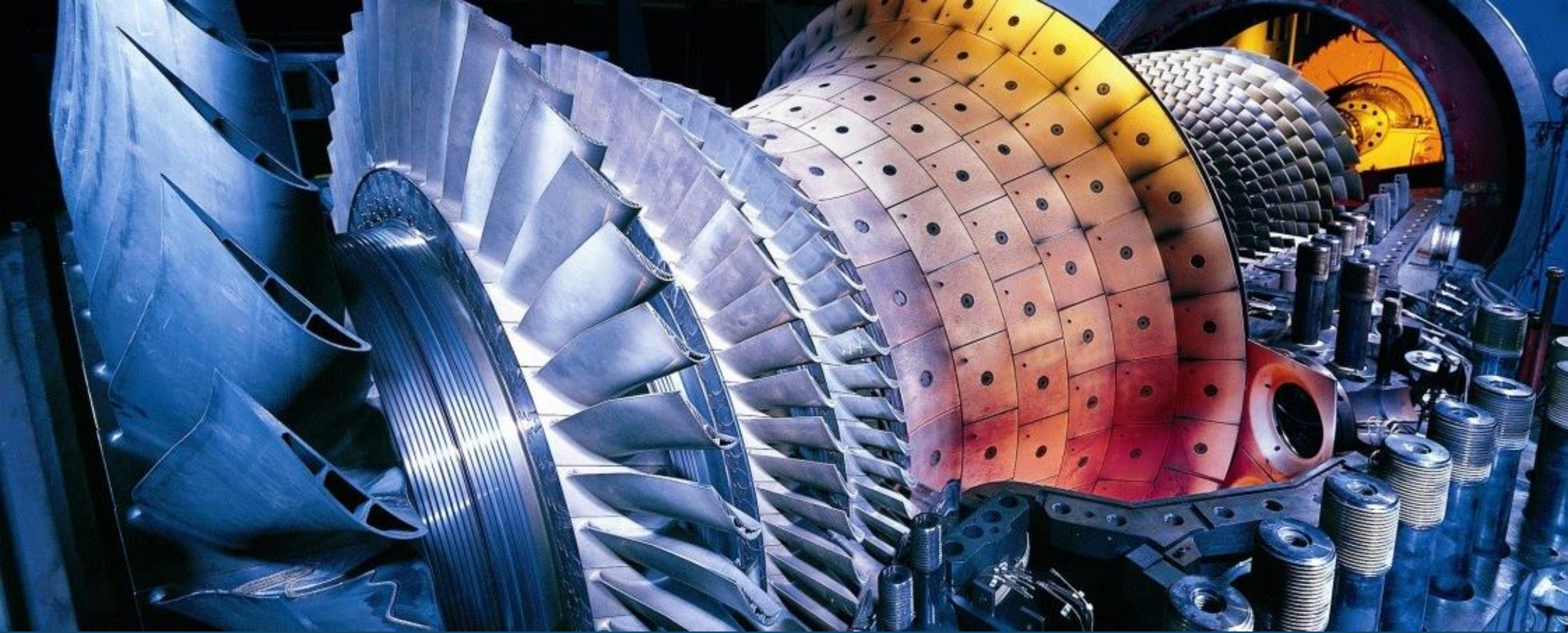
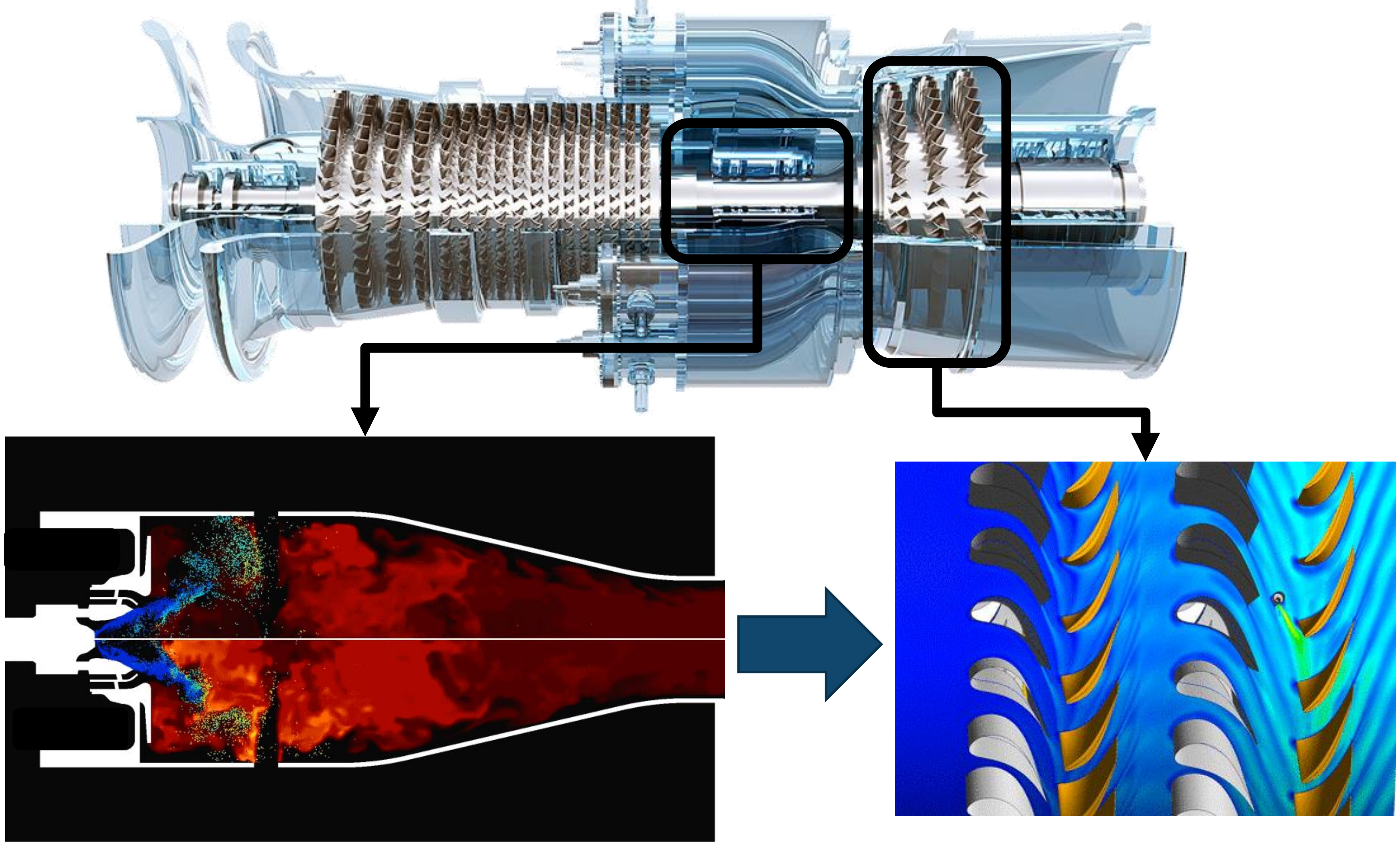


Characterization of Unsteady Boundary Layer Response to Free-Stream Variations

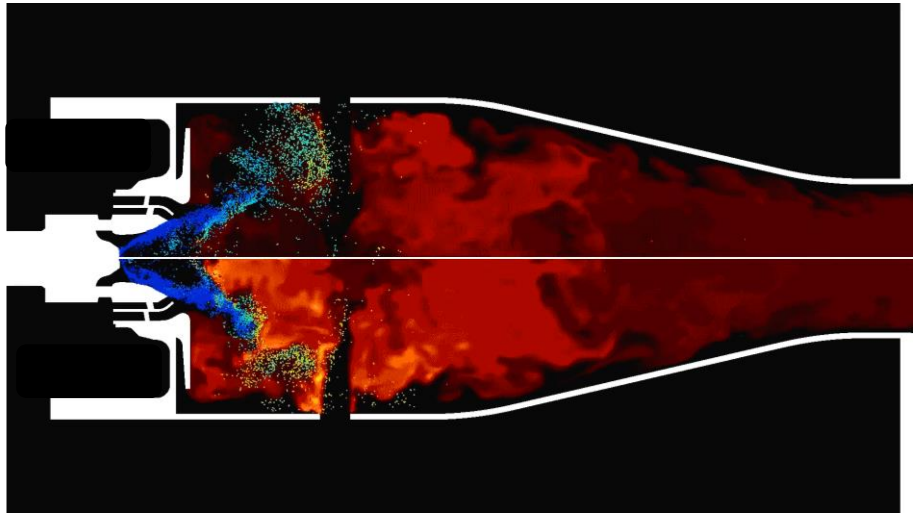
Kanika Gakhar

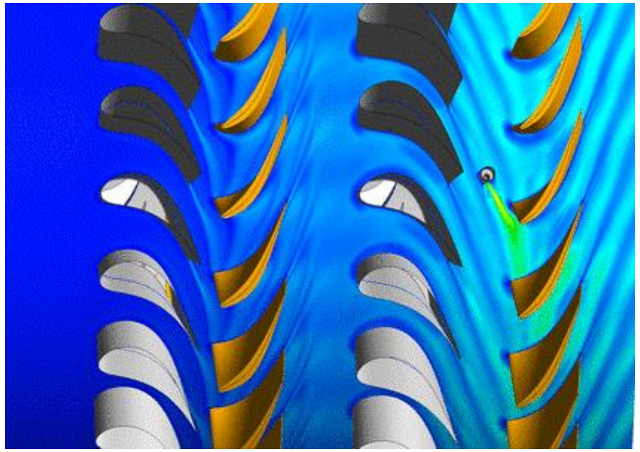
2.29 Final Project

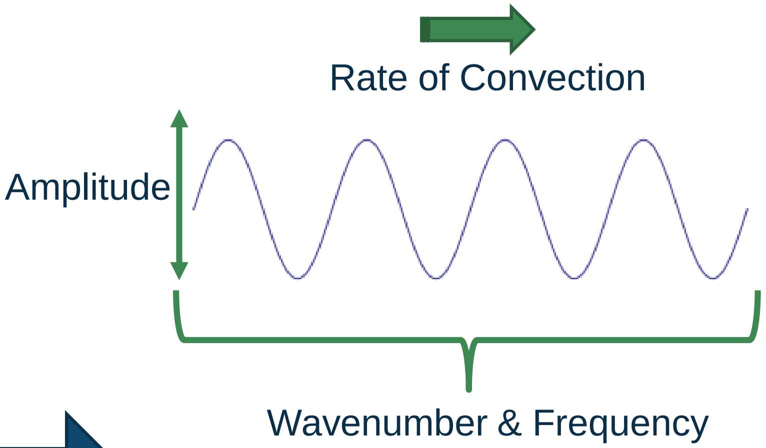
Motivation: Effect of Combustor Turbulence on Boundary Layer Dissipation for High-Pressure Turbine Blades

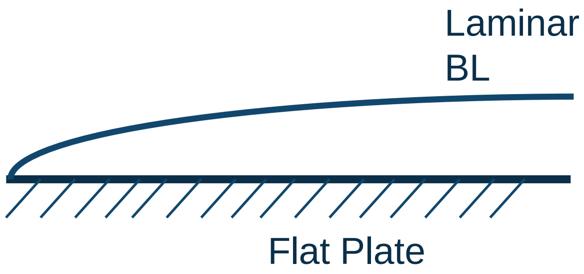


Project Objectives



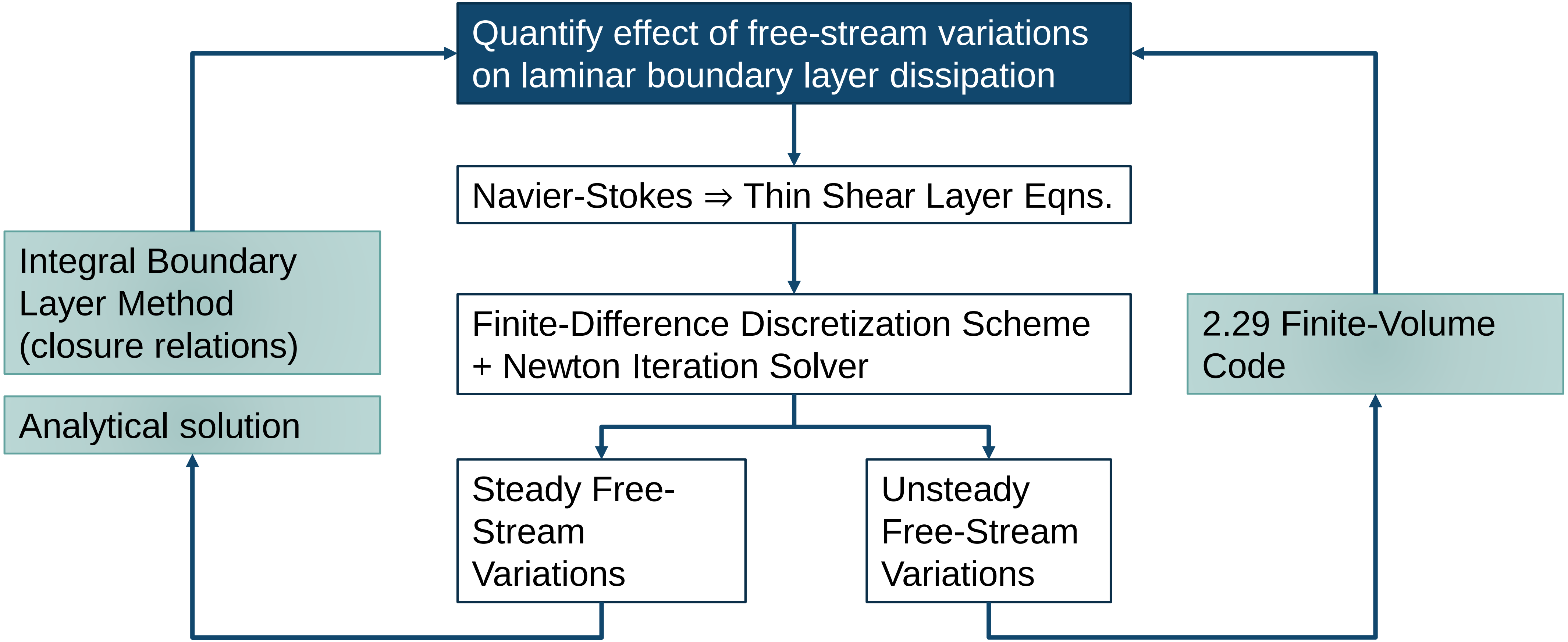






- Model Free Stream Turbulence (FST) as a **periodic time and space varying free-stream velocity**
  - ➢ Define variables to isolate and model effect of various FST parameters on BL response
- Characterize **unsteady** response of **laminar boundary layer** over a semi-infinite **flat plate**

Project Scope & Overview



Boundary Layers: Brief Background

- Most aerodynamic flows have high Reynolds numbers
- Solution of Navier Stokes equations exhibits boundary layers at solid-surface boundaries → magnitude of $V(r)$ rapidly drops from the bulk-flow velocity down to $V = 0$ at the surface
- Origin of boundary layer behavior → highest-derivative term $\nabla^2 V$ being multiplied by small viscosity coefficient, ν

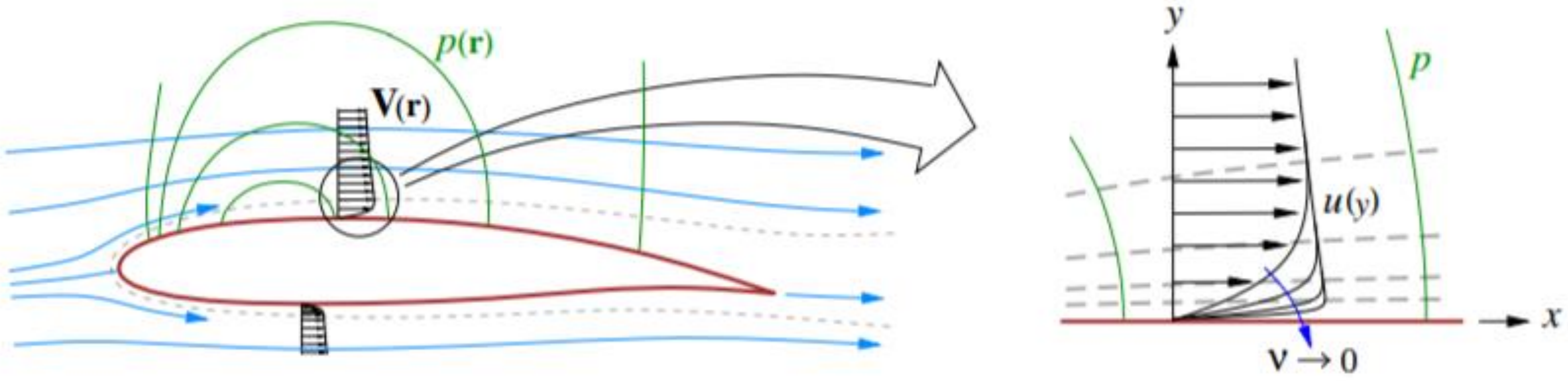


Image Credits: Mark Drela, Aerodynamics of Viscous Fluids

Thin-Shear Layer Equation

- Step 1: Write pressure term in terms of inviscid boundary-layer edge velocity

$$\frac{\partial(\rho u)}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho u \frac{\partial u}{\partial x} = \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2}\right)$$

Thin-Shear Layer Equation

- Step 1: Write pressure term in terms of inviscid boundary-layer edge velocity

$$\frac{\partial(\rho u)}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho u \frac{\partial u}{\partial x} = \boxed{\frac{\partial p}{\partial x}} + \mu\left(\frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial x^2}}\right)$$

$$\left[\frac{\rho_e \partial u_e}{\partial t} + \rho_e u_e \frac{\partial u_e}{\partial x}\right]$$

Thin-Shear Layer Equation

- Step 1: Write pressure term in terms of inviscid boundary-layer edge velocity

$$\frac{\partial(\rho u)}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\left[\frac{\rho_e\partial u_e}{\partial t}+\rho_e u_e\frac{\partial u_e}{\partial x}\right]+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)$$

Thin-Shear Layer Equation

- Step 1: Write pressure term in terms of inviscid boundary-layer edge velocity

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- Step 2: Write u and v in terms of stream-function, ψ, to impose continuity

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

Thin-Shear Layer Equation

- Step 1: Write pressure term in terms of inviscid boundary-layer edge velocity

$$\frac{\partial(\rho u)}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \left[\frac{\rho_e \partial u_e}{\partial t} + \rho u_e \frac{\partial u_e}{\partial x}\right] + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

- Step 2: Write u and v in terms of stream-function, ψ, to impose continuity

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad \Rightarrow \quad \rho u = \frac{\partial \psi}{\partial y}, \qquad \rho v = -\frac{\partial \psi}{\partial x}$$

Thin-Shear Layer Equation

- Step 1: Write pressure term in terms of inviscid boundary-layer edge velocity

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Thin-Shear Layer Equation

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$$\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial \psi}{\partial y}\frac{\partial u}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial u}{\partial y} = \left[\frac{\rho_e \partial u_e}{\partial t} + \rho_e u_e \frac{\partial u_e}{\partial x}\right] + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

Thin-Shear Layer Equation

➢ Step 1: Write pressure term in terms of inviscid boundary-layer edge velocity

$$\frac{\partial(\rho u)}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \left[\frac{\rho_e \partial u_e}{\partial t} + \rho u_e \frac{\partial u_e}{\partial x}\right] + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

➢ Step 2: Write u and v in terms of stream-function, ψ, to impose continuity

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad \Rightarrow \quad \rho u = \frac{\partial \psi}{\partial y}, \qquad \rho v = -\frac{\partial \psi}{\partial x}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial \psi}{\partial y}\frac{\partial u}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial u}{\partial y} = \left[\frac{\rho_e \partial u_e}{\partial t} + \rho_e u_e \frac{\partial u_e}{\partial x}\right] + \frac{\partial \tau}{\partial y}$$

Thin-Shear Layer Equation

- Step 1: Write pressure term in terms of inviscid boundary-layer edge velocity

$$\frac{\partial(\rho u)}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \left[\frac{\rho_e \partial u_e}{\partial t} + \rho u_e \frac{\partial u_e}{\partial x}\right] + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

- Step 2: Write u and v in terms of stream-function, ψ, to impose continuity

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BC at edge: $u = u_e$

BC on solid body: $u = 0, \psi = 0$

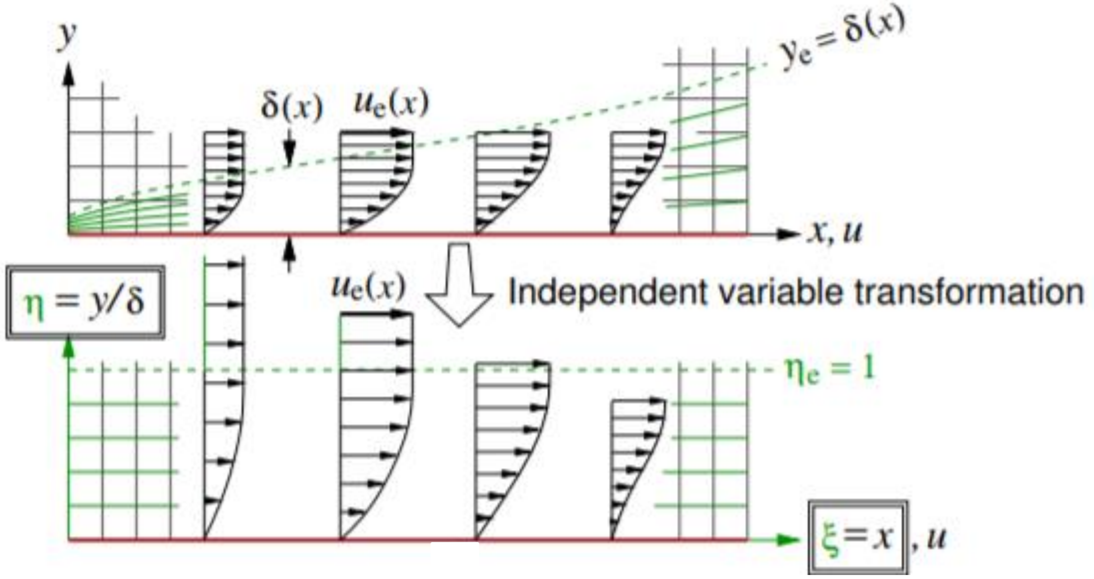
Local Scaling Transformation: Independent Variable Transformation

Non-dimensionalize x w.r.t. arbitrary plate length, L and y w.r.t. boundary layer thickness scale, $\delta(x,t) = \sqrt{\frac{\nu x}{u_e(x,t)}}$:

$$\xi = \frac{x}{L} \qquad \eta = \frac{y}{\delta(x,t)}$$

$$-\frac{1}{\delta}\frac{\partial\psi}{\partial\eta}\frac{\partial\delta}{\partial t} + \frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial\eta}\right) + \frac{1}{L}\frac{\partial\psi}{\partial\eta}\frac{\partial u}{\partial\xi} - \frac{1}{L}\frac{\partial\psi}{\partial\xi}\frac{\partial u}{\partial\eta}$$

$$= \frac{\rho_e u_e \delta}{L}\frac{\partial u_e}{\partial\xi} + \rho_e\delta\frac{\partial u_e}{\partial t} + \frac{\partial\tau}{\partial\eta}$$



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Local Scaling Transformation: Dependent Variable Transformation

Non-dimensionalize u w.r.t. edge velocity, u_e; ψ w.r.t. mass-flow scale, m; τ w.r.t. edge dynamic pressure :

$$F = \frac{\psi}{m} \qquad U = \frac{u}{u_e} \qquad S = \frac{\tau}{\rho u_e^2}\left(\frac{\xi L}{\delta}\right)$$

