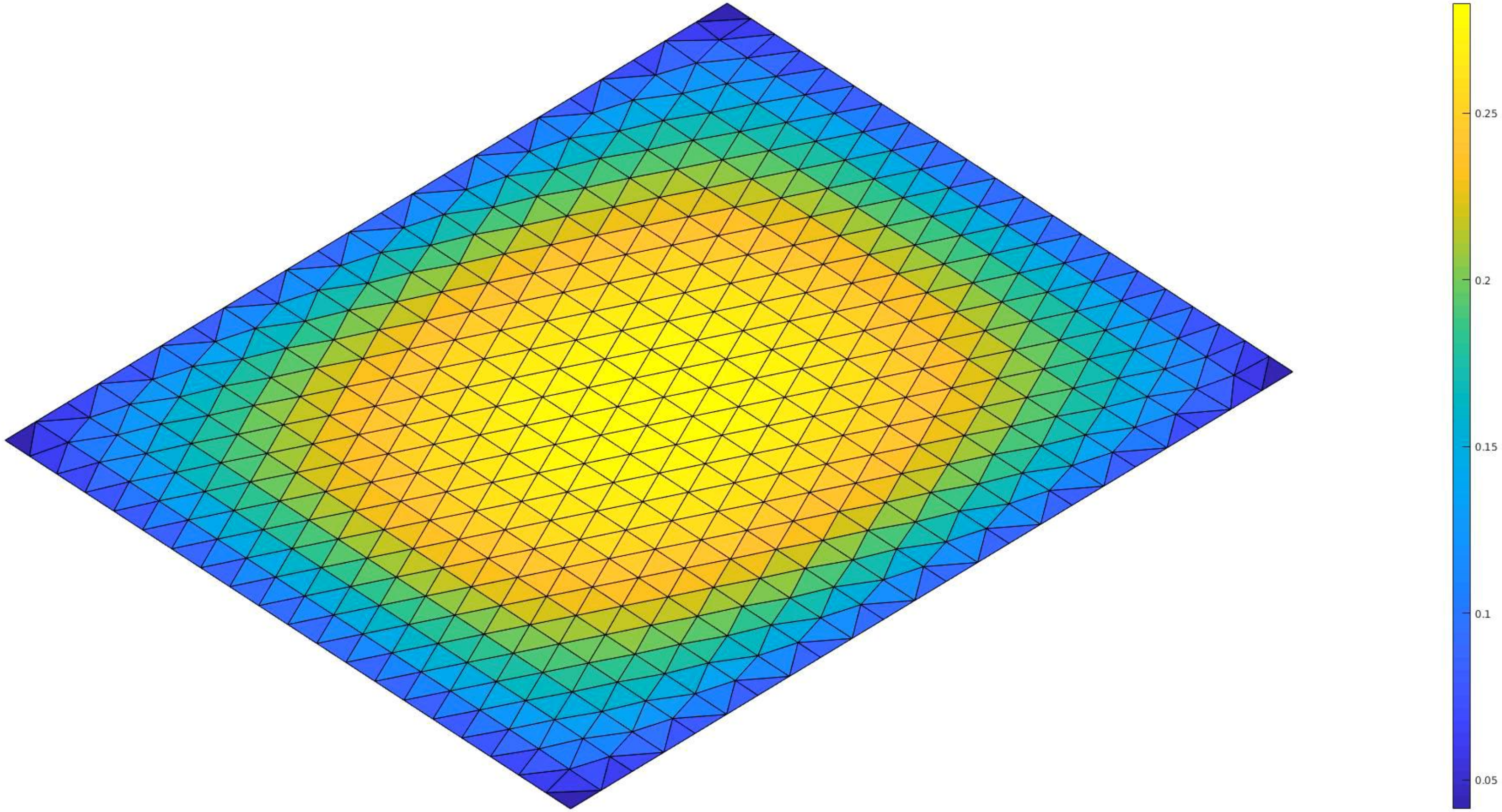


Qualitatively correct wake dynamics





Weakly Coupled FSI



Thank You! Questions?