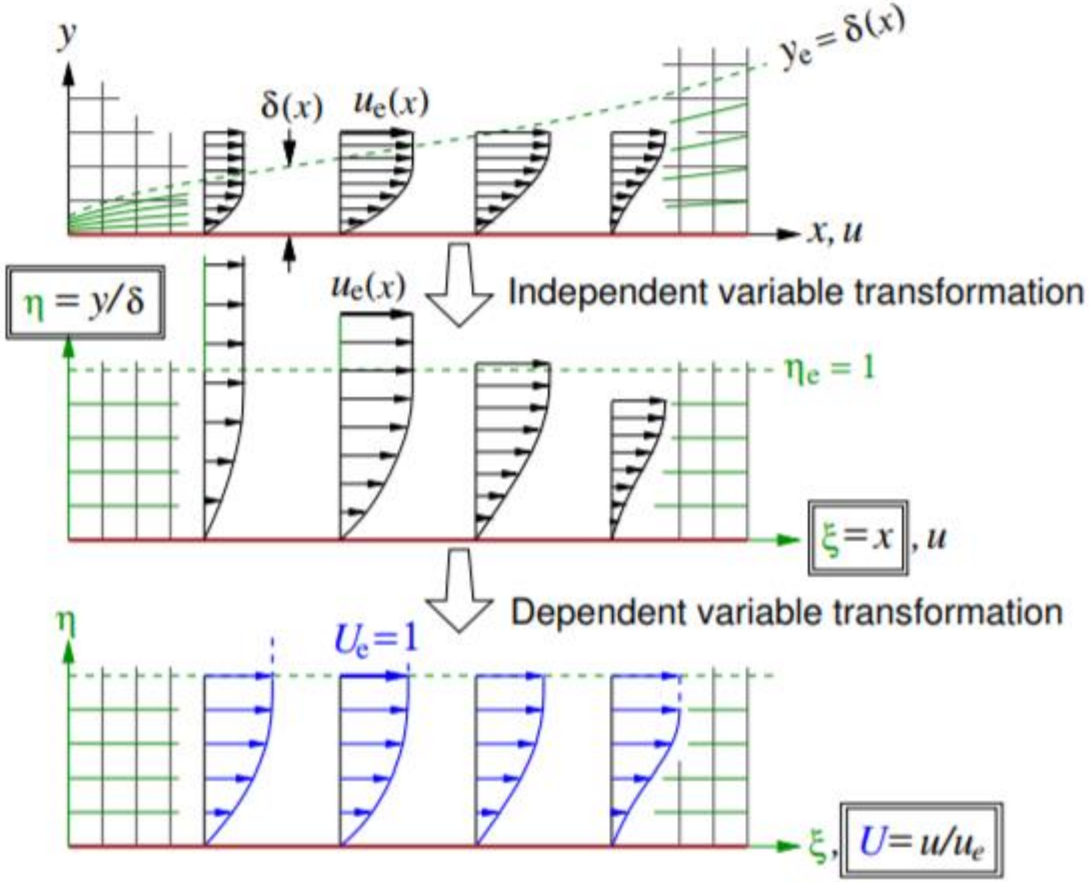
$$m = \rho_e u_e \delta$$

$$\xi\left[\frac{L}{u_e}\left(\frac{\partial U}{\partial t} + \frac{U-1}{u_e}\frac{\partial u_e}{\partial t}\right) + \frac{\partial F}{\partial \eta}\frac{\partial u}{\partial \xi} - \frac{\partial F}{\partial \xi}\frac{\partial U}{\partial \eta}\right]$$

$$= \beta_m F\frac{\partial U}{\partial \eta} + \beta_u\left(1 - U\frac{\partial F}{\partial \eta}\right) + \frac{\partial S}{\partial \eta}$$

$$\beta_u = \frac{\xi}{u_e}\frac{du_e}{d\xi}$$

$$\beta_m = \frac{dln(m)}{dln(\xi)}$$



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Local Scaling Transformation Equations

Three non-linear, first-order PDEs:

- $U = \frac{\partial F}{\partial \eta}$
- $S = \frac{\nu \xi L}{u_e \delta^2} \frac{\partial U}{\partial \eta}$
- $\xi \left[\frac{L}{u_e} \left(\frac{\partial U}{\partial t} + \frac{U-1}{u_e} \frac{\partial u_e}{\partial t} \right) + \frac{\partial F}{\partial \eta} \frac{\partial u}{\partial \xi} - \frac{\partial F}{\partial \xi} \frac{\partial U}{\partial \eta} \right] = \beta_m F \frac{\partial U}{\partial \eta} + \beta_u \left(1 - U \frac{\partial F}{\partial \eta} \right) + \frac{\partial S}{\partial \eta}$

Three BCs:

BC at edge: $U(\eta = 1) = 1$

BC at body: $U(\eta = 0) = 0$

$F(\eta = 0) = 0$

Discretized Equations: Finite-Difference Scheme

Three non-linear, first-order PDEs:

- $U = \dfrac{\partial F}{\partial \eta}$

- $S = \dfrac{\nu \xi L}{u_e \delta^2} \dfrac{\partial U}{\partial \eta}$

- $$\xi \left[\frac{L}{u_e} \left(\frac{\partial U}{\partial t} + \frac{U-1}{u_e} \frac{\partial u_e}{\partial t} \right) + \frac{\partial F}{\partial \eta} \frac{\partial u}{\partial \xi} - \frac{\partial F}{\partial \xi} \frac{\partial U}{\partial \eta} \right] = \beta_m F \frac{\partial U}{\partial \eta} + \beta_u \left(1 - U \frac{\partial F}{\partial \eta} \right) + \frac{\partial S}{\partial \eta}$$

Three non-linear residual equations:

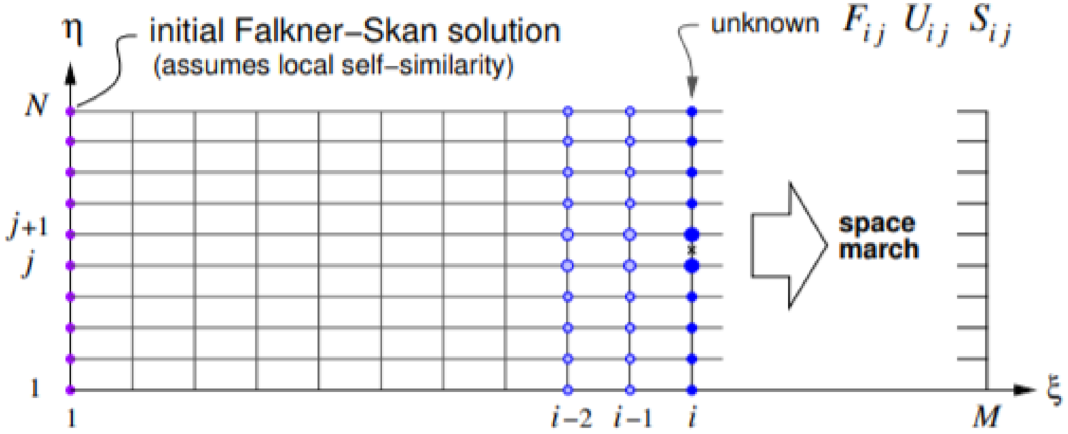
- $R_{F_j} = F^k_{i,j+1} - F^k_{i,j} - \frac{1}{2}\left(U^k_{i,j+1} + U^k_{i,j}\right)\Delta\eta_j = 0$

- $R_{U_j} = U^k_{i,j+1} - U^k_{i,j} - \dfrac{u^k_{e_i}\left(\delta^k_i\right)^2}{\nu \xi_i L} \dfrac{1}{2}\left(S^k_{i,j+1} + S^k_{i,j}\right)\Delta\eta_j = 0$

- $$\begin{aligned} R_{S_j} = {} & S^k_{i,j+1} - S^k_{i,j} + \beta^k_{m_i} \tfrac{1}{2} (F^k_{i,j+1} + F^k_{i,j})(U^k_{i,j+1} - U^k_{i,j}) \\ & + \beta^k_{u_i} \Big[\Delta\eta_j - \tfrac{1}{2} (F^k_{i,j+1} - F^k_{i,j})(U^k_{i,j+1} + U^k_{i,j}) \Big] + \frac{\xi_i}{\Delta\xi} (\tfrac{1}{2} (F^k_{i,j+1} - \\ & F^k_{i-1,j+1} + F^k_{i,j} - F^k_{i-1,j})(U^k_{i,j+1} - U^k_{i,j}) - \tfrac{1}{2} (U^k_{i,j+1} - U^k_{i-1,j+1} + \\ & U^k_{i,j} - U^k_{i-1,j})(F^k_{i,j+1} - F^k_{i,j}) - \frac{L \xi_i \Delta\eta_j}{\left(u^k_{e_i}\right)^2 \Delta t} \Big[\frac{u^k_{e_i}}{2} (U^k_{i,j+1} - U^{k-1}_{i,j+1} + \\ & U^k_{i,j} - U^{k-1}_{i,j}) + \Big\{ \tfrac{1}{2} (U^k_{i,j+1} + U^k_{i,j}) - 1 \Big\} (u^k_{e_i} - u^k_{e_i}) \Big] = 0 \end{aligned}$$

- t: Fully implicit, 1st order Backward Euler
- η: 1st order Forward FD, Trapezoidal Integral
- ξ: 1st order Backward FD, Trapezoidal Integral

Discretized Equations: Finite-Difference Scheme



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Three non-linear residual equations:

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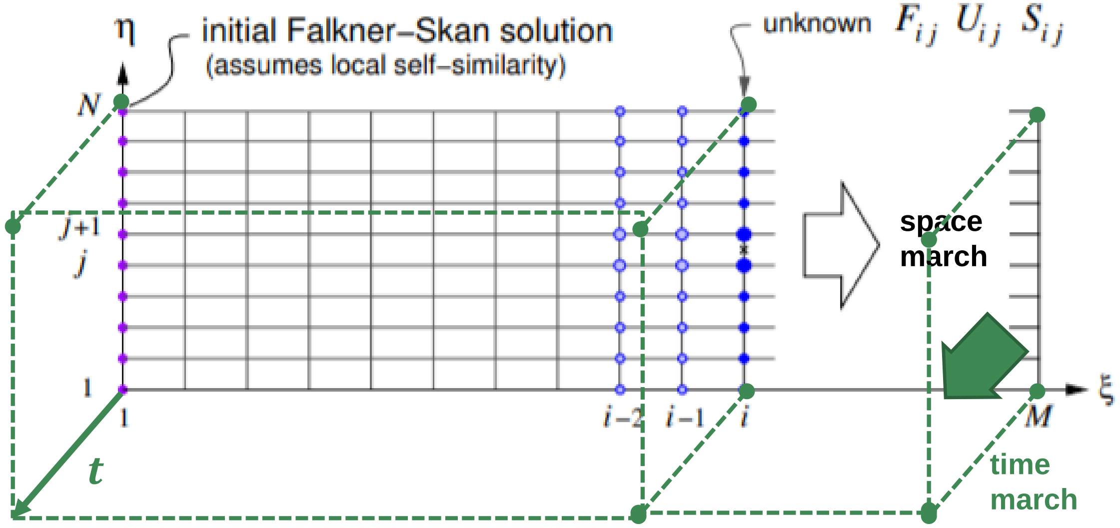
- $R_{U_j} = U^k_{i,j+1} - U^k_{i,j} - \frac{u^k_{e_i}\left(\delta^k_i\right)^2}{\nu\xi_i L}\frac{1}{2}\left(S^k_{i,j+1} + S^k_{i,j}\right)\Delta\eta_j = 0$
- $R_{S_j} = S^k_{i,j+1} - S^k_{i,j} + \beta^k_{m_i}\frac{1}{2}\left(F^k_{i,j+1} + F^k_{i,j}\right)\left(U^k_{i,j+1} - U^k_{i,j}\right) + \beta^k_{u_i}\left[\Delta\eta_j - \frac{1}{2}\left(F^k_{i,j+1} - F^k_{i,j}\right)\left(U^k_{i,j+1} + U^k_{i,j}\right)\right] + \frac{\xi_i}{\Delta\xi}\left(\frac{1}{2}\left(F^k_{i,j+1} - F^k_{i-1,j+1} + F^k_{i,j} - F^k_{i-1,j}\right)\left(U^k_{i,j+1} - U^k_{i,j}\right) - \frac{1}{2}\left(U^k_{i,j+1} - U^k_{i-1,j+1} + U^k_{i,j} - U^k_{i-1,j}\right)\left(F^k_{i,j+1} - F^k_{i,j}\right) - \frac{L\xi_i\Delta\eta_j}{\left(u^k_{e_i}\right)^2\Delta t}\left[\frac{u^k_{e_i}}{2}\left(U^k_{i,j+1} - U^{k-1}_{i,j+1} + U^k_{i,j} - U^{k-1}_{i,j}\right) + \left\{\frac{1}{2}\left(U^k_{i,j+1} + U^k_{i,j}\right) - 1\right\}\left(u^k_{e_i} - u^k_{e_i}\right)\right] = 0$

Discretized Equations: Finite-Difference Scheme



- t: Fully implicit, 1st order Backward Euler
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Three non-linear residual equations:

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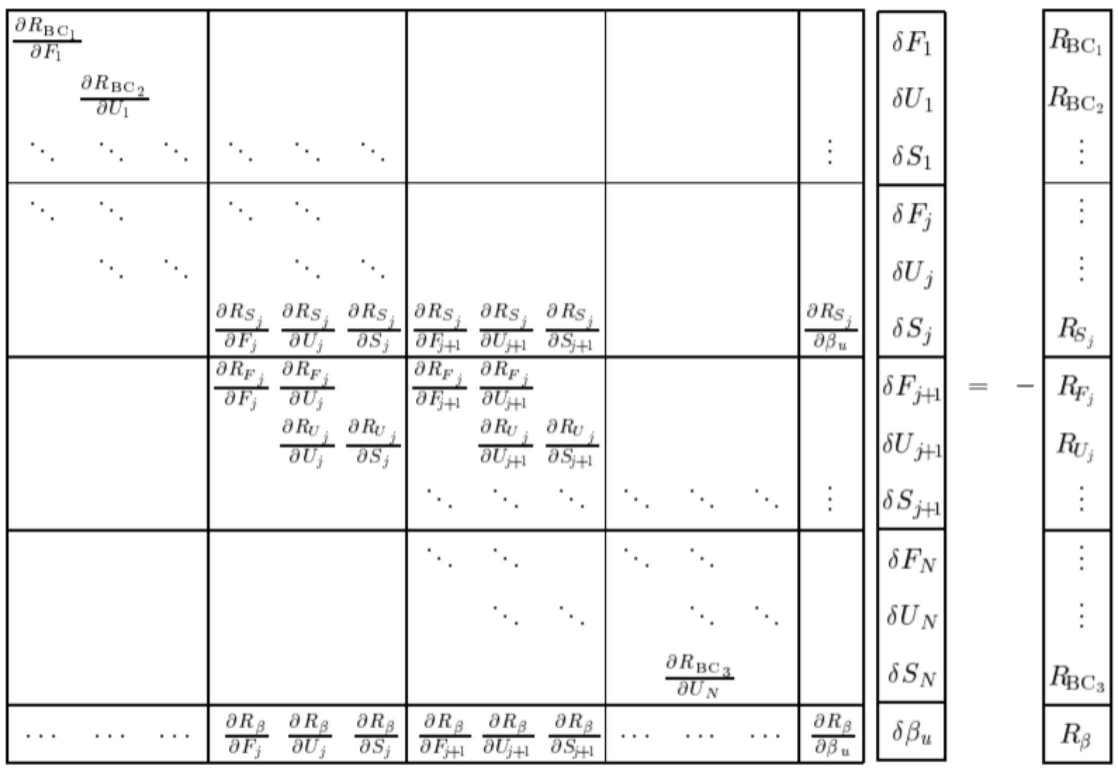
- $R_{U_j} = U^k_{i,j+1} - U^k_{i,j} - \frac{u^k_{e_i}\left(\delta^k_i\right)^2}{\nu\xi_i L}\frac{1}{2}\left(S^k_{i,j+1} + S^k_{i,j}\right)\Delta\eta_j = 0$
- $R_{S_j} = S^k_{i,j+1} - S^k_{i,j} + \beta^k_{m_i}\frac{1}{2}\left(F^k_{i,j+1} + F^k_{i,j}\right)\left(U^k_{i,j+1} - U^k_{i,j}\right) + \beta^k_{u_i}\left[\Delta\eta_j - \frac{1}{2}\left(F^k_{i,j+1} - F^k_{i,j}\right)\left(U^k_{i,j+1} + U^k_{i,j}\right)\right] + \frac{\xi_i}{\Delta\xi}\Big(\frac{1}{2}\big(F^k_{i,j+1} - F^k_{i-1,j+1} + F^k_{i,j} - F^k_{i-1,j}\big)\left(U^k_{i,j+1} - U^k_{i,j}\right) - \frac{1}{2}\big(U^k_{i,j+1} - U^k_{i-1,j+1} + U^k_{i,j} - U^k_{i-1,j}\big)\left(F^k_{i,j+1} - F^k_{i,j}\right) - \frac{L\xi_i\Delta\eta_j}{\left(u^k_{e_i}\right)^2\Delta t}\left[\frac{u^k_{e_i}}{2}\left(U^k_{i,j+1} - U^{k-1}_{i,j+1} + U^k_{i,j} - U^{k-1}_{i,j}\right) + \left\{\frac{1}{2}\left(U^k_{i,j+1} + U^k_{i,j}\right) - 1\right\}\left(u^k_{e_i} - u^k_{e_i}\right)\right] = 0$

Solving Non-Linear Residual Equations: Newton Iteration with Underrelaxation



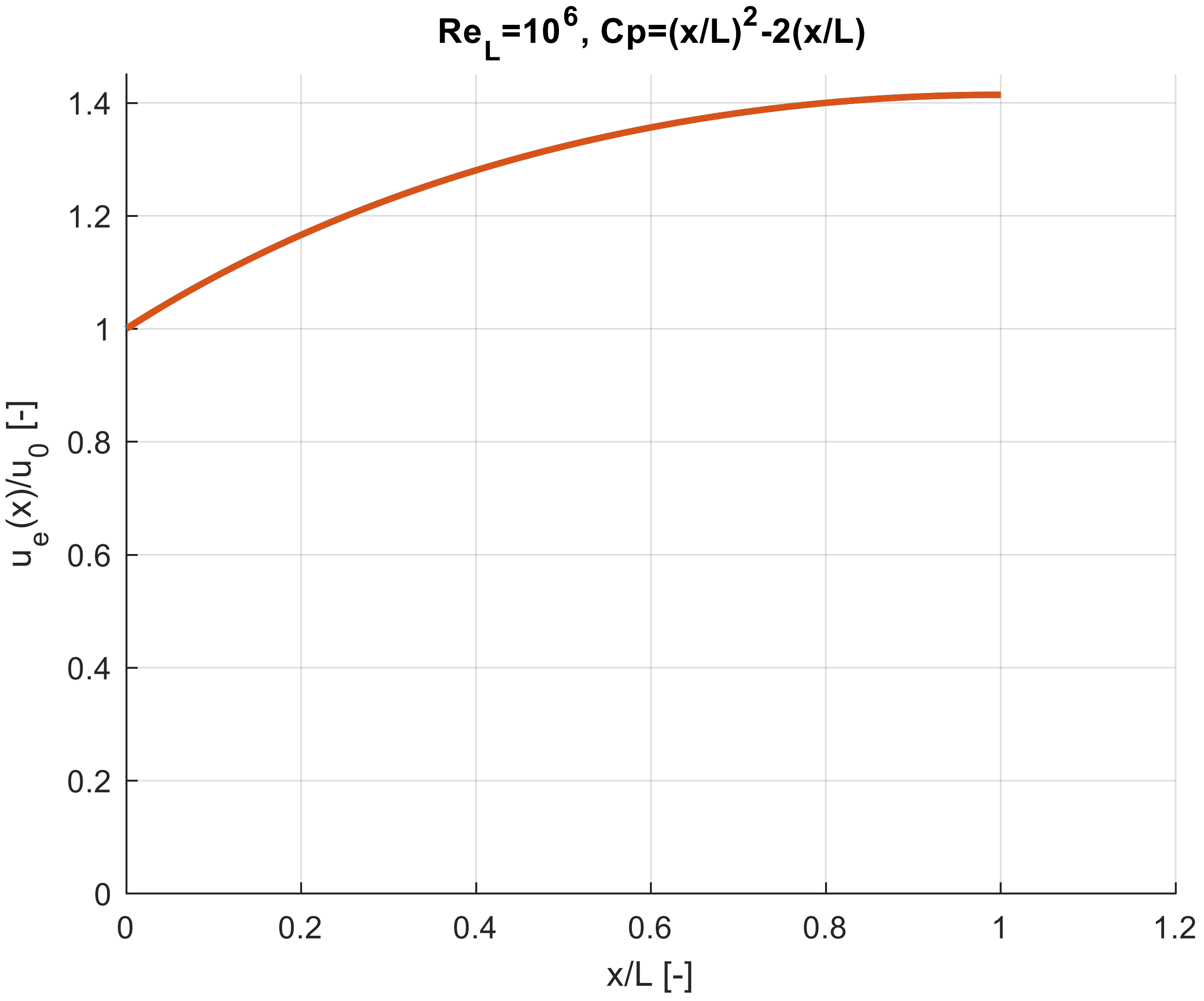
- **Sparse, banded matrix** → can reduce operation count from $O[N^3]$ to $O[N]$ by using special sparse-matrix methods, such as banded or block-tridiagonal solvers that exploit zeros

- **Underrelaxation** → prevent possible divergence: $\omega \leq 1$

$$F_j^{n+1} = F_j^n + \omega\, \delta F_j$$
$$U_j^{n+1} = U_j^n + \omega\, \delta F_j$$
$$S_j^{n+1} = S_j^n + \omega\, \delta F_j$$
$$\beta_{u\,j}^{\;n+1} = \beta_{u\,j}^{\;n} + \omega\, \delta F_j$$

- **Convergence** → $\max_j\left(\left|\delta F_j\right|, \left|\delta U_j\right|, \left|\delta S_j\right|\right) < \epsilon$

Results: Validation for Steady Case



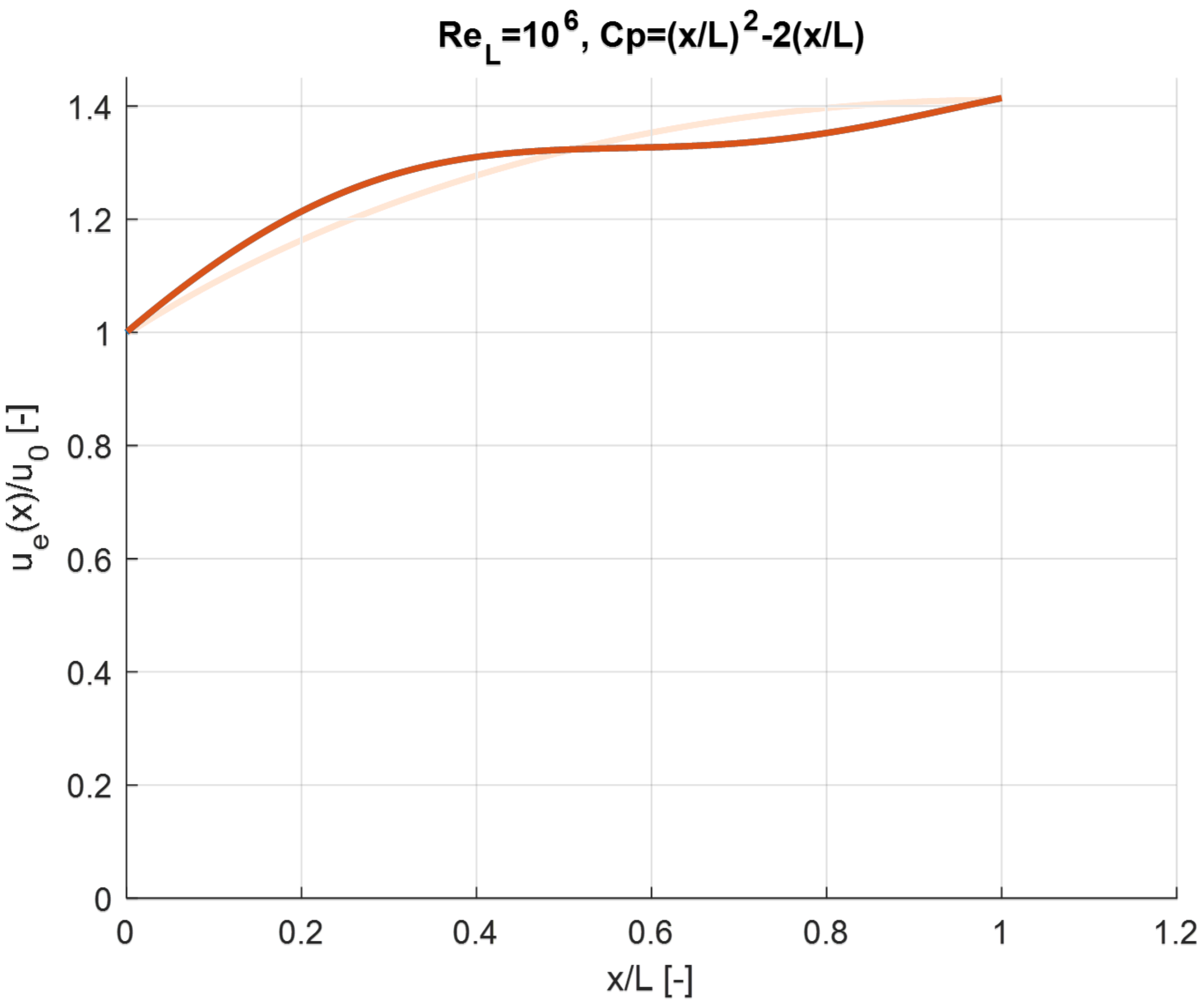
Test case#1:

- 2D steady, laminar, flat-plate boundary layer
- Favorable pressure gradient:

$$C_p = \left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)$$

- $Re_L = 10^6$

Results: Validation for Steady Case *with Spatial Variations*



Test case#1:

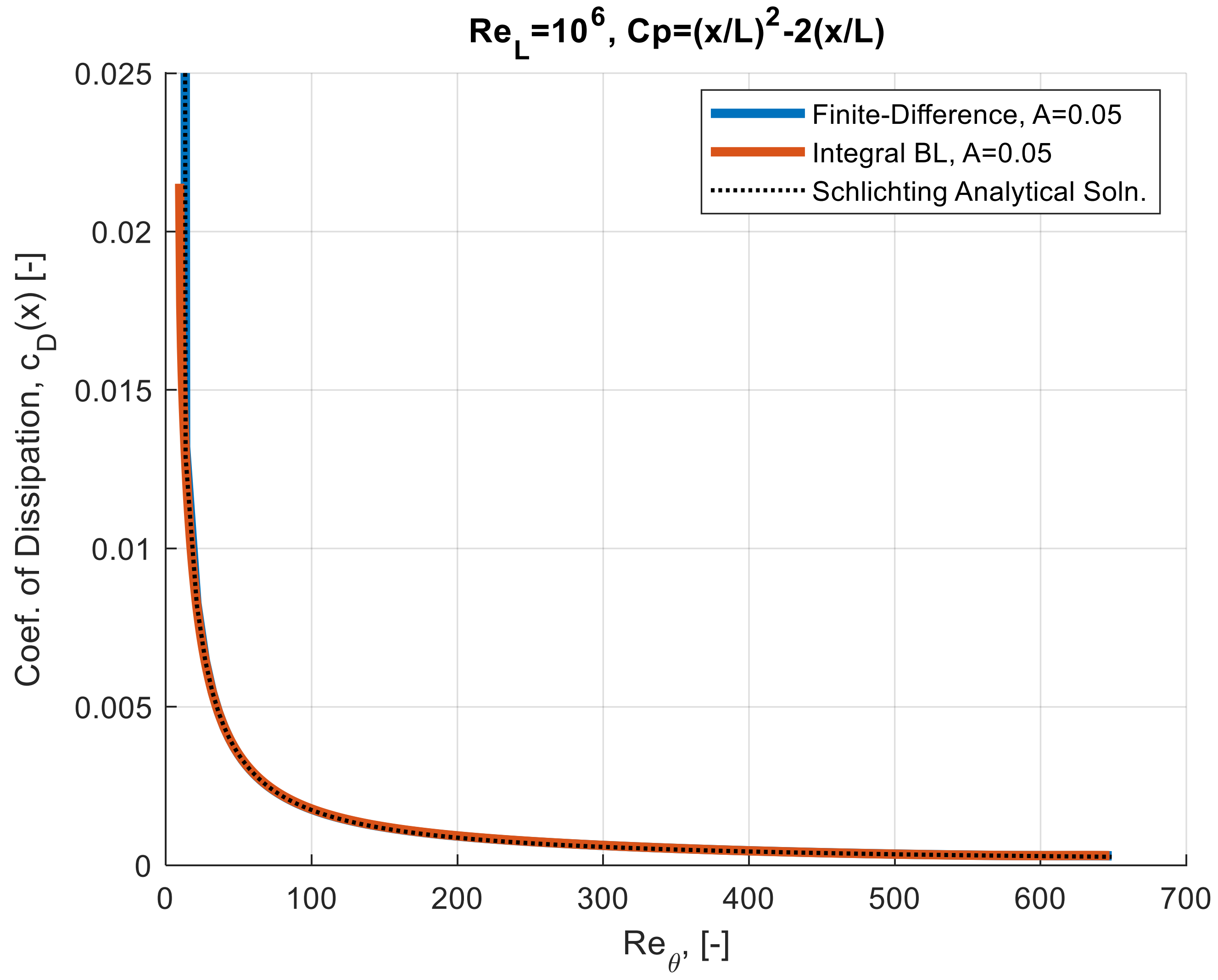
- 2D steady, laminar, flat-plate boundary layer
- Favorable pressure gradient:

$$C_p = \left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)$$

- $Re_L = 10^6$
- Added variations in free-stream velocity with amplitude, $A = 0.05$, such that:

$$\frac{u_e}{u_0} = \sqrt{1 - C_p} + A * \sin\left(2\pi\left(\frac{x}{L}\right)\left(\frac{L}{\lambda}\right)\right)$$

Results: Validation for Steady Case with Spatial Variations



Test case#1:

Compared solutions obtained from:

1. Finite-difference Boundary Layer Code
2. Integral Boundary Layer Method using Drela's closure relations
3. Schlichting's Analytical solution
   - C_d as a function of Re_θ and Pohlausen pressure gradient parameter, λ

- Solutions for Finite-Difference & Integral Boundary Layer Codes in Agreement with Analytical Soln.

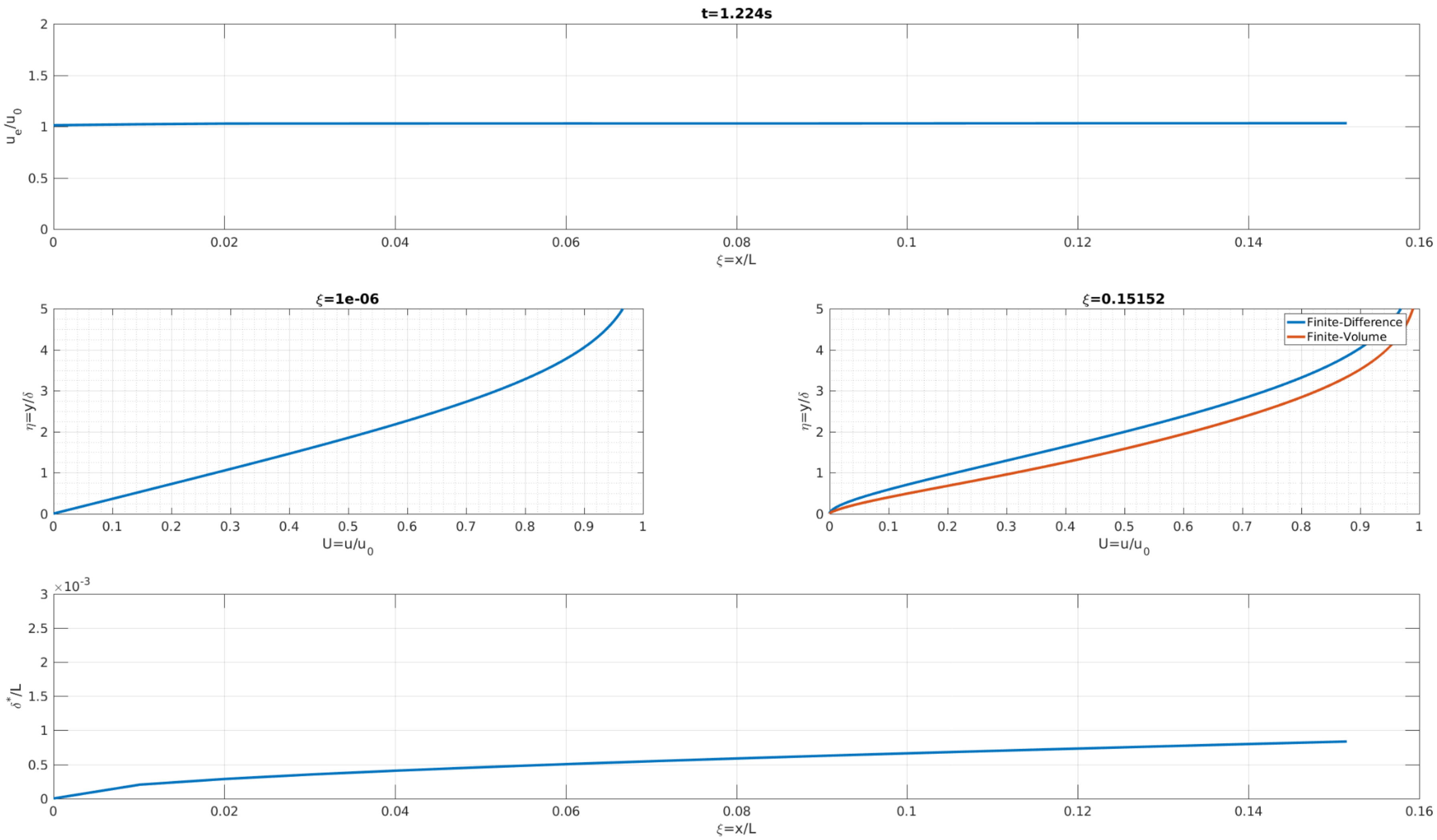
Results: Validation for Uniform Unsteady Case

Test case#2:

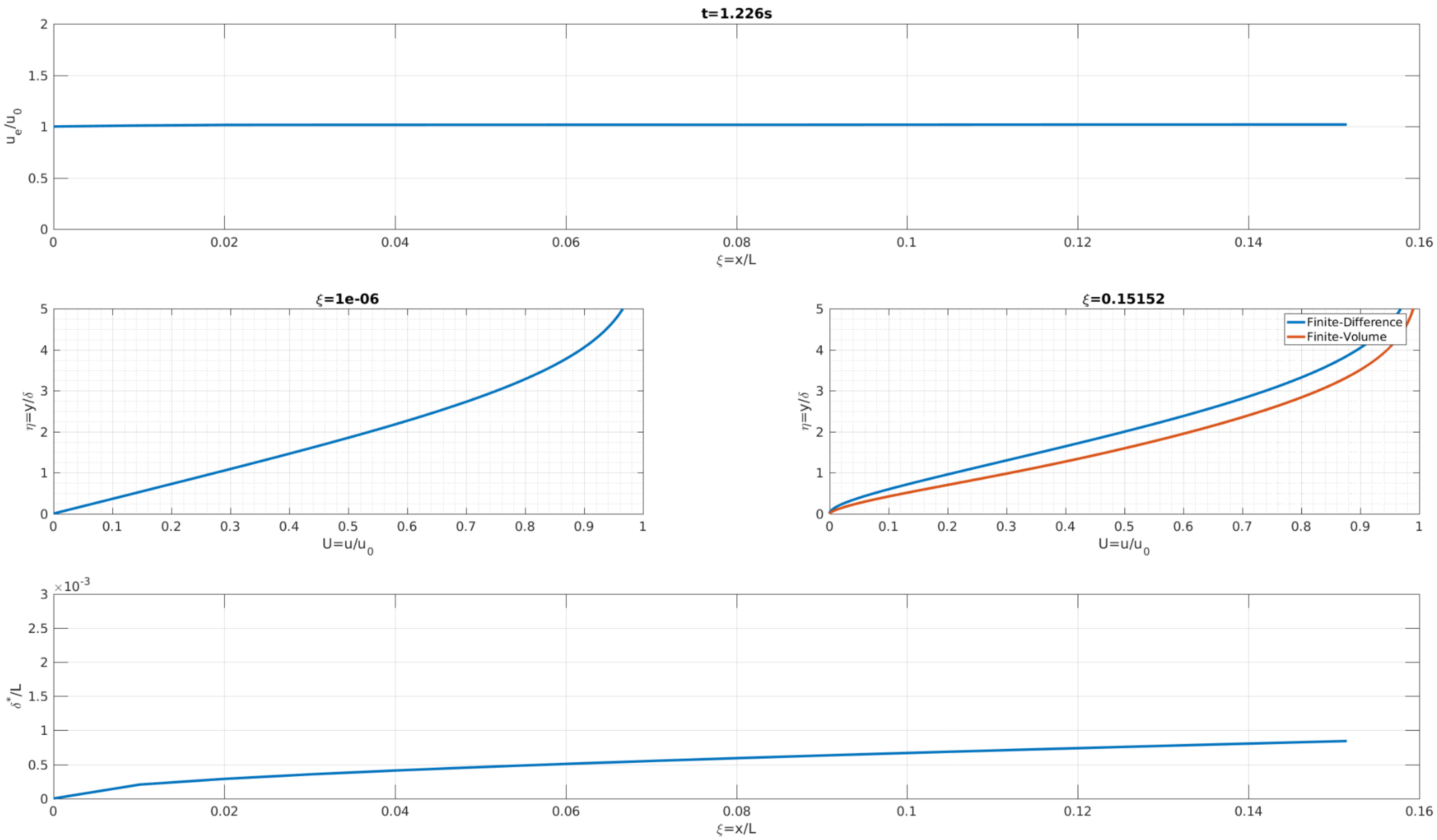
- 2D unsteady, laminar, flat-plate boundary layer
- Uniform pressure distribution with no spatial variation
- $Re_L = 10^6$
- Oscillating inlet conditions, such that:

$$\left.\frac{u}{u_0}\right|_{inlet} = 1 + A * \cos(\omega t)$$

Results: Validation for Uniform Unsteady Case



Results: Validation for Uniform Unsteady Case



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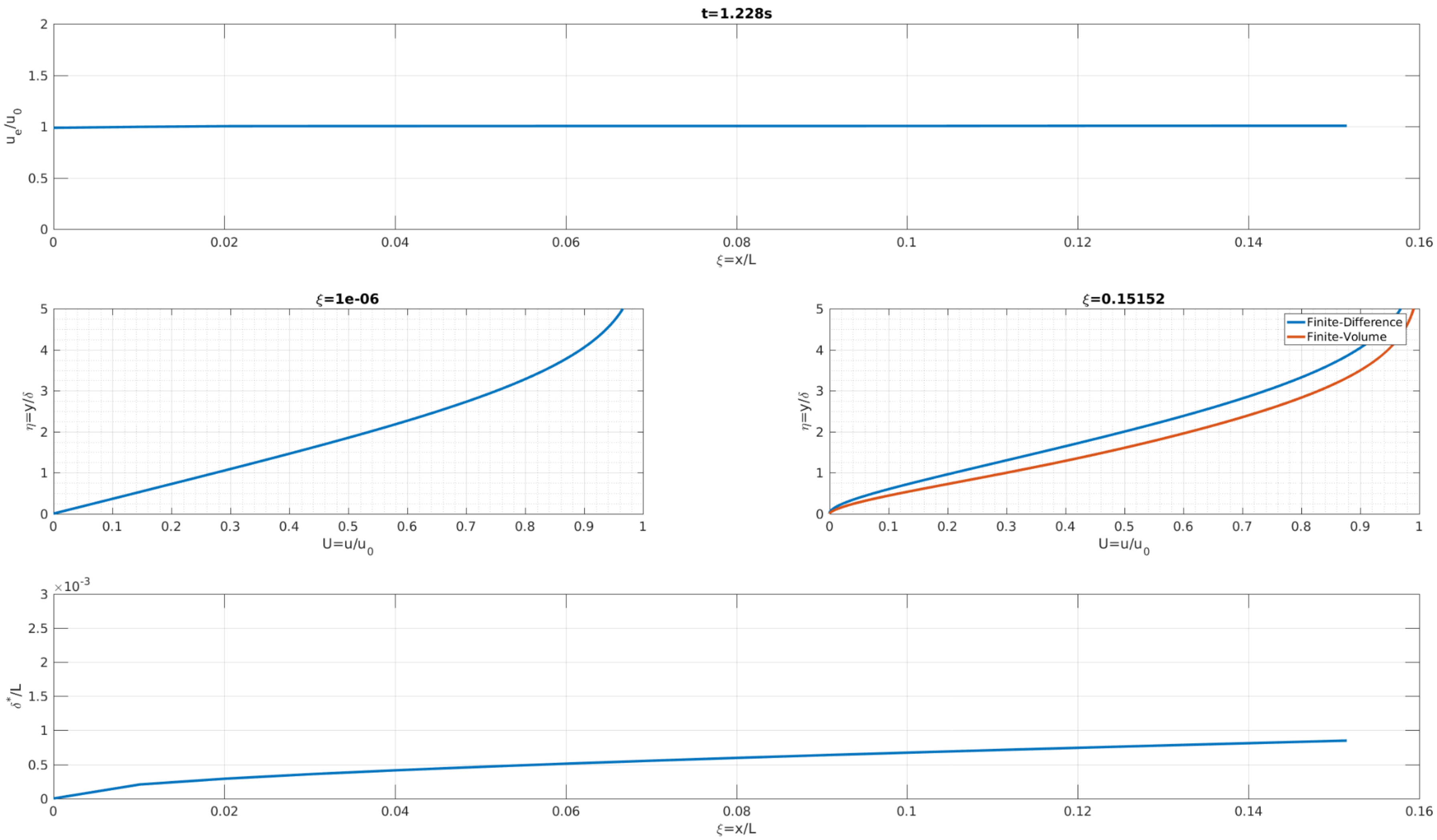

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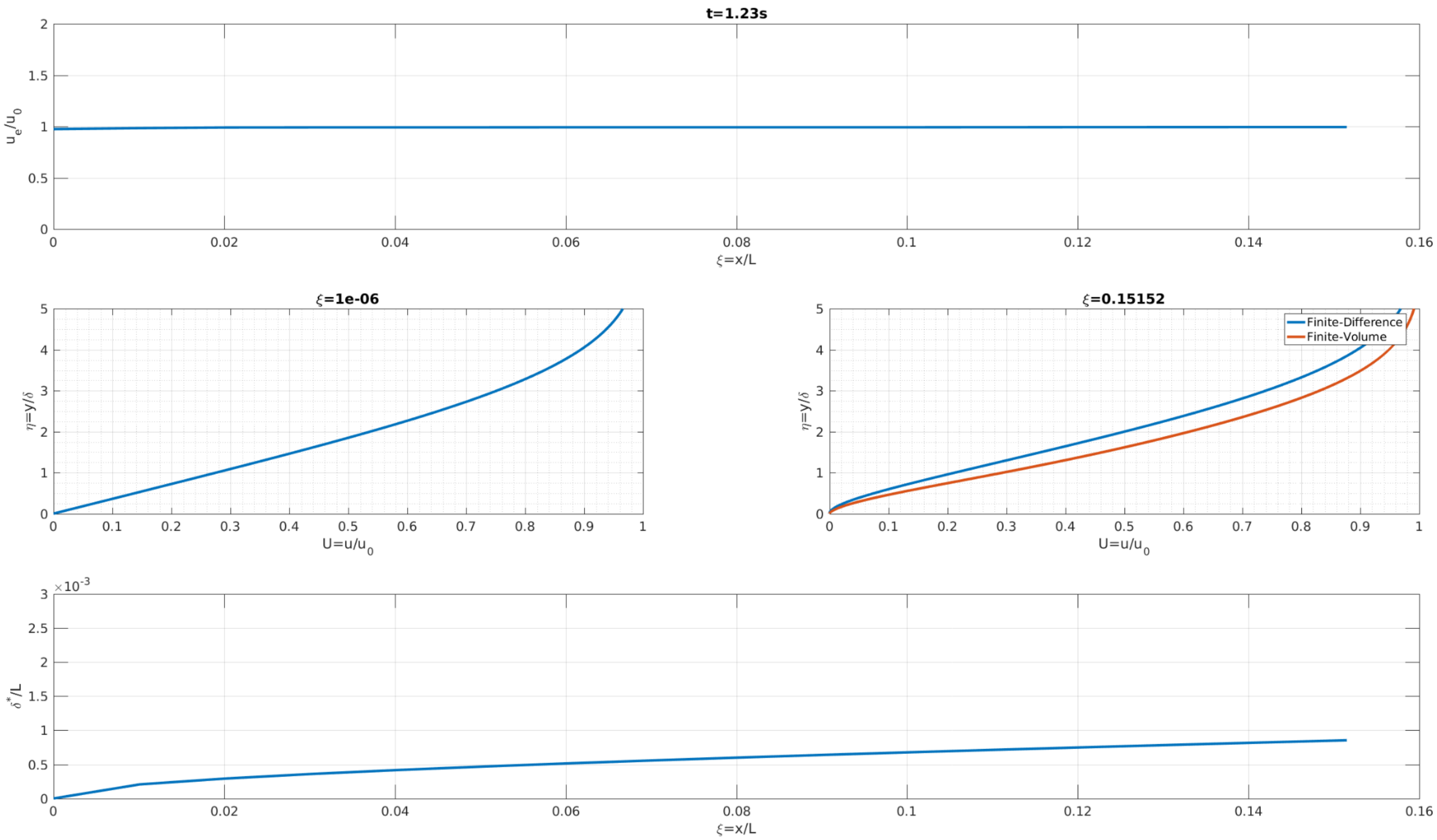

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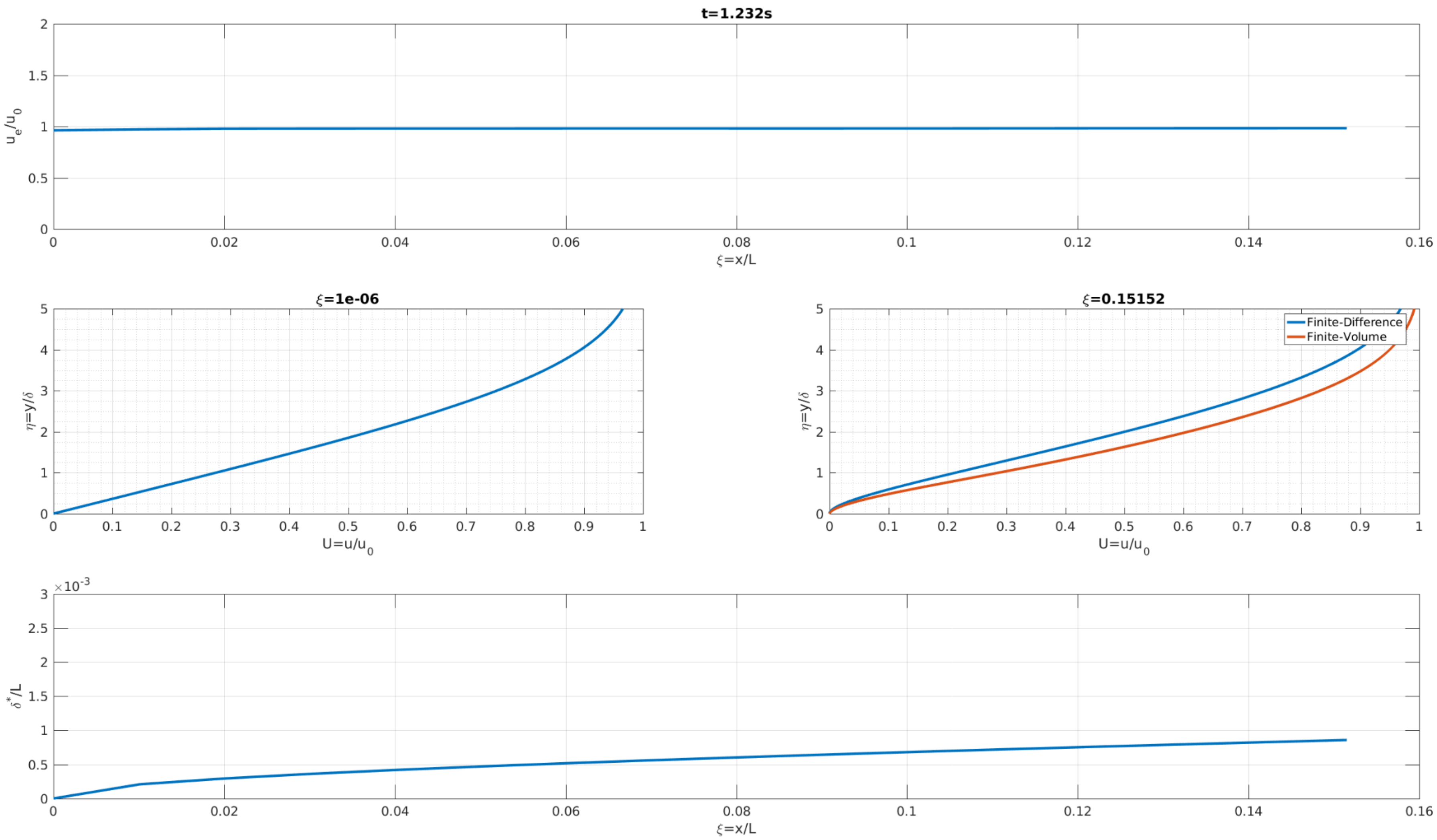

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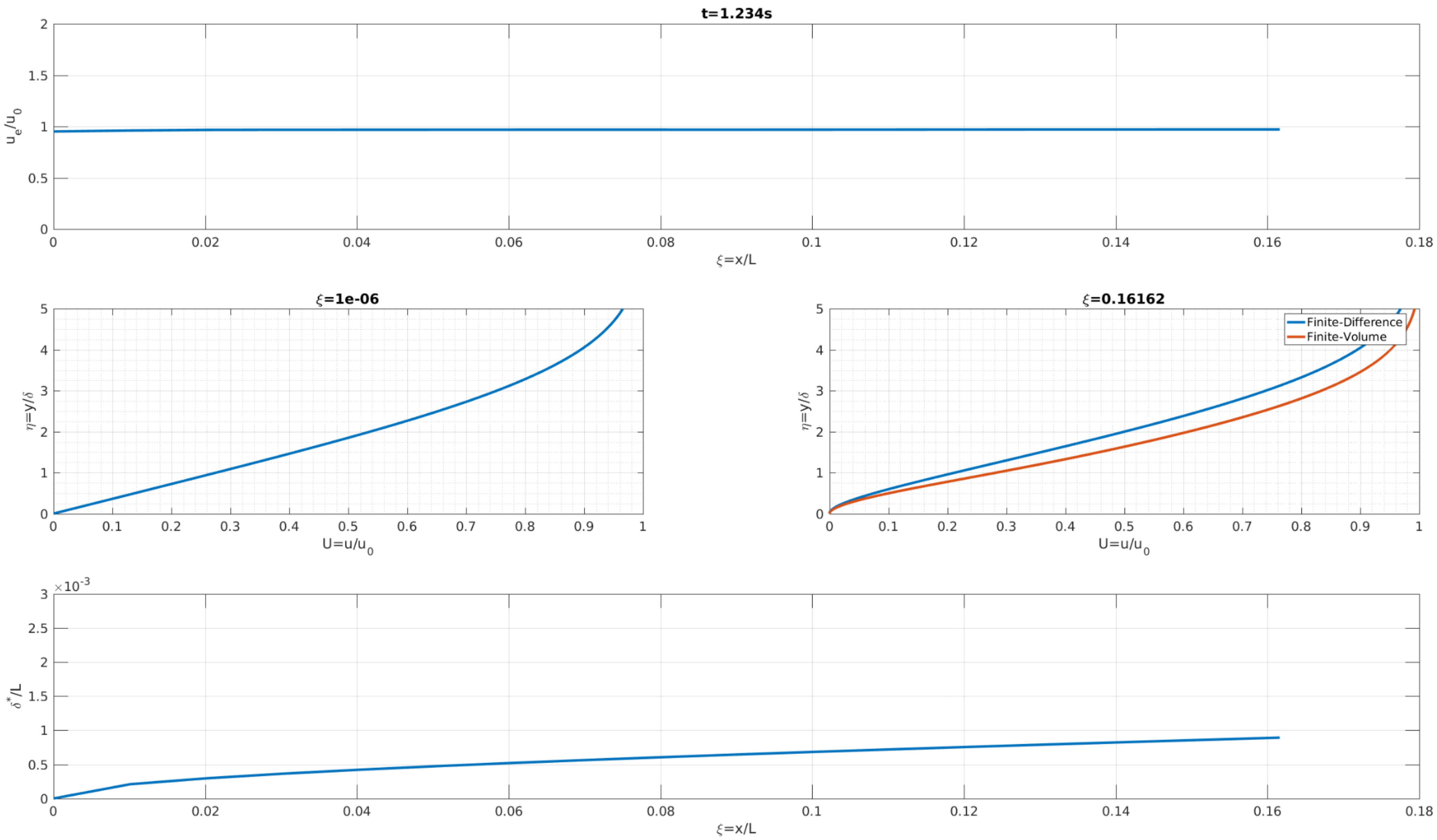

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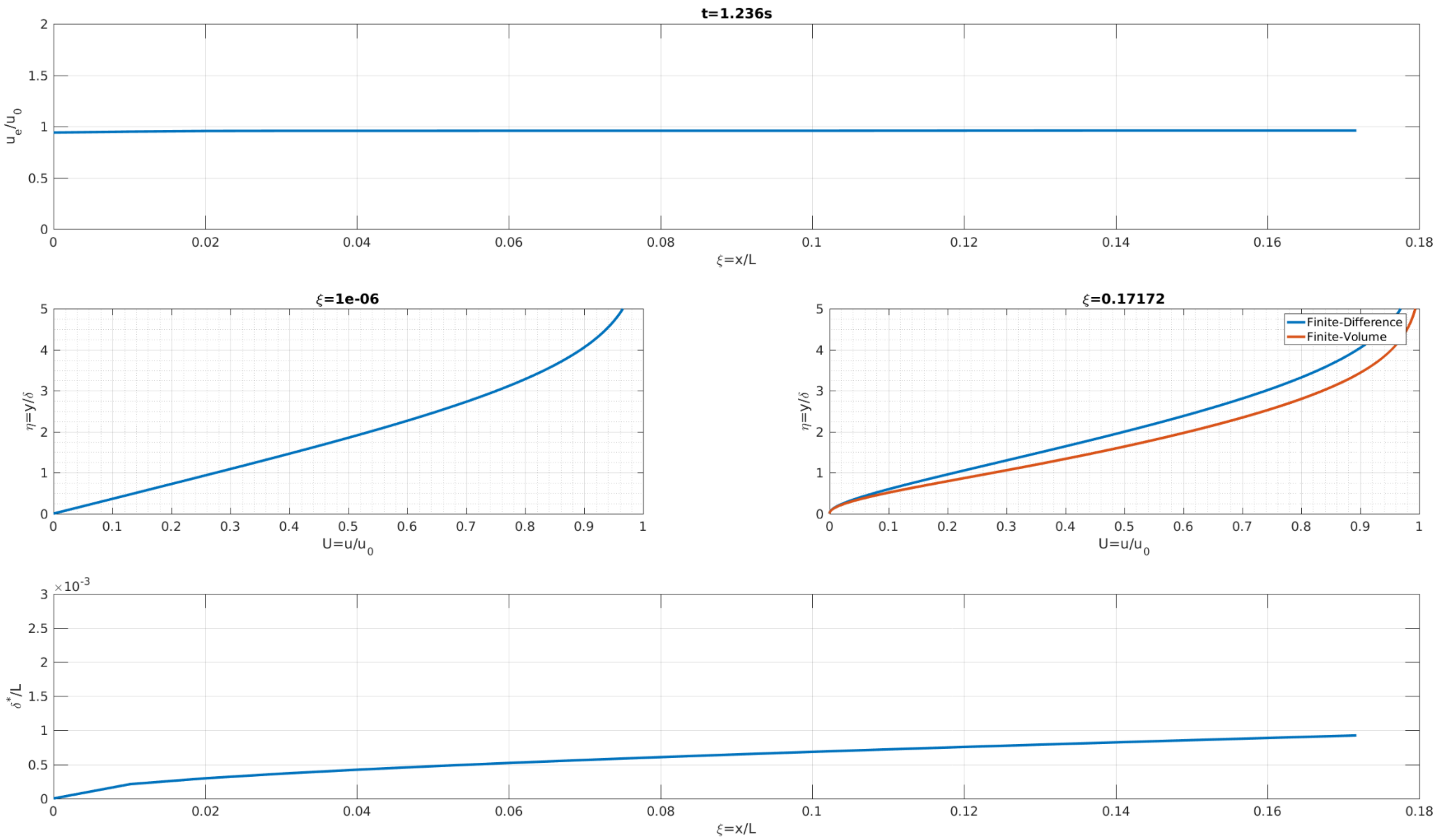

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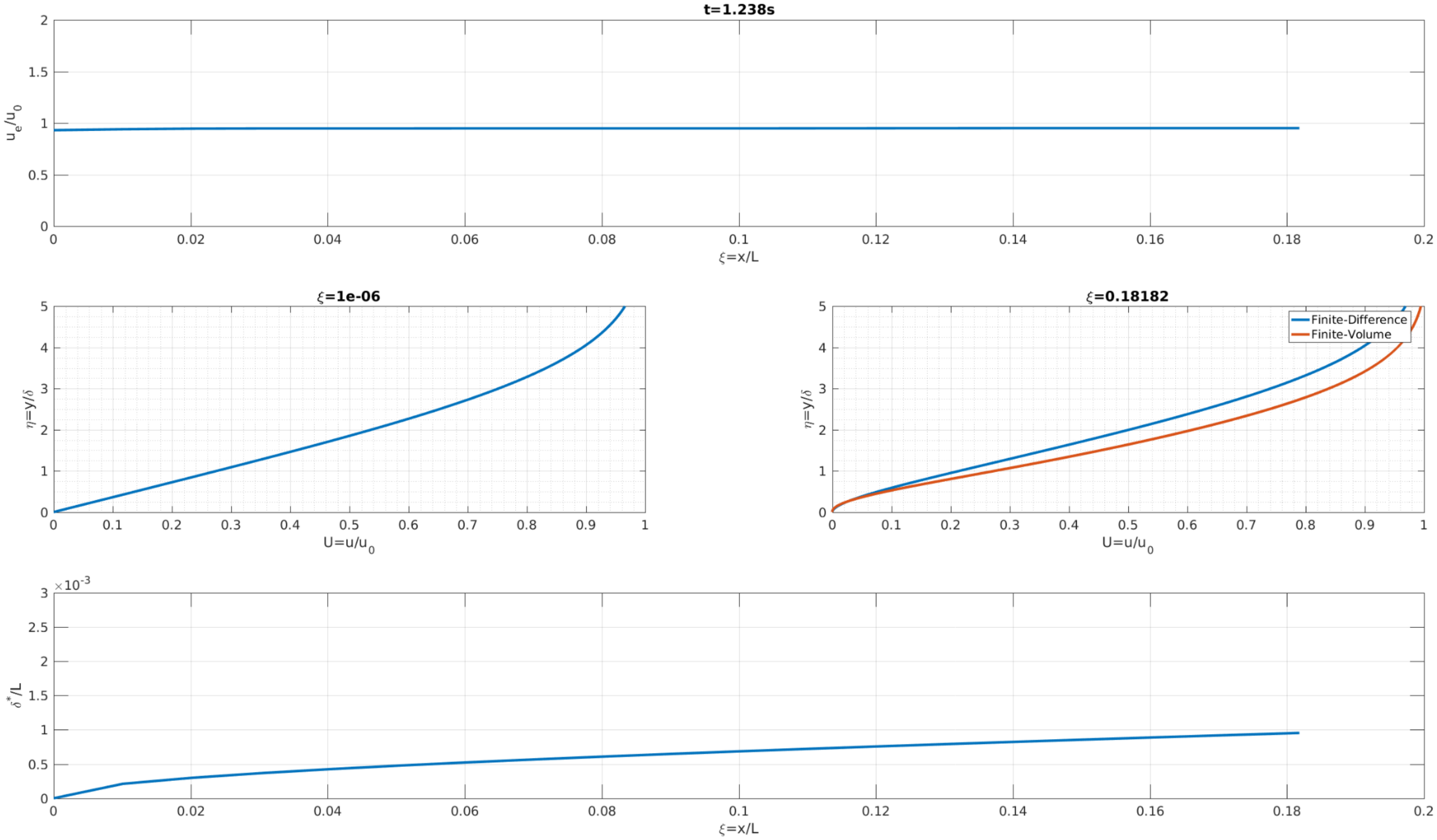

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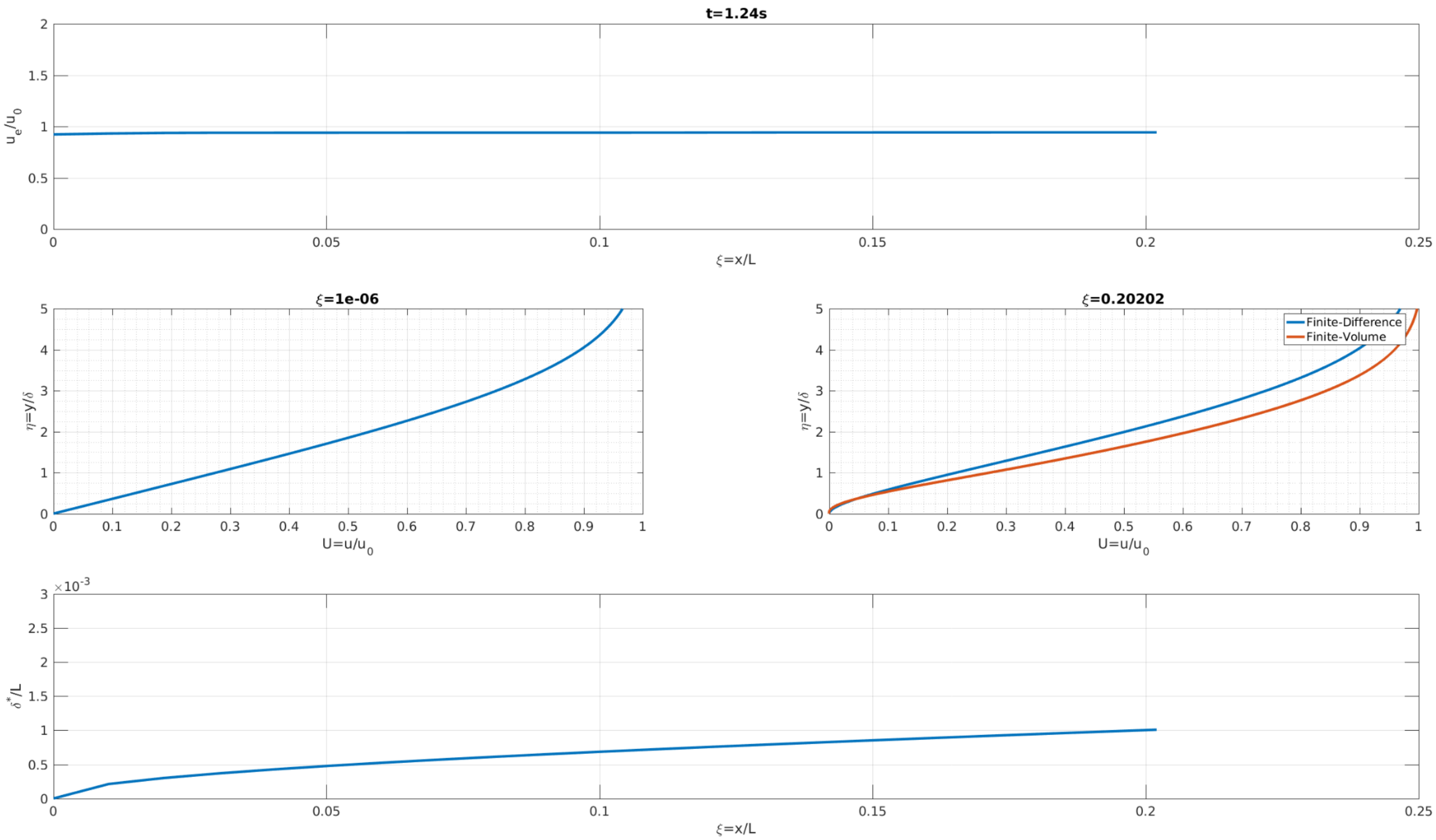

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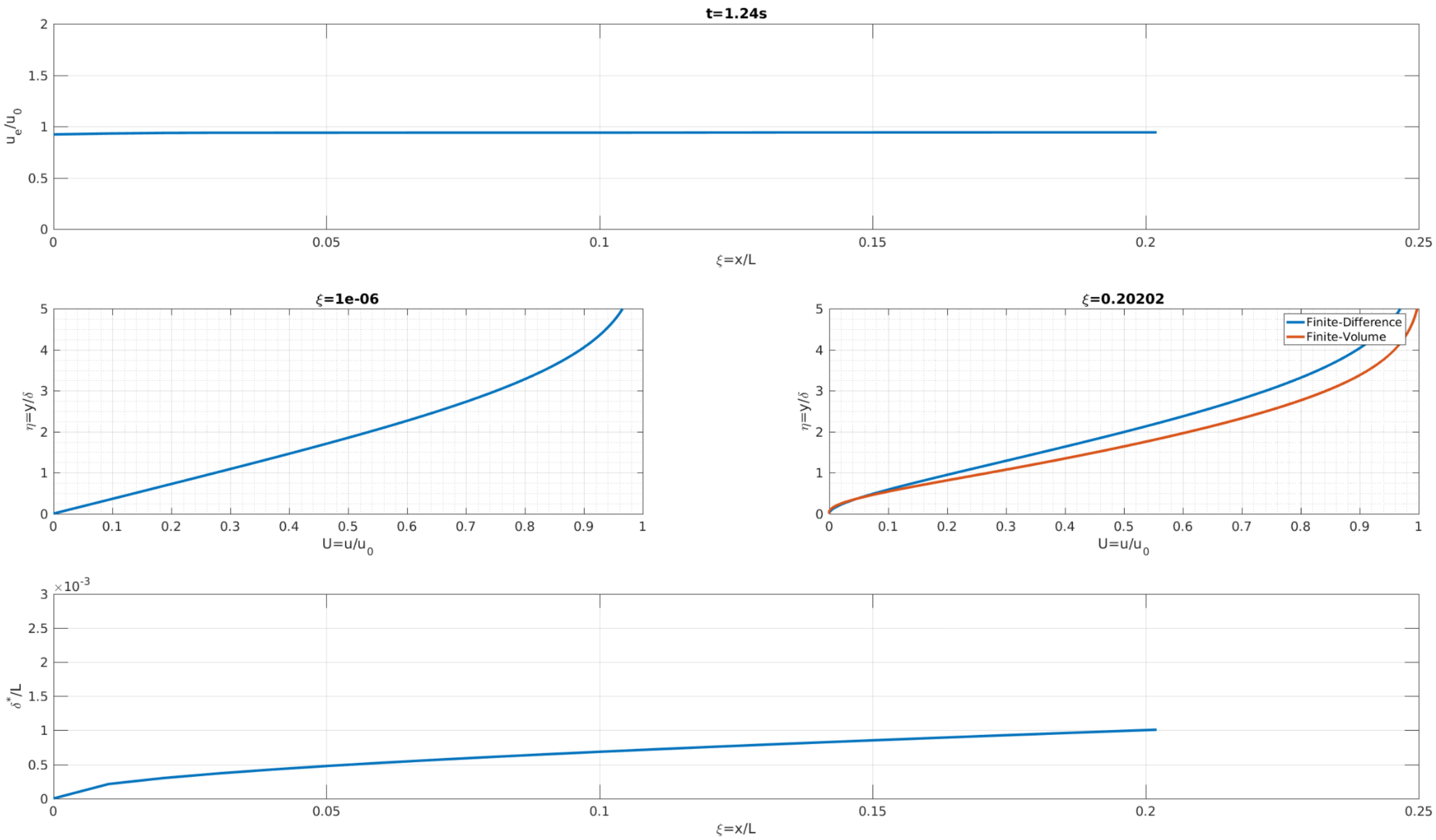

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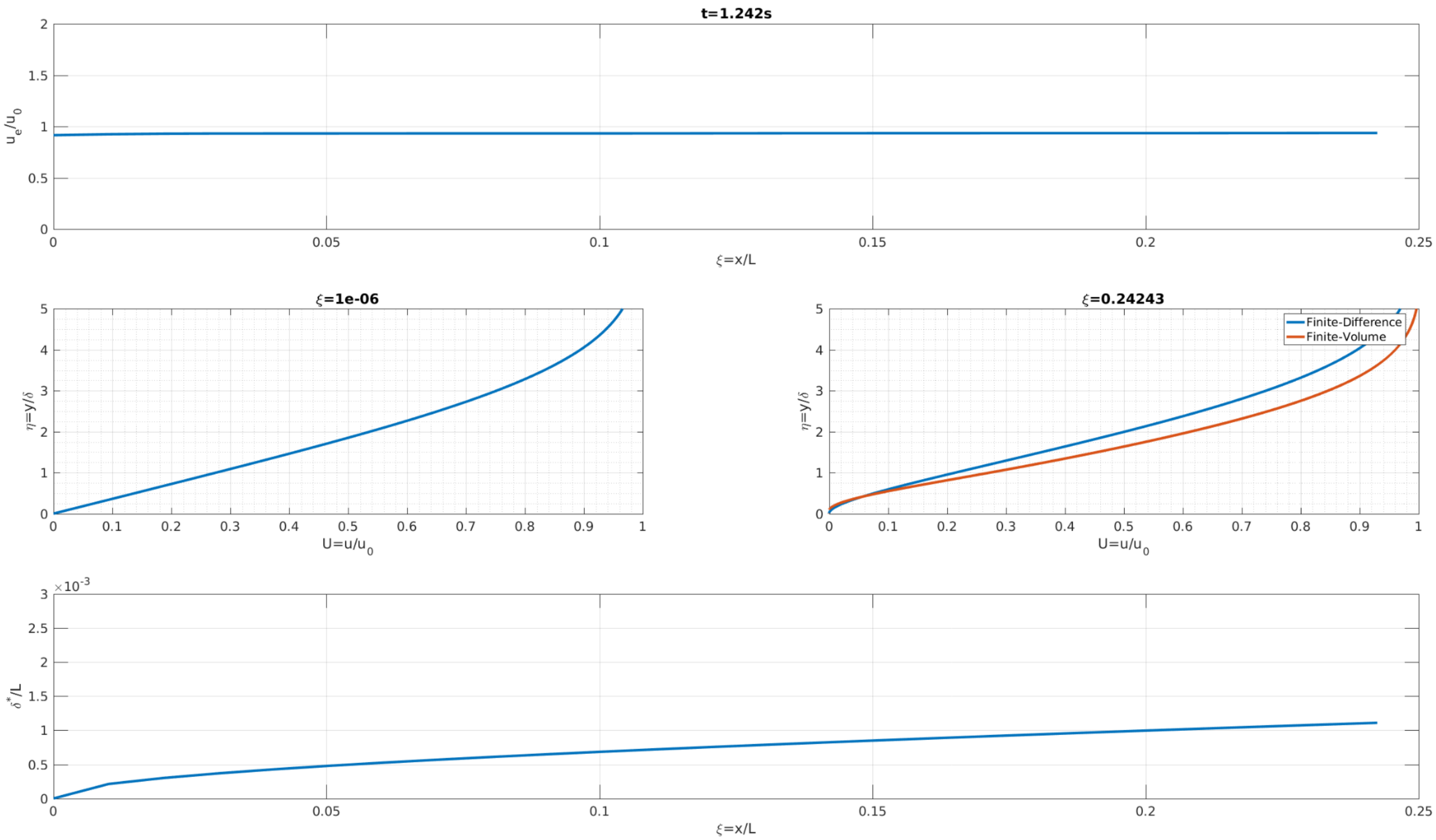

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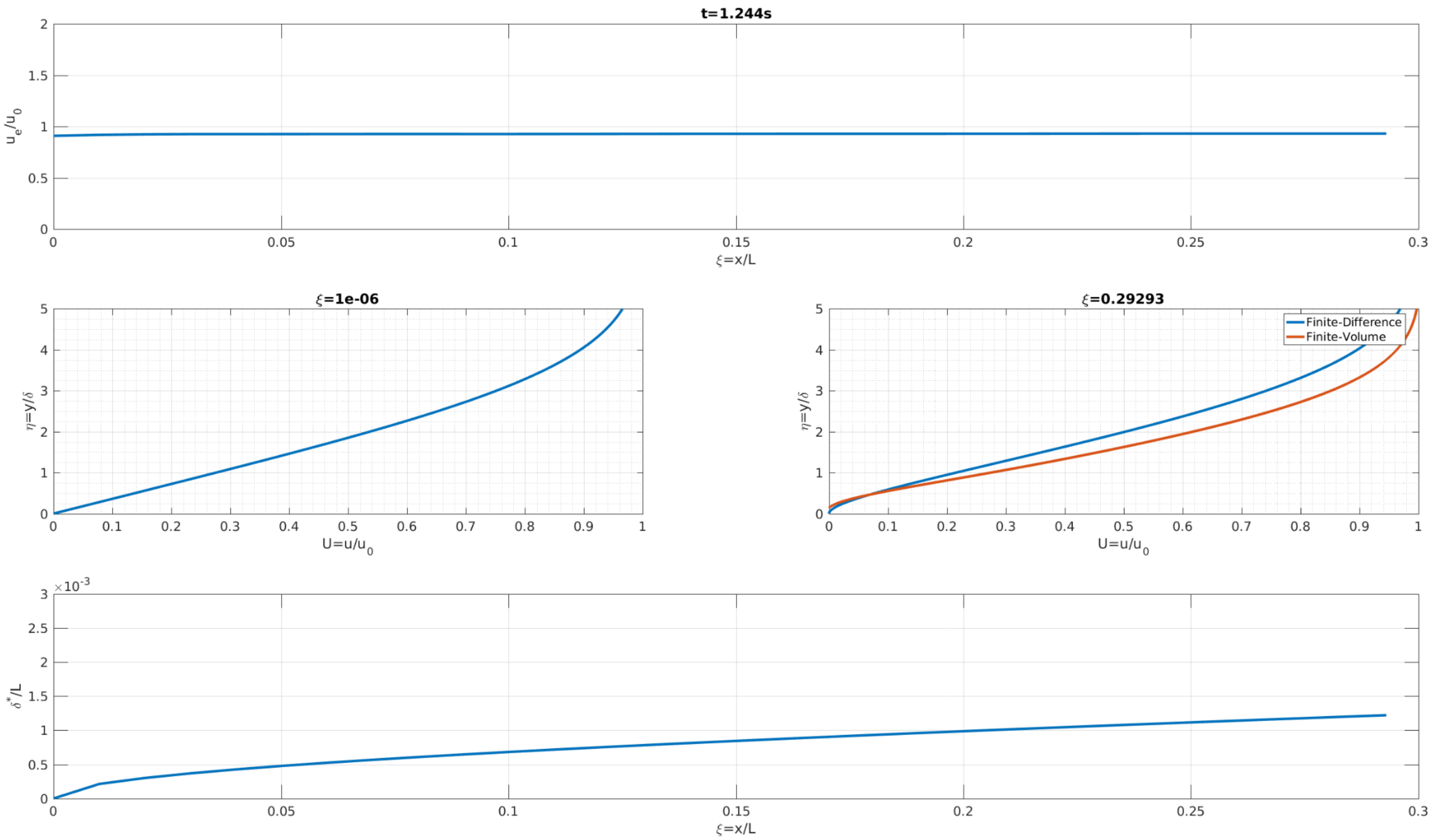

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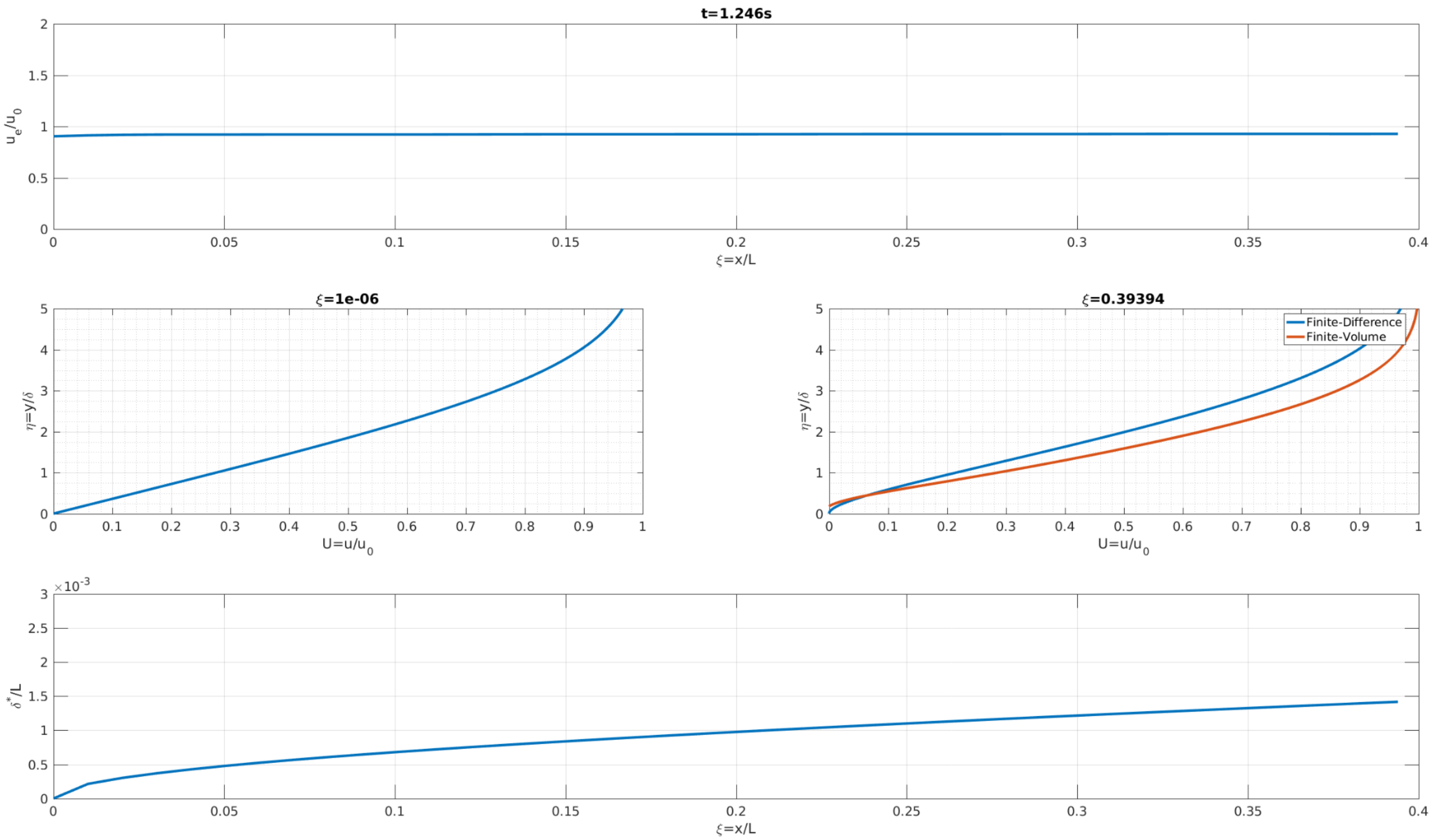

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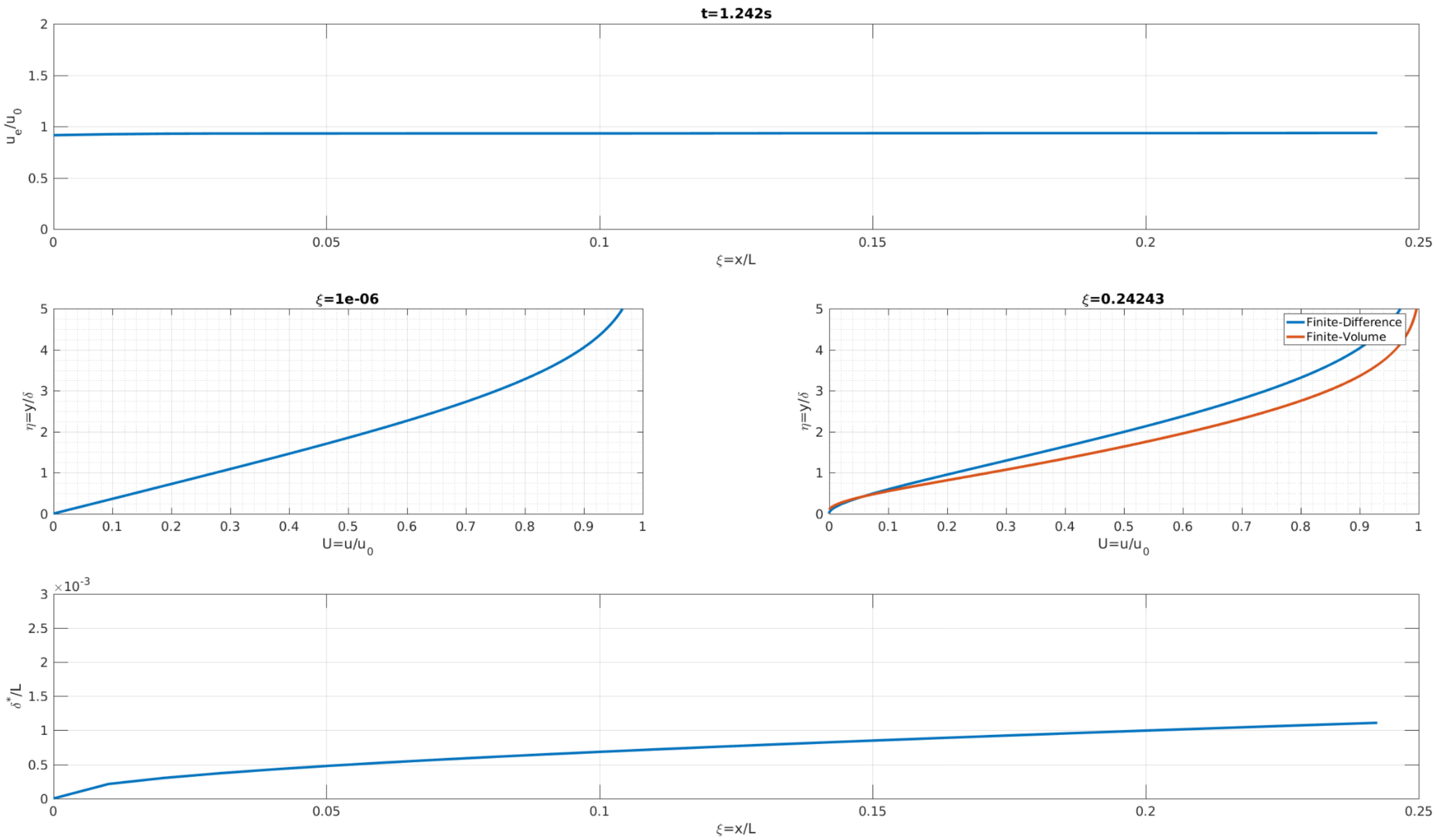

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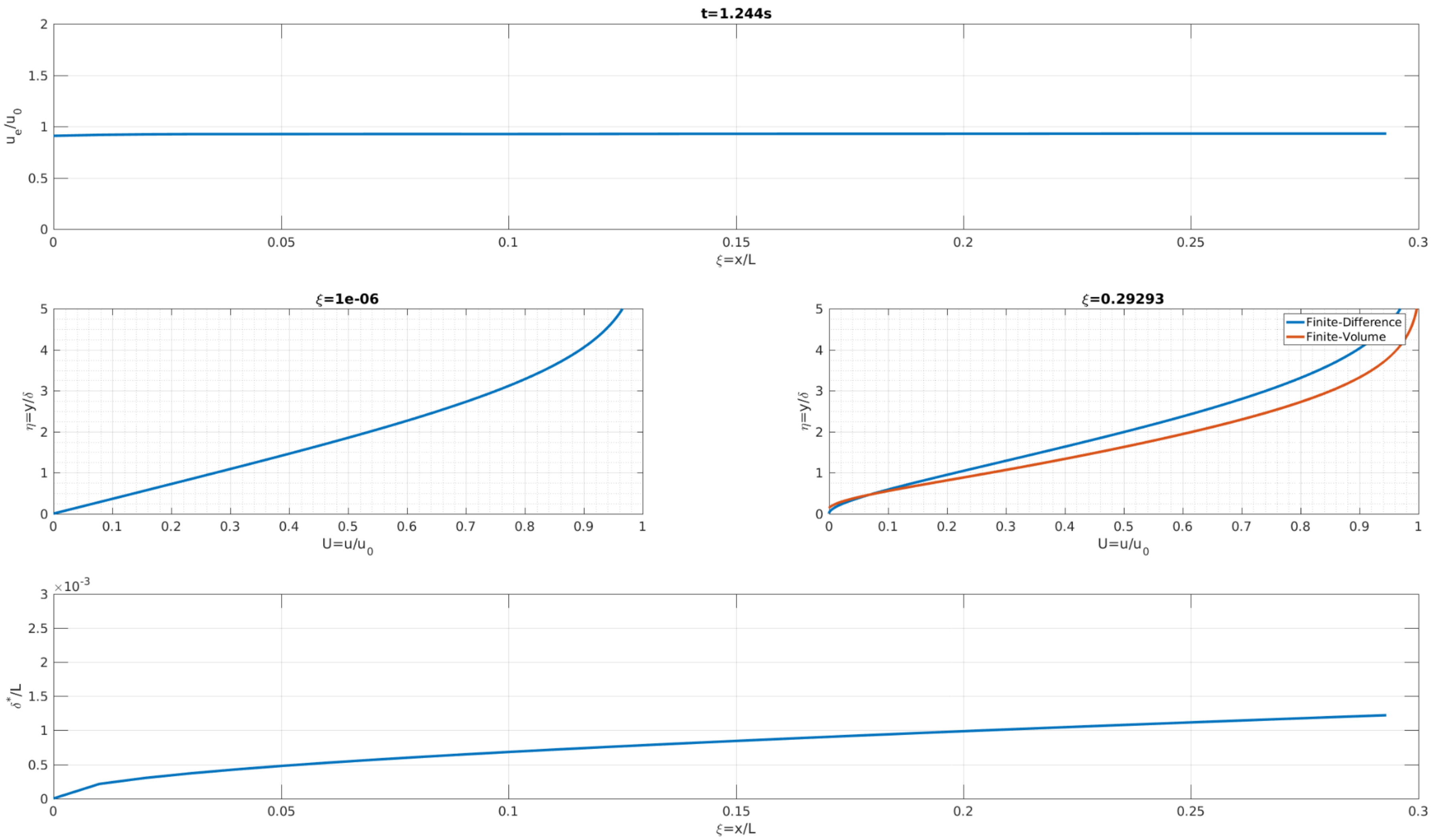

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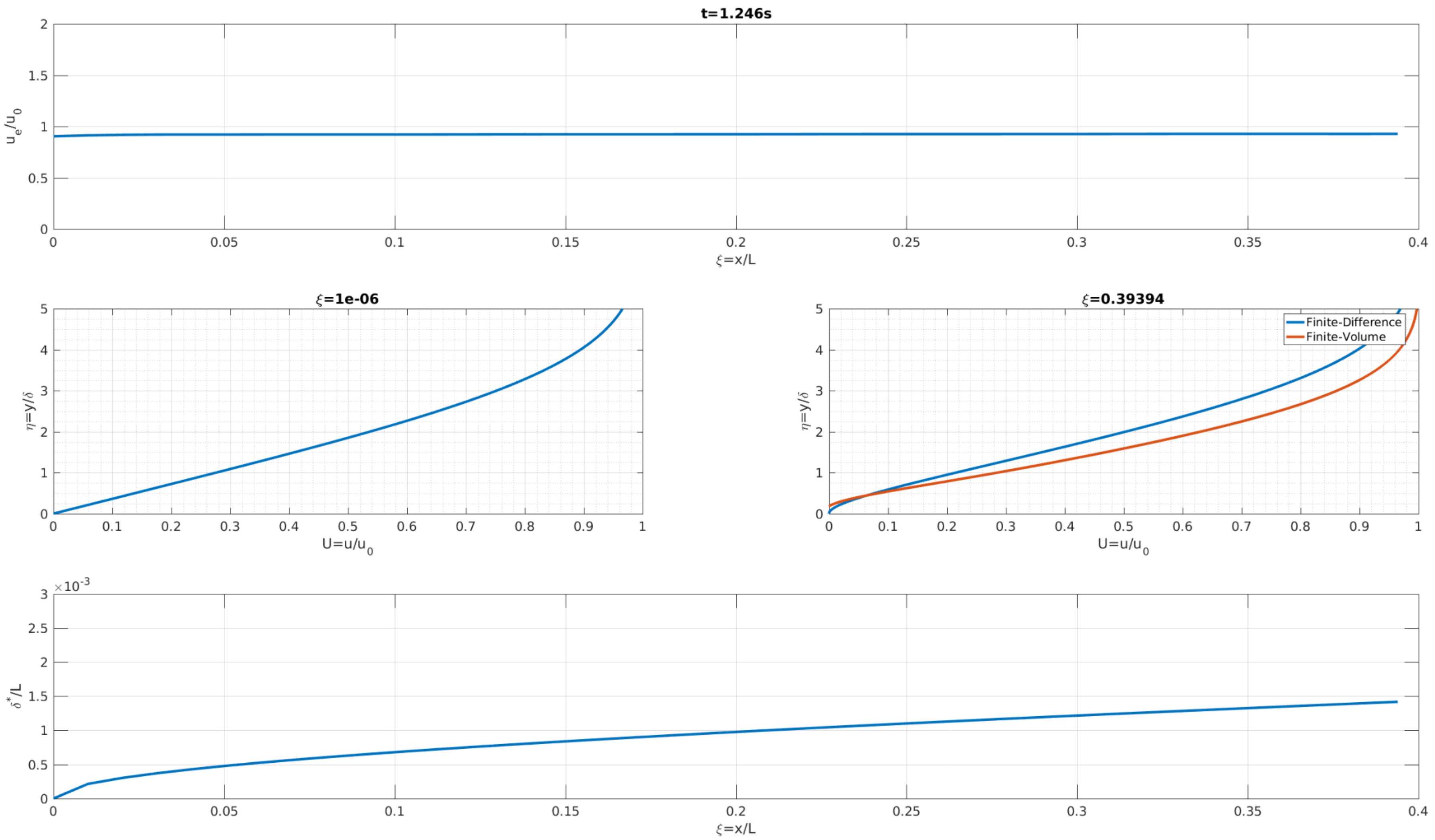

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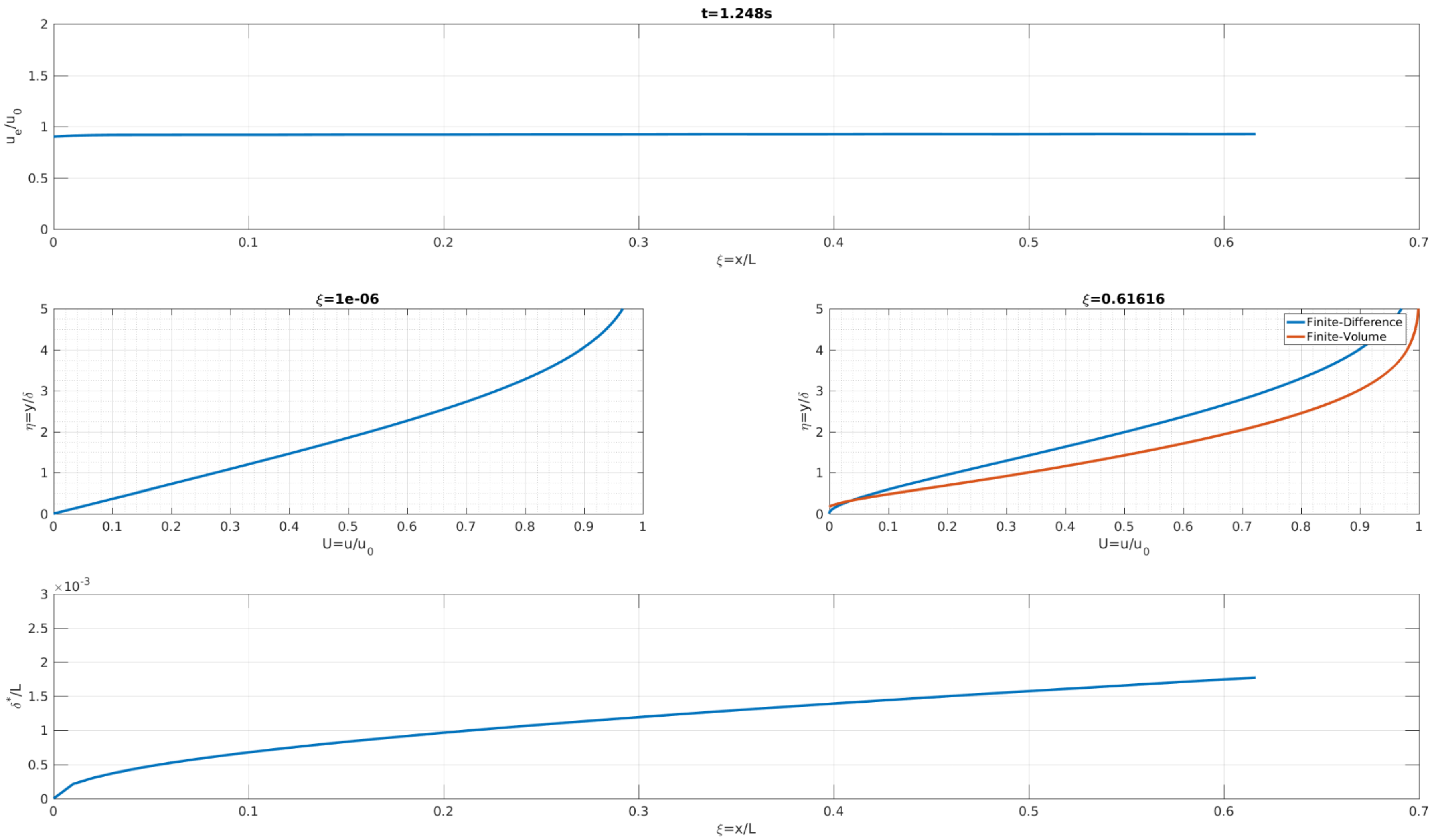

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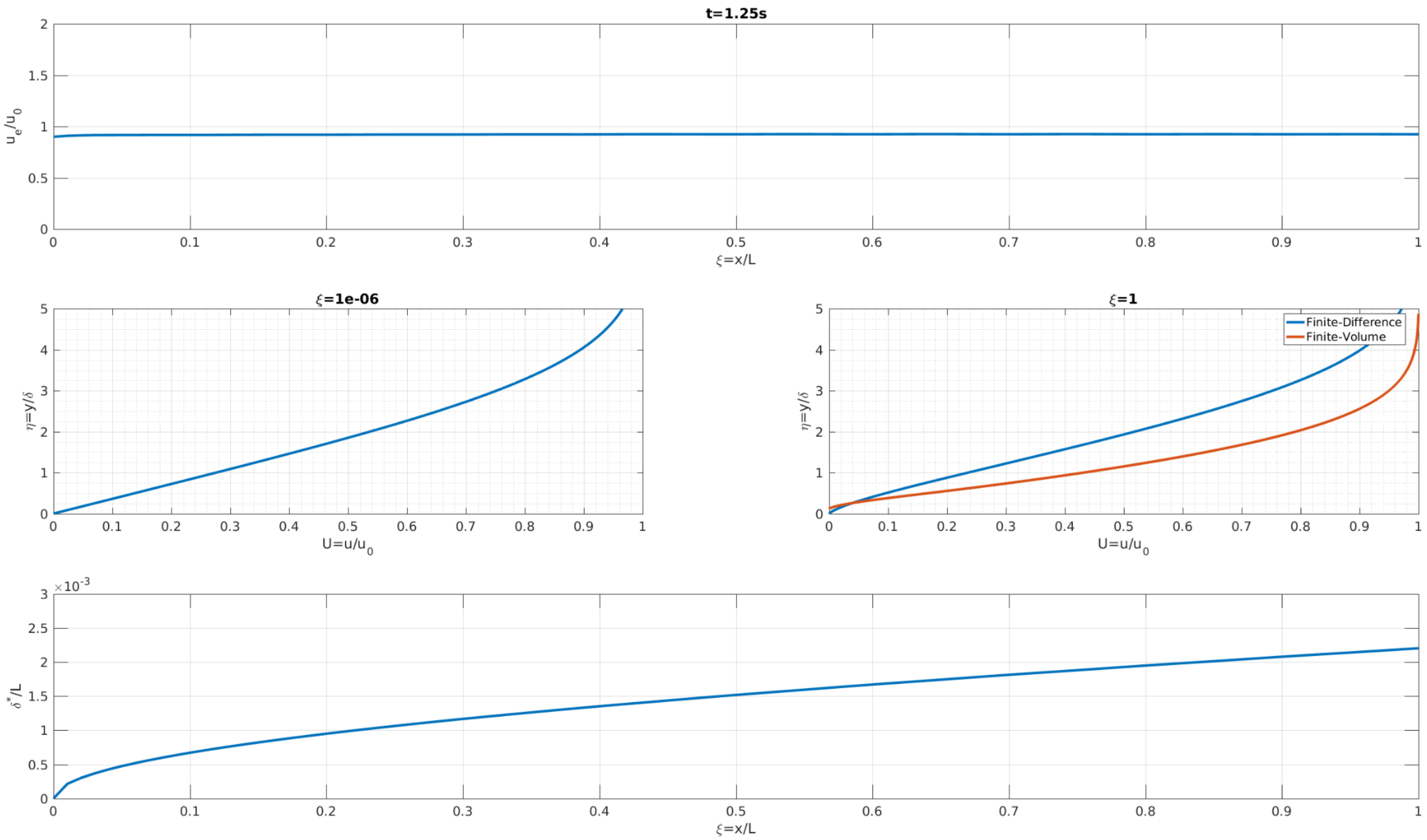

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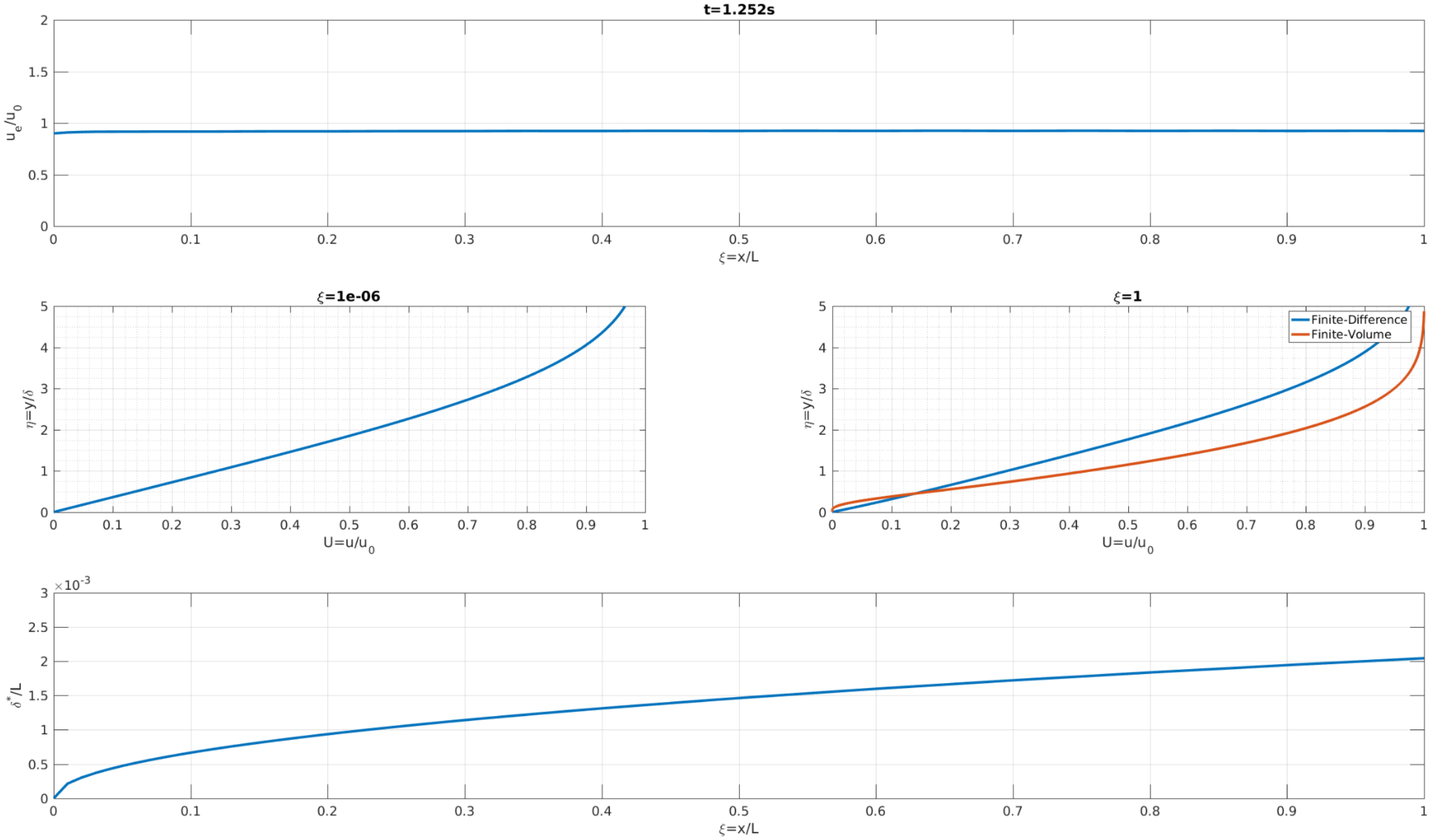

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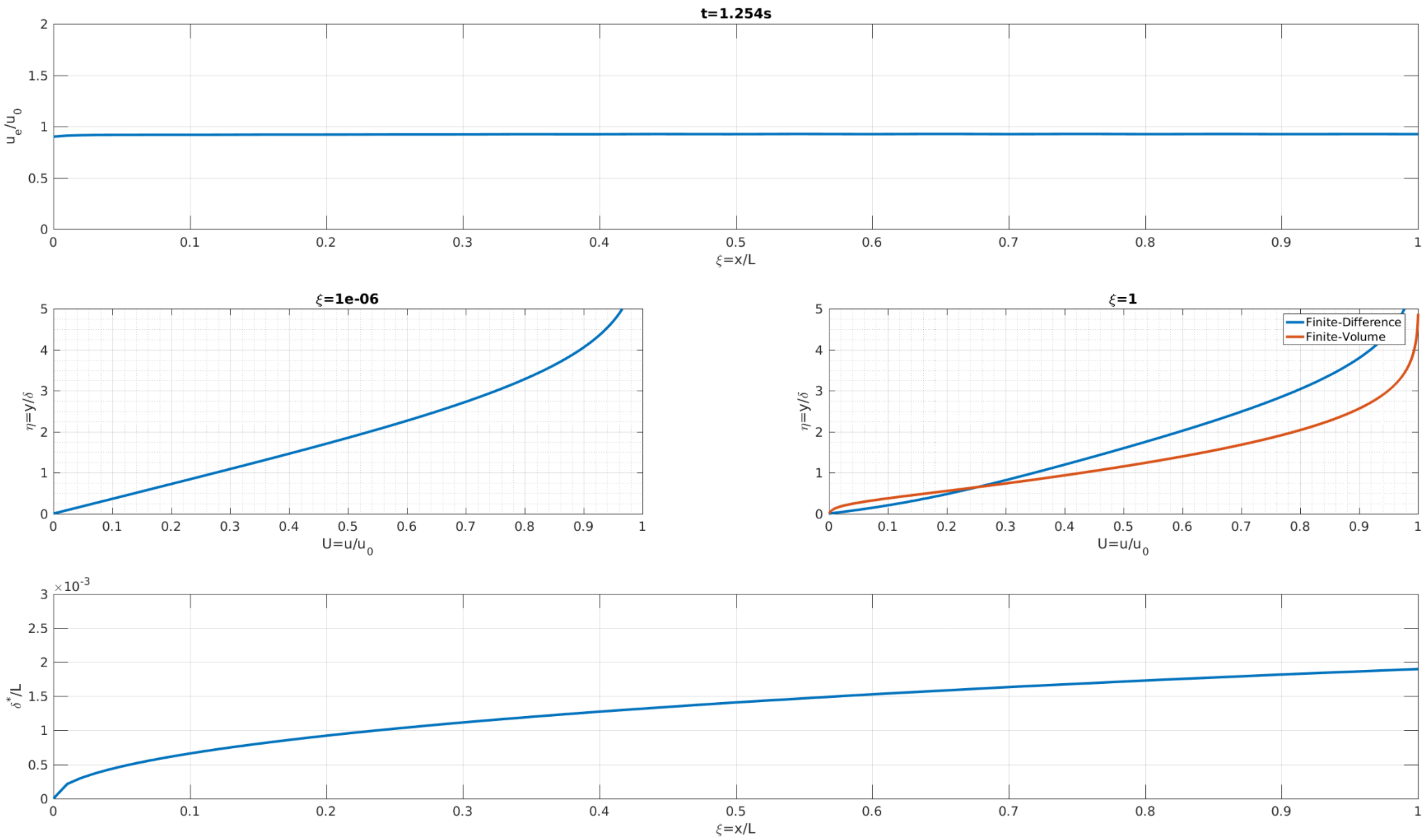

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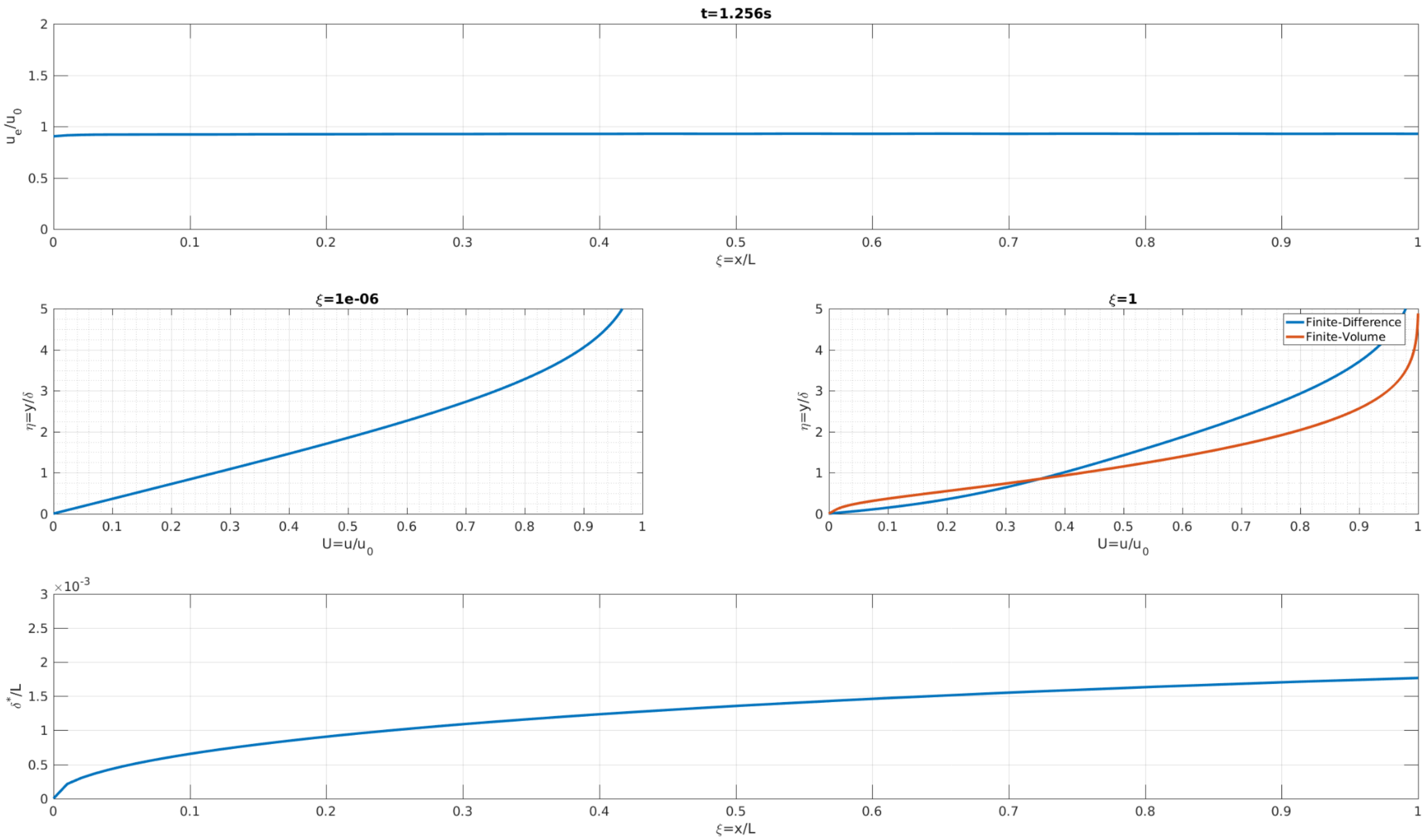

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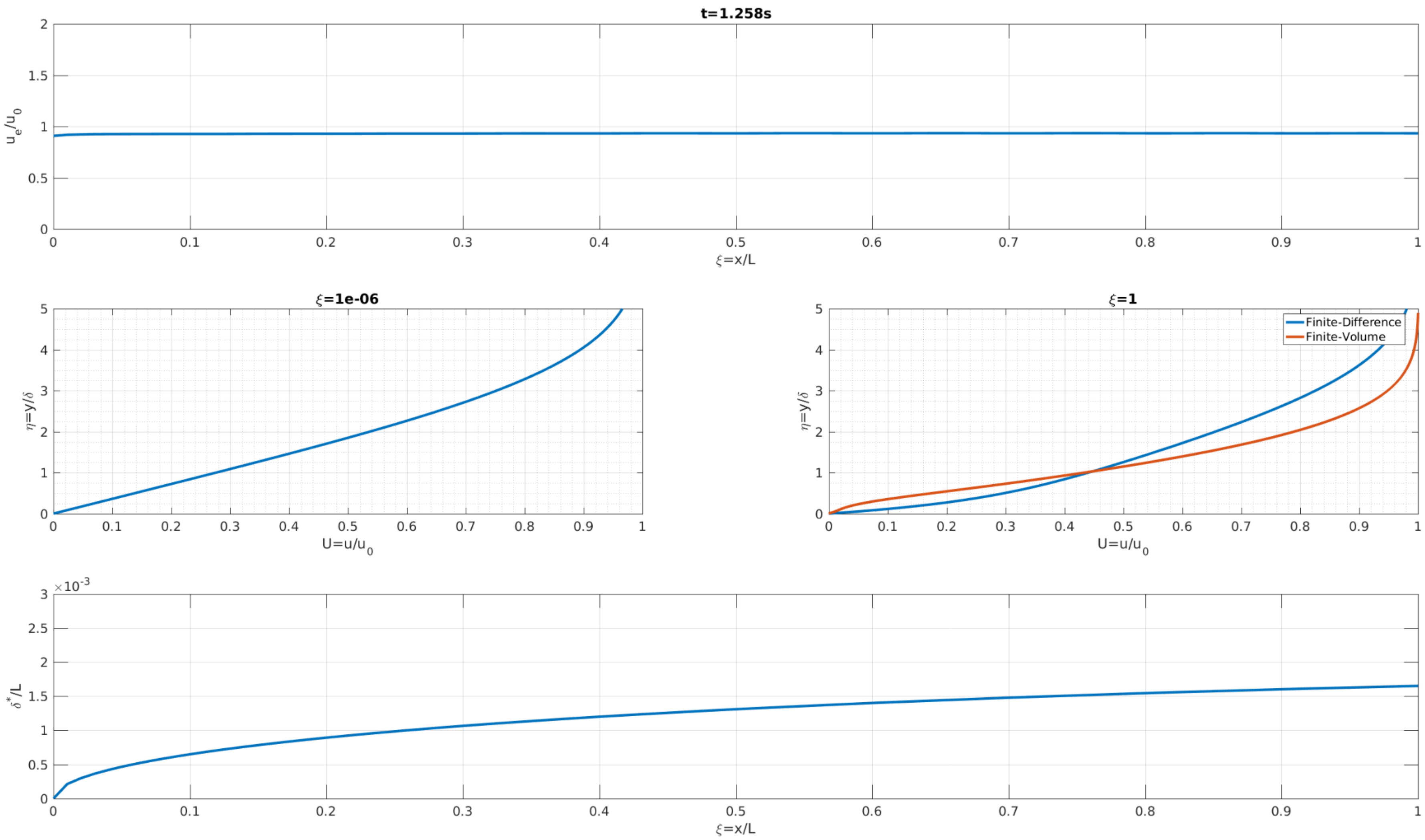

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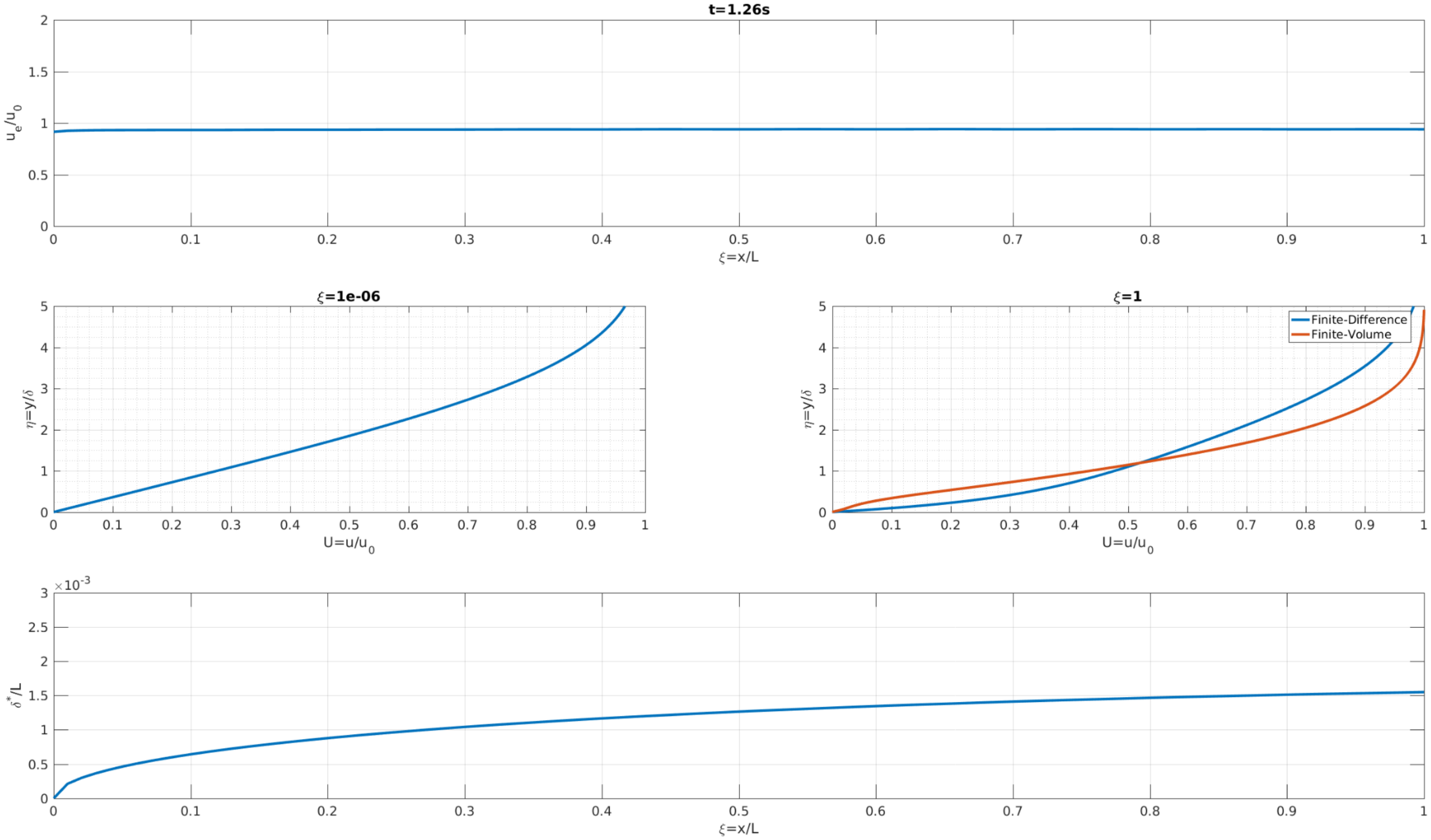

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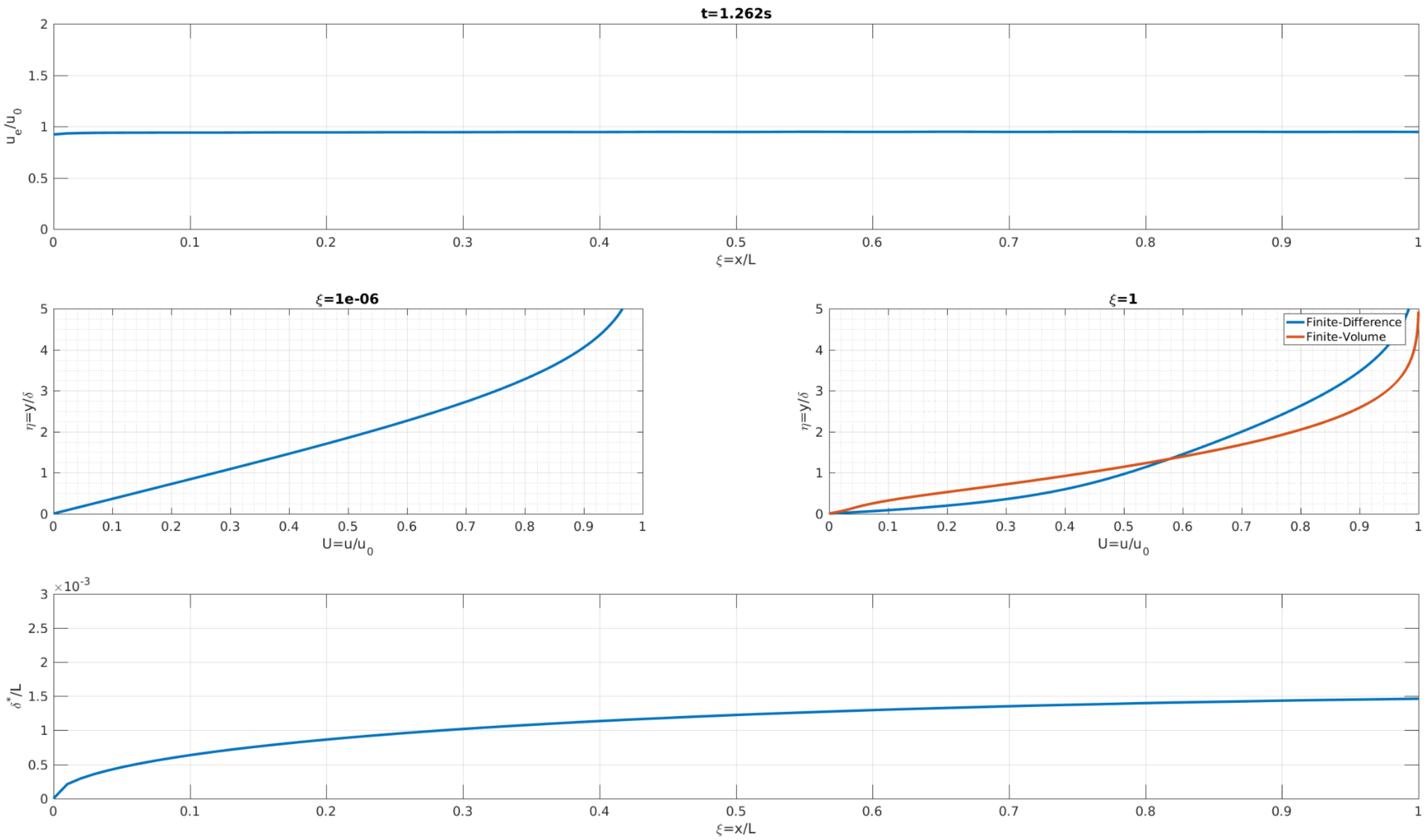

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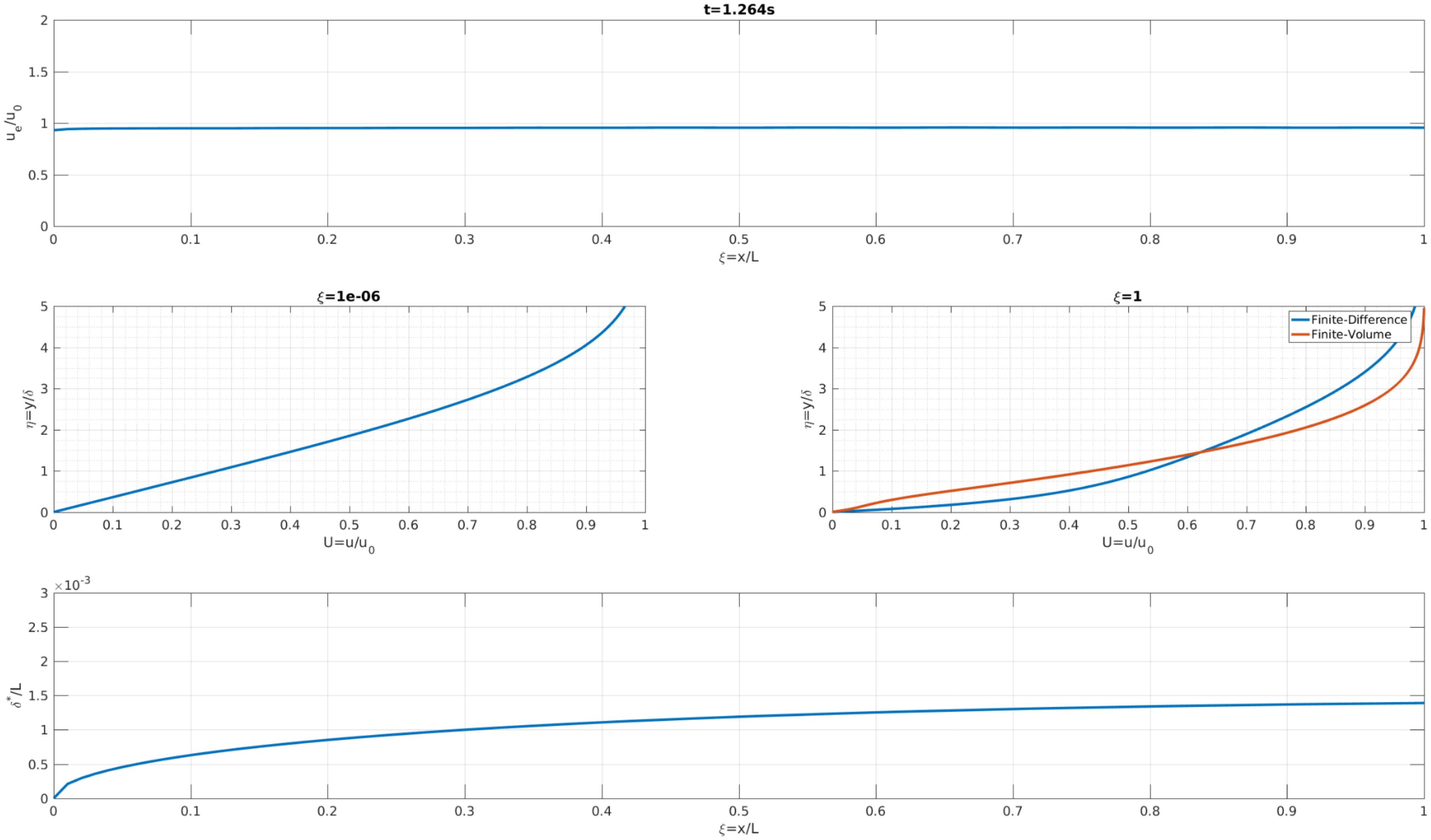

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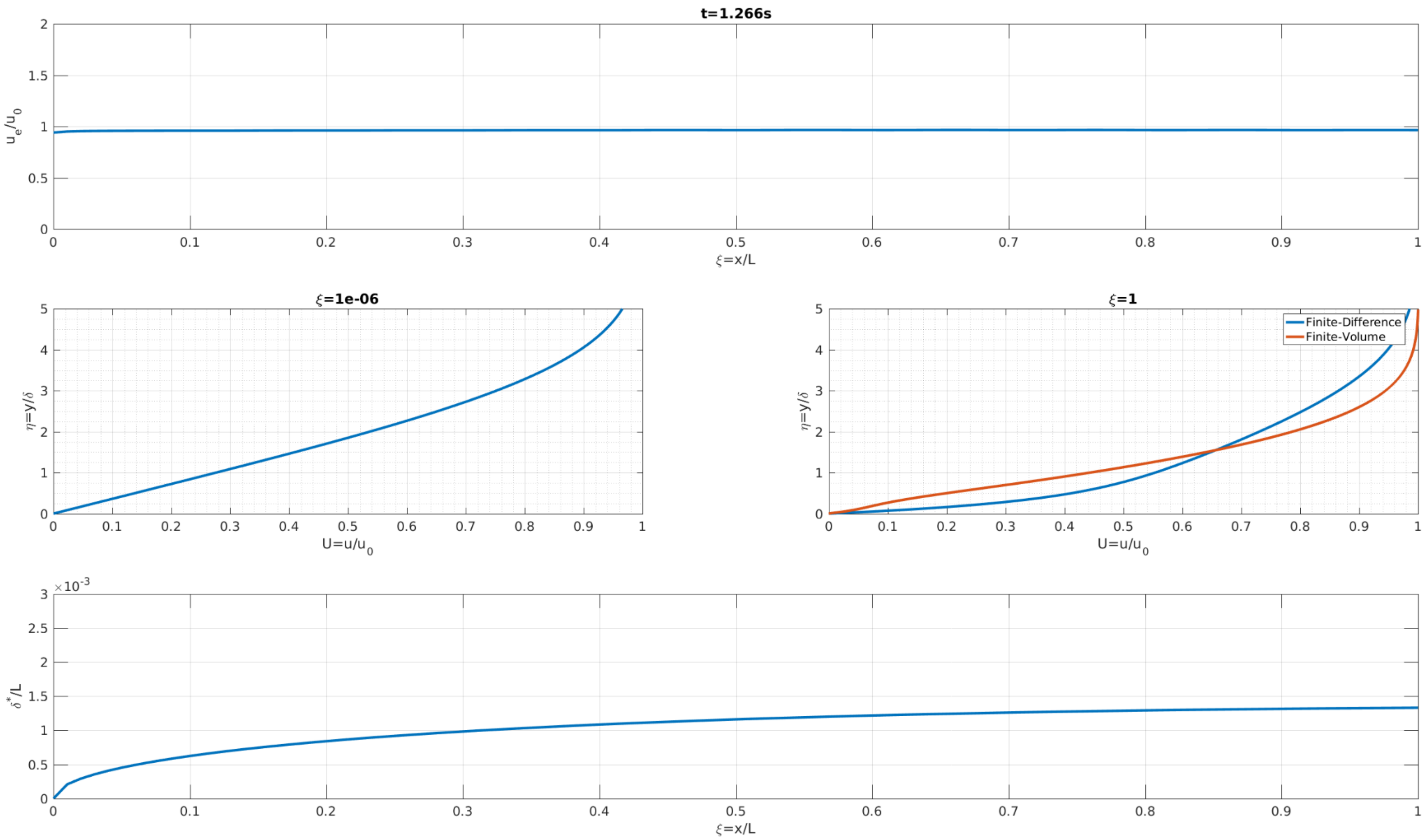

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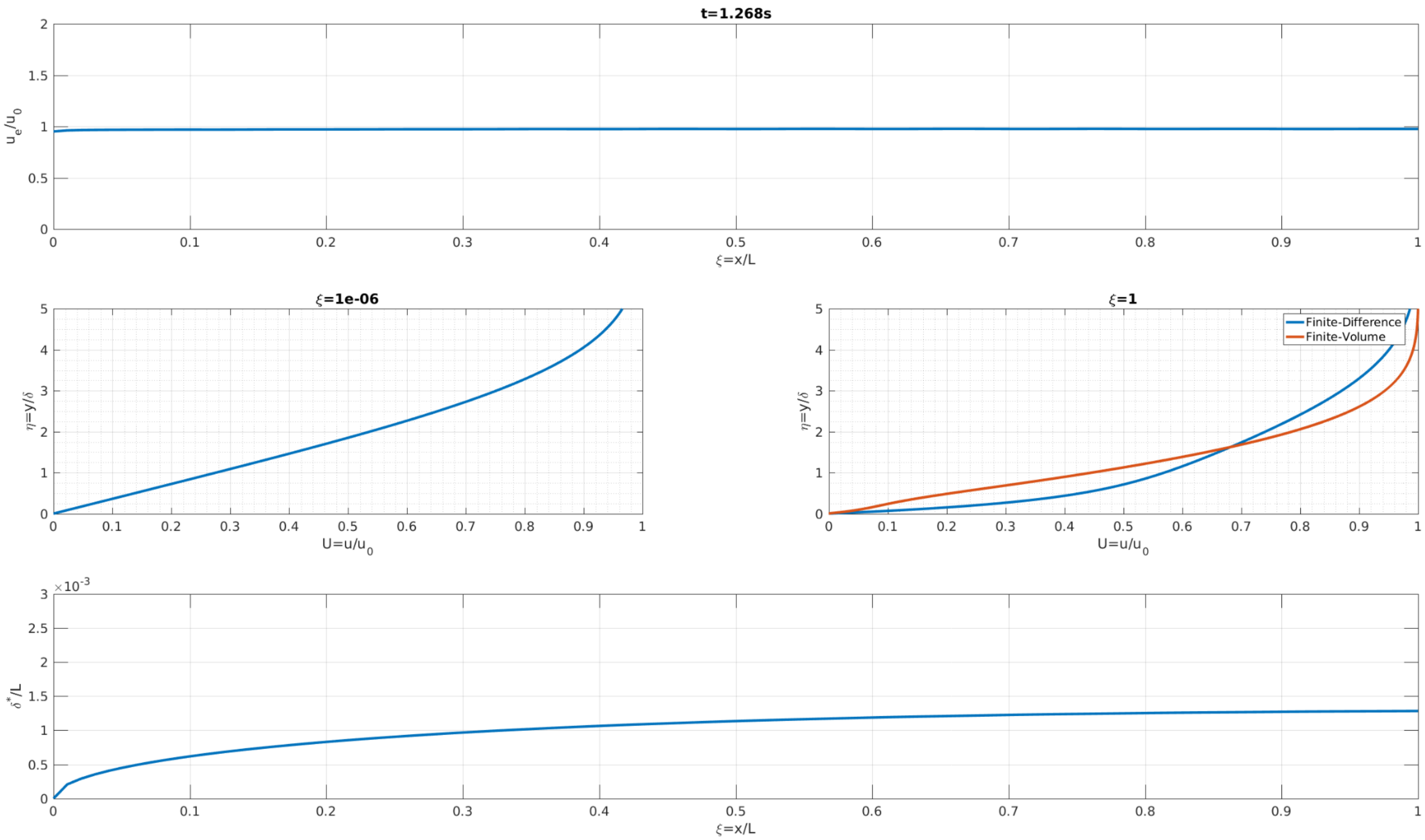

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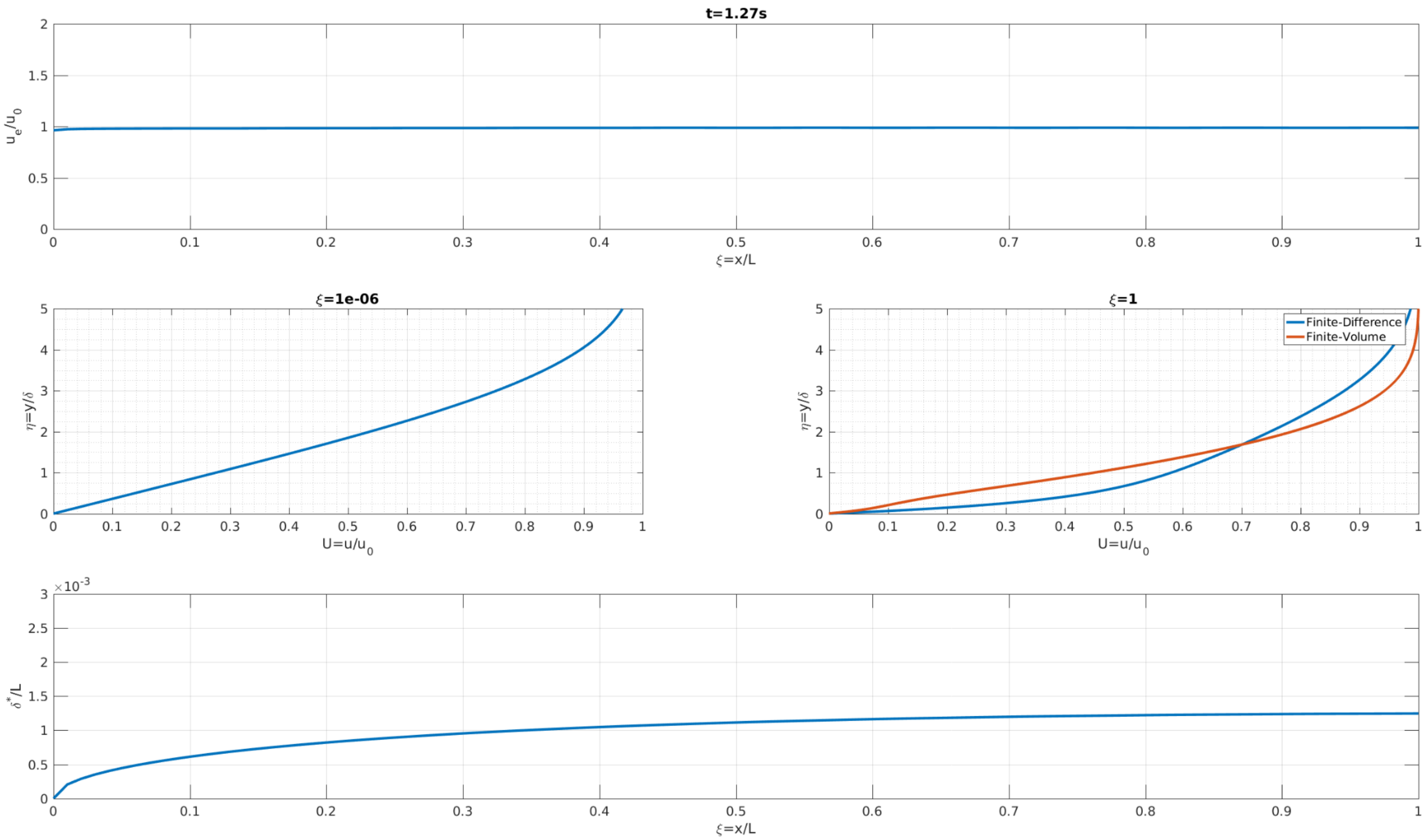

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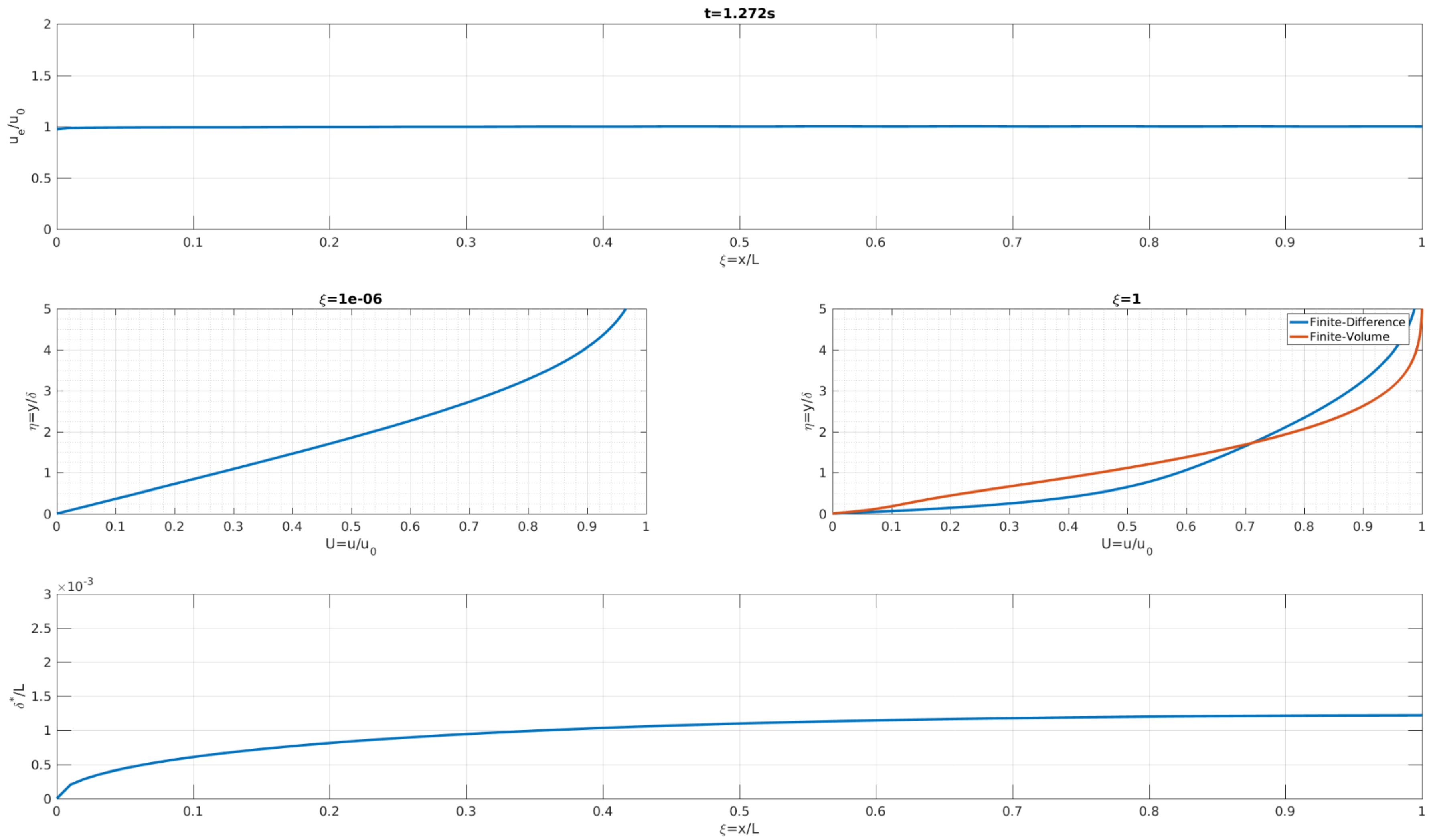


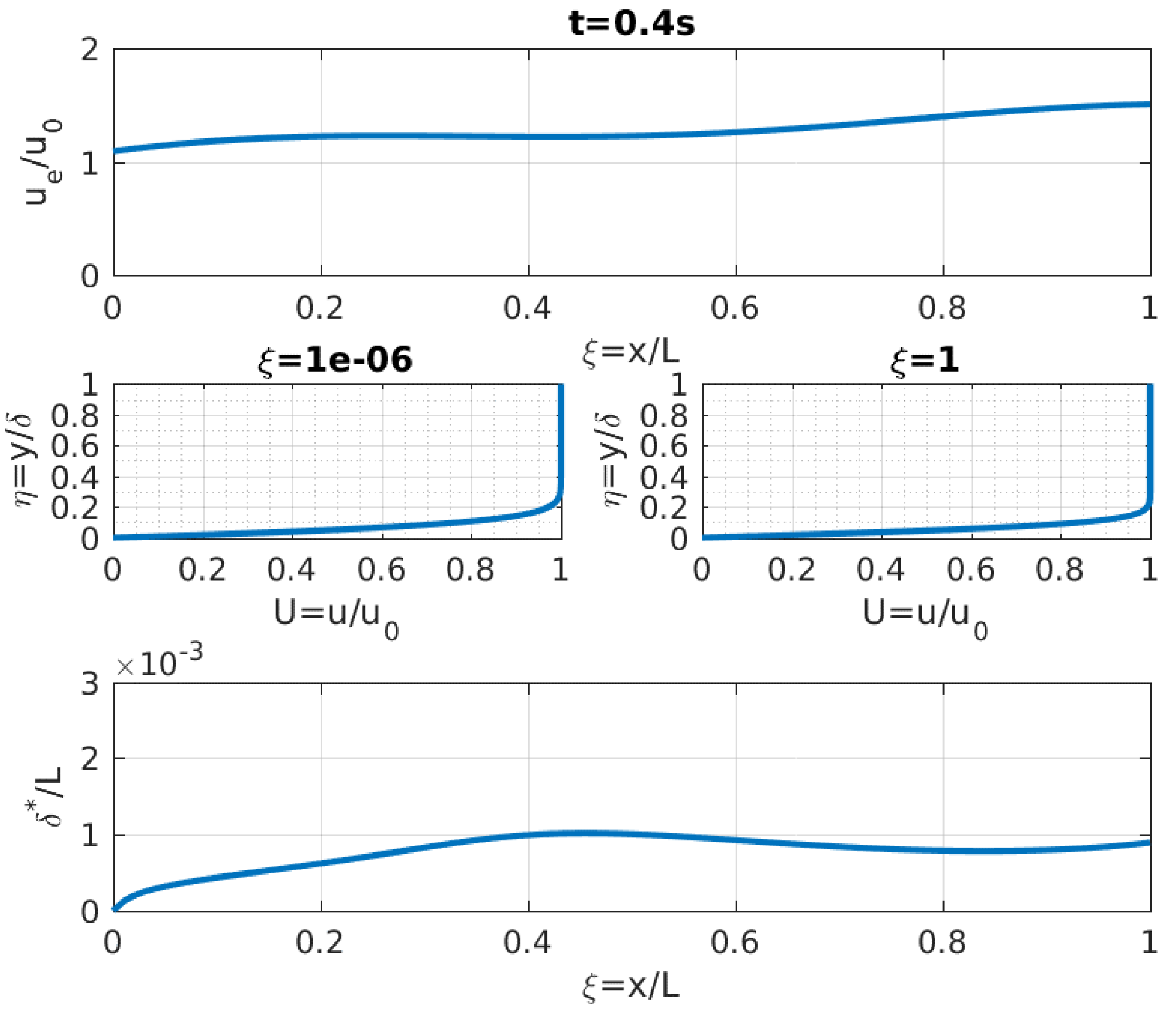
Results: Validation for Unsteady Case with Spatial Variations

Test case#3:

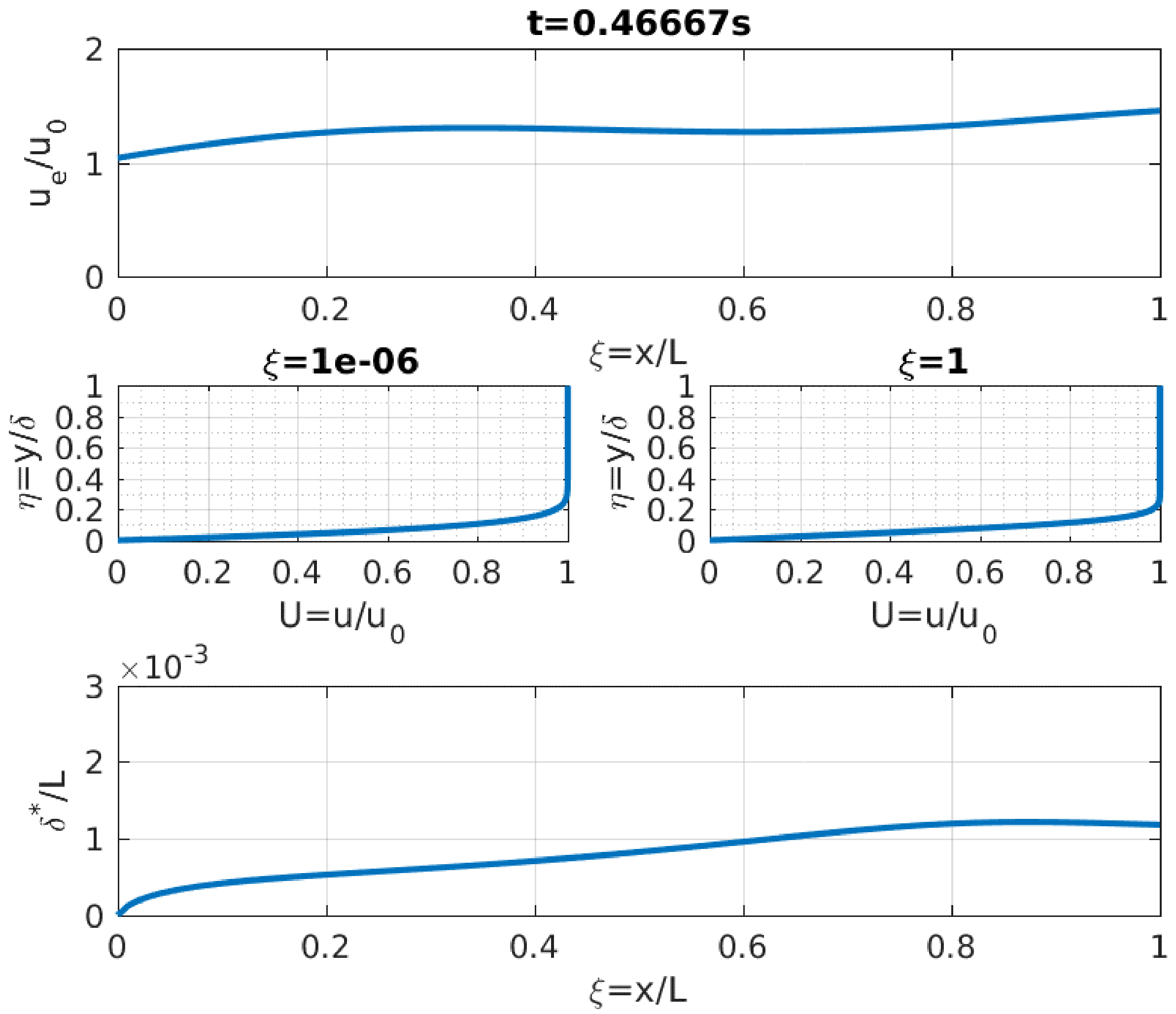
- 2D unsteady, laminar, flat-plate boundary layer
- $Re_L = 10^6$
- Traveling wave conditions, such that:

$$\frac{u_e}{u_0} = 1 + A * \cos(\omega t - kx)$$

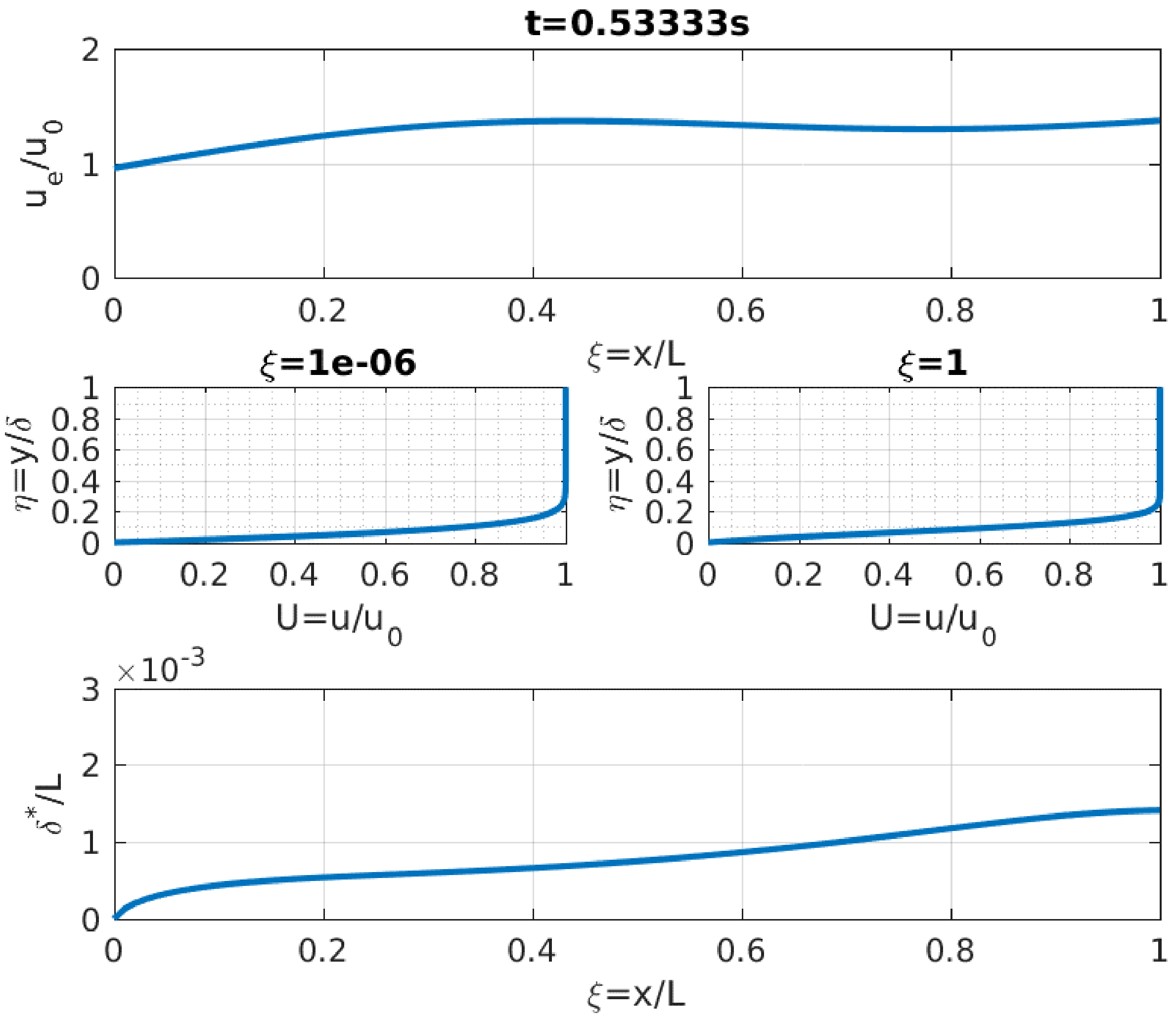
Results: Validation for Unsteady Case with Spatial Variations – Finite Difference



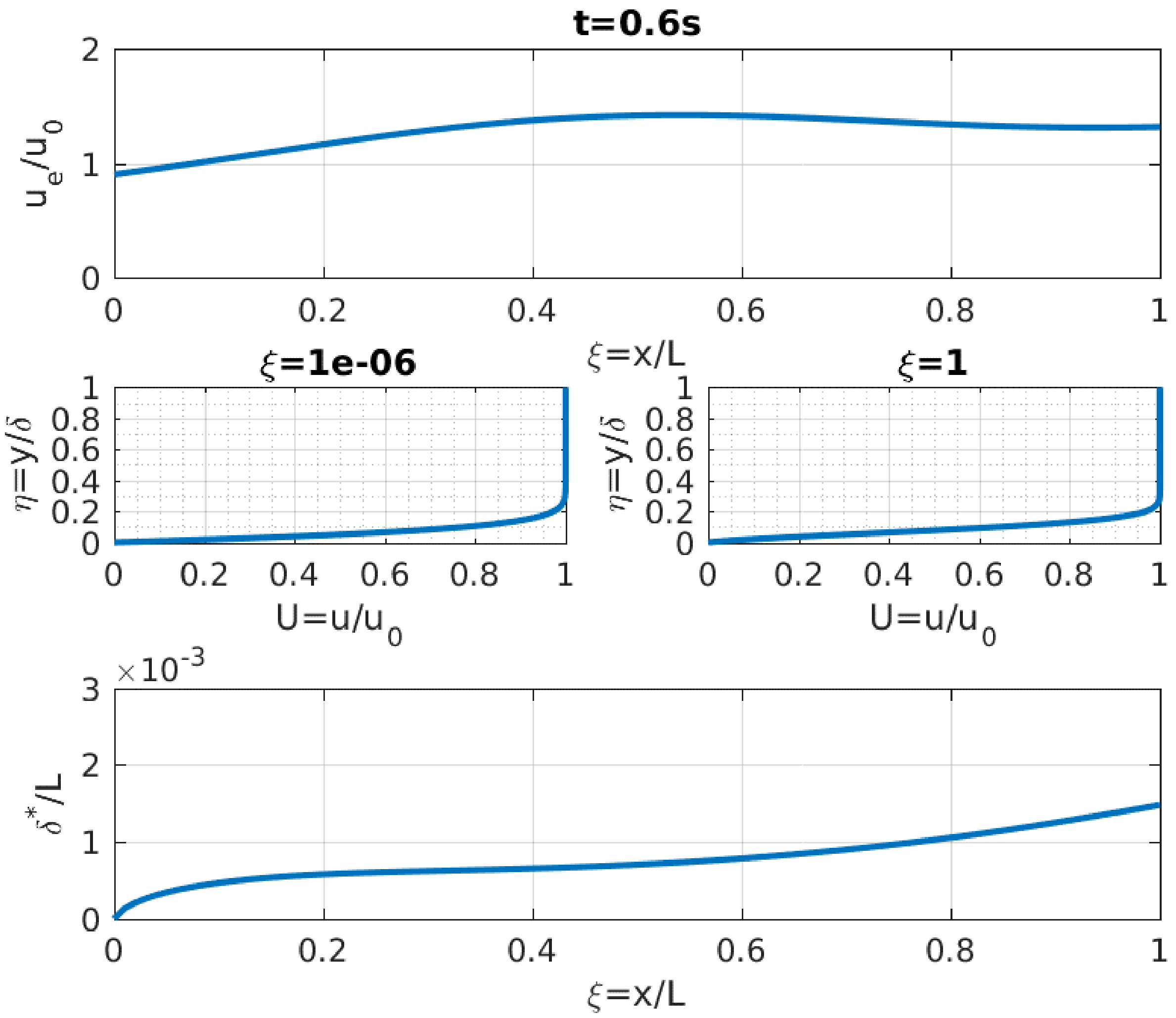
Results: Validation for Unsteady Case with Spatial Variations – Finite Difference



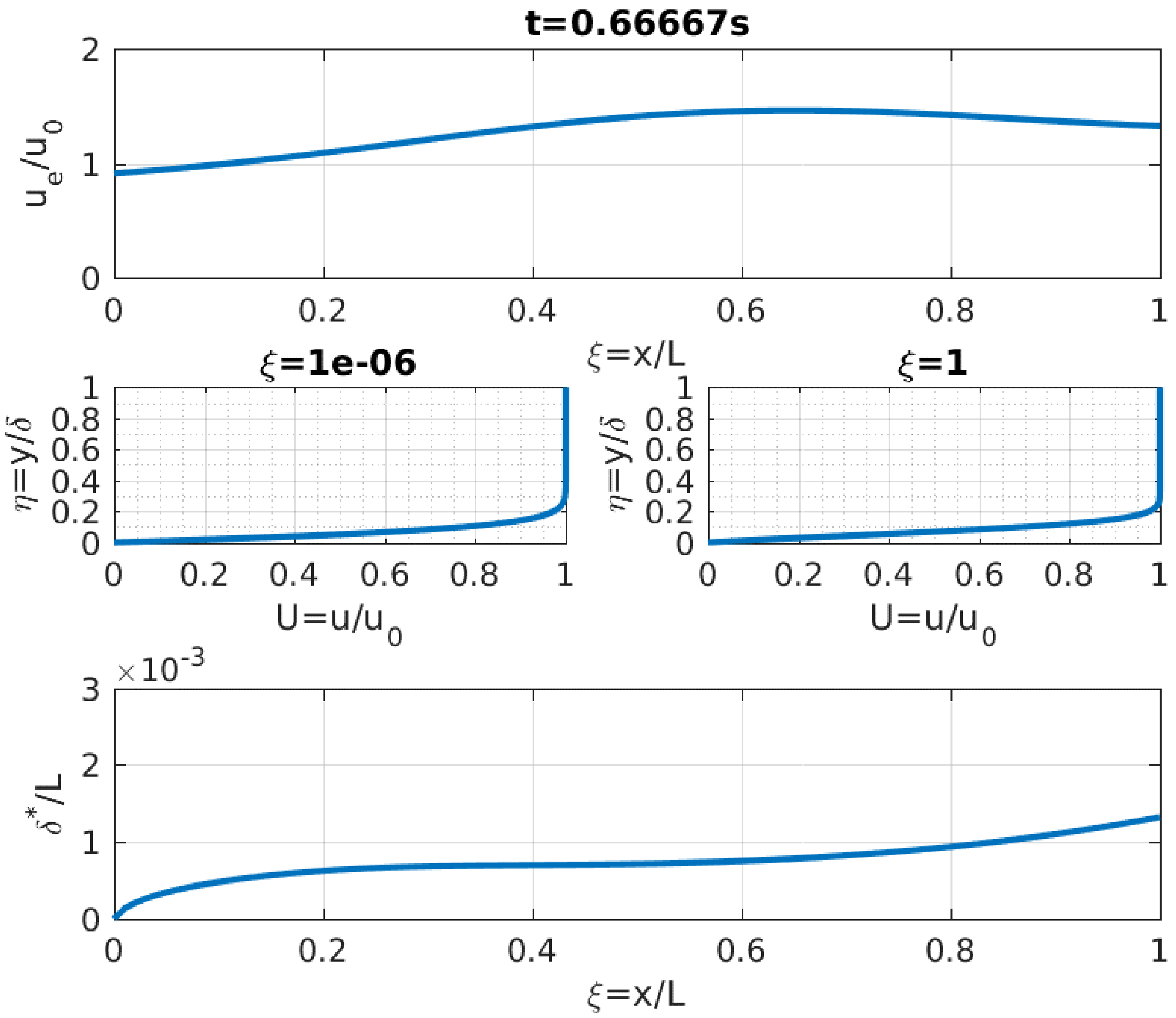
Results: Validation for Unsteady Case with Spatial Variations – Finite Difference



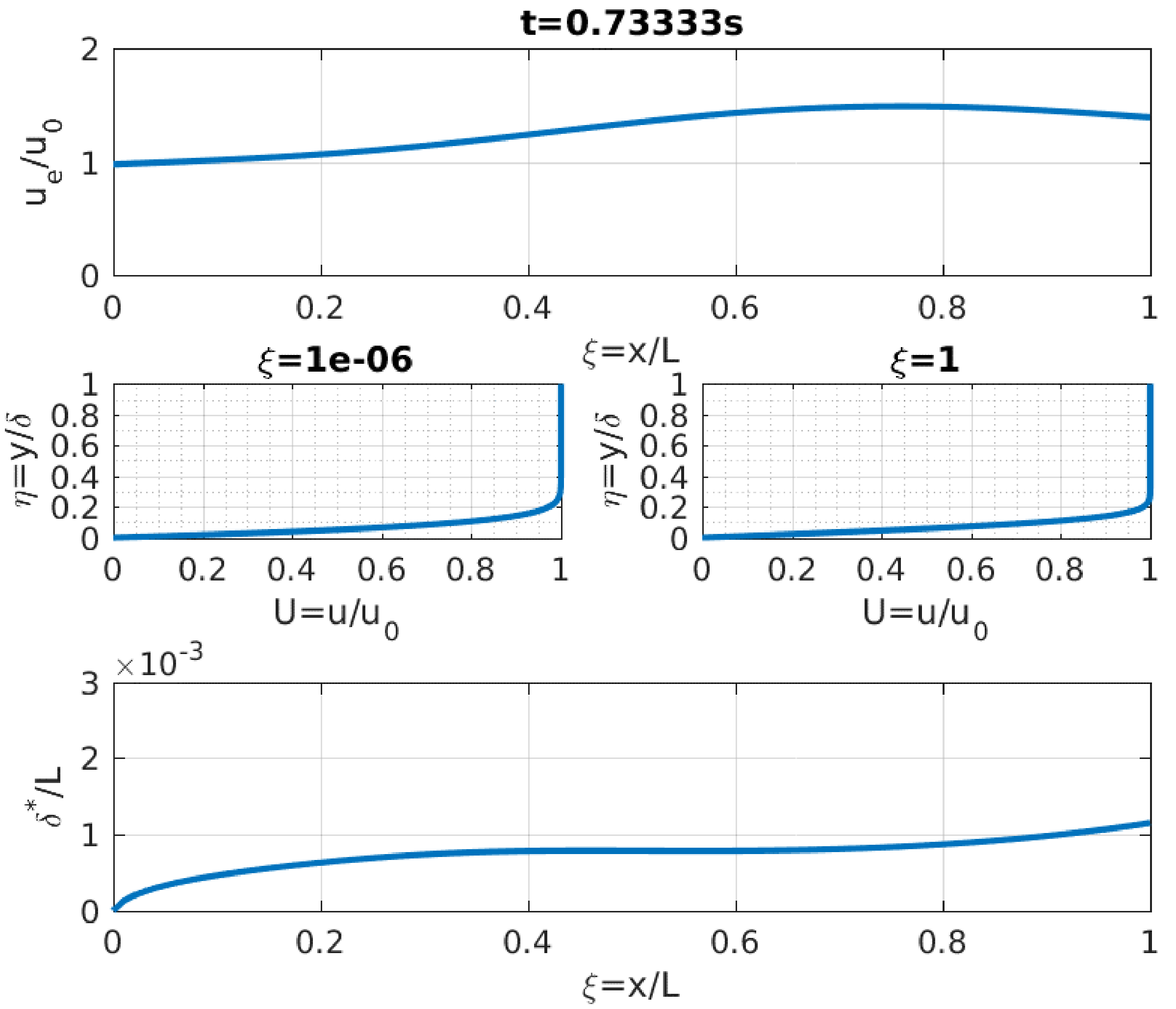
Results: Validation for Unsteady Case with Spatial Variations – Finite Difference



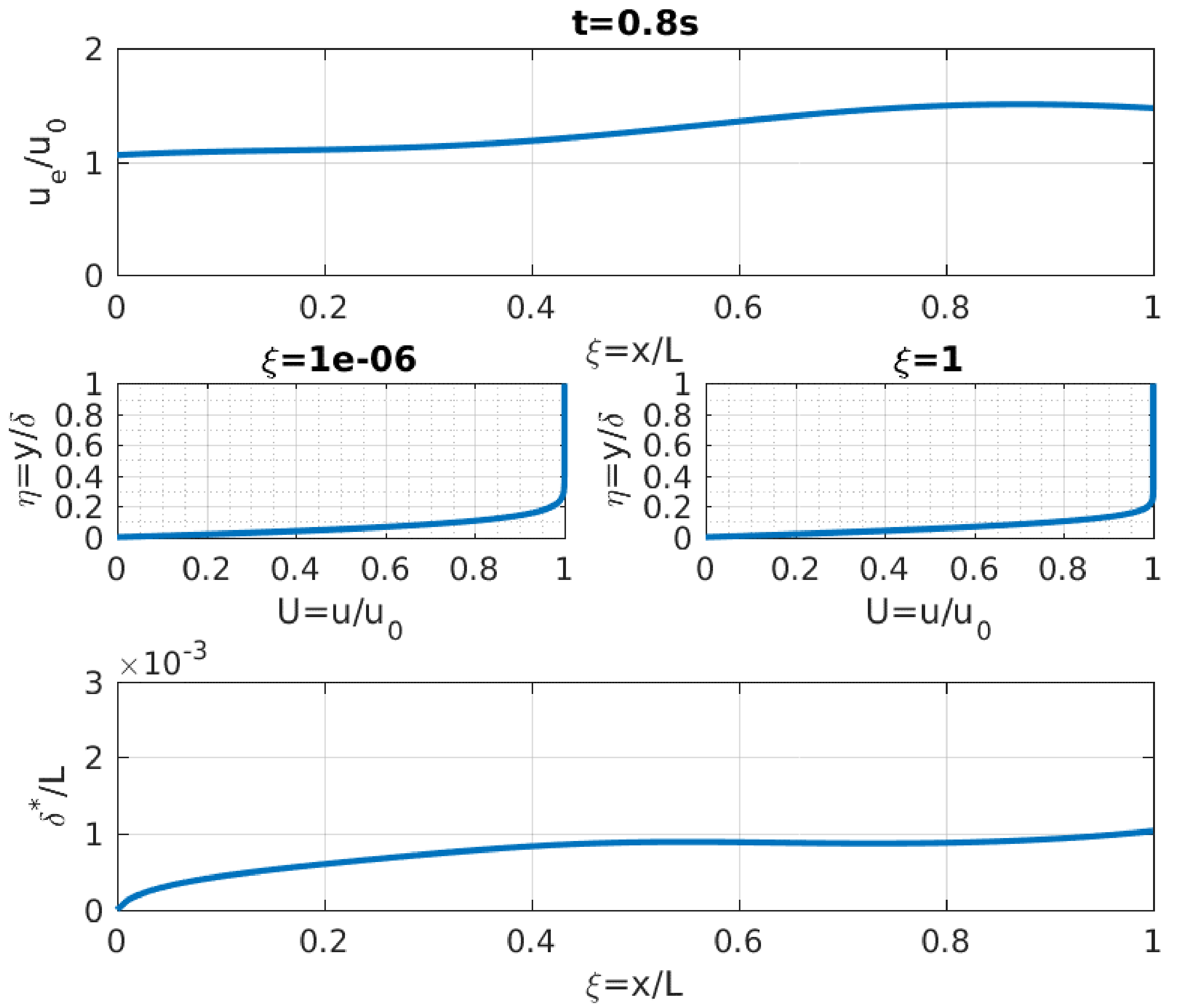
Results: Validation for Unsteady Case with Spatial Variations – Finite Difference



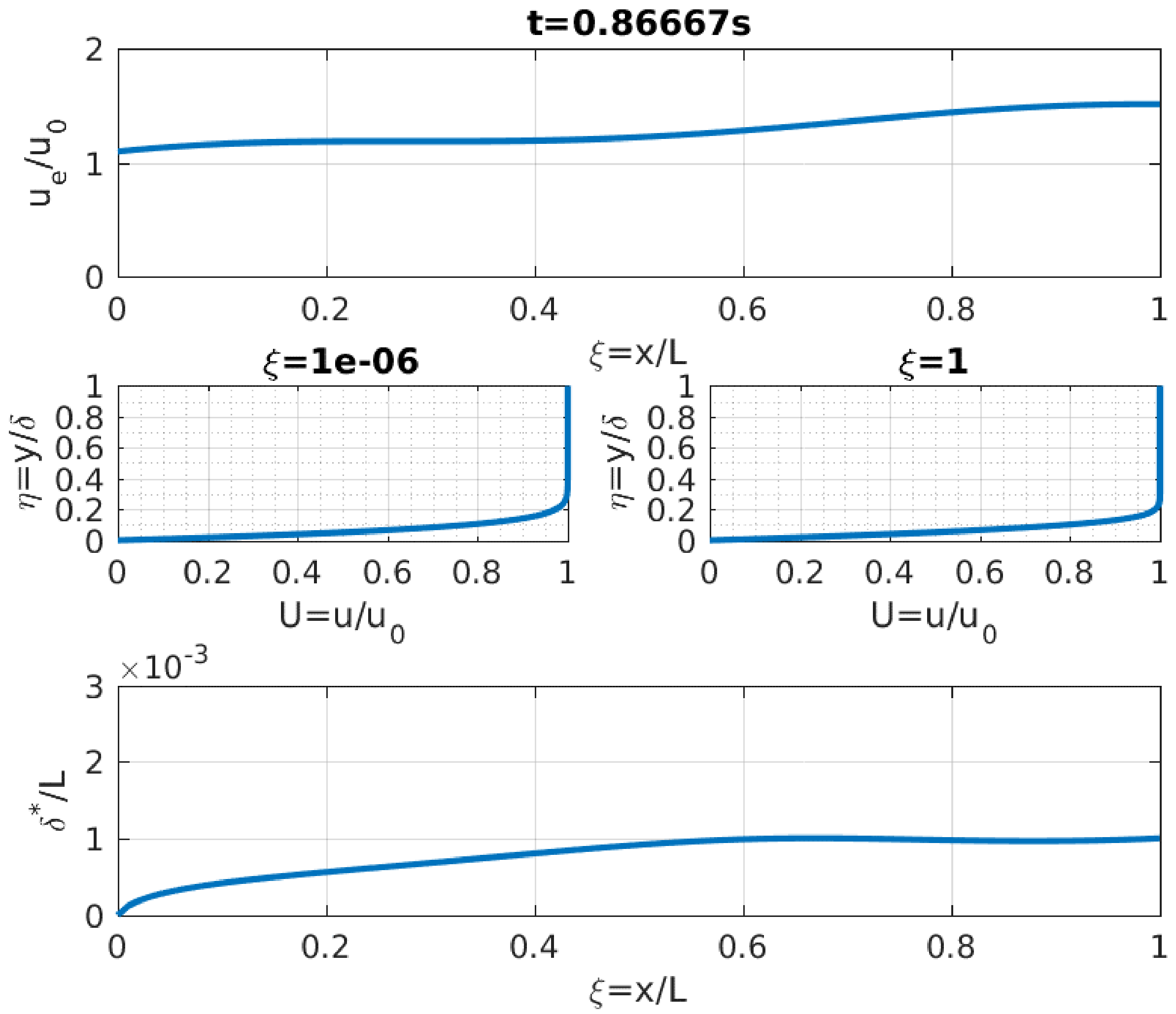
Results: Validation for Unsteady Case with Spatial Variations – Finite Difference



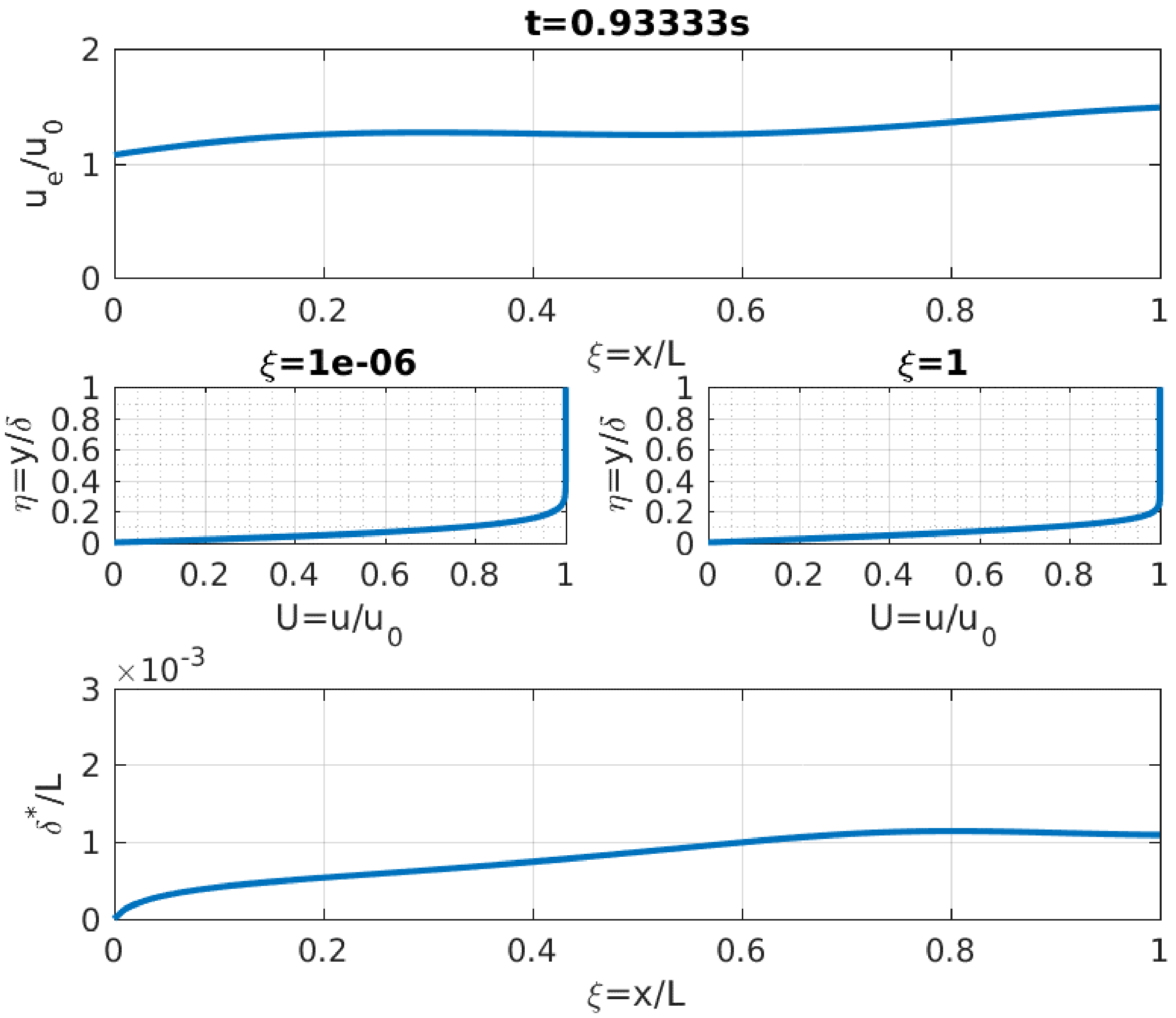
Results: Validation for Unsteady Case with Spatial Variations – Finite Difference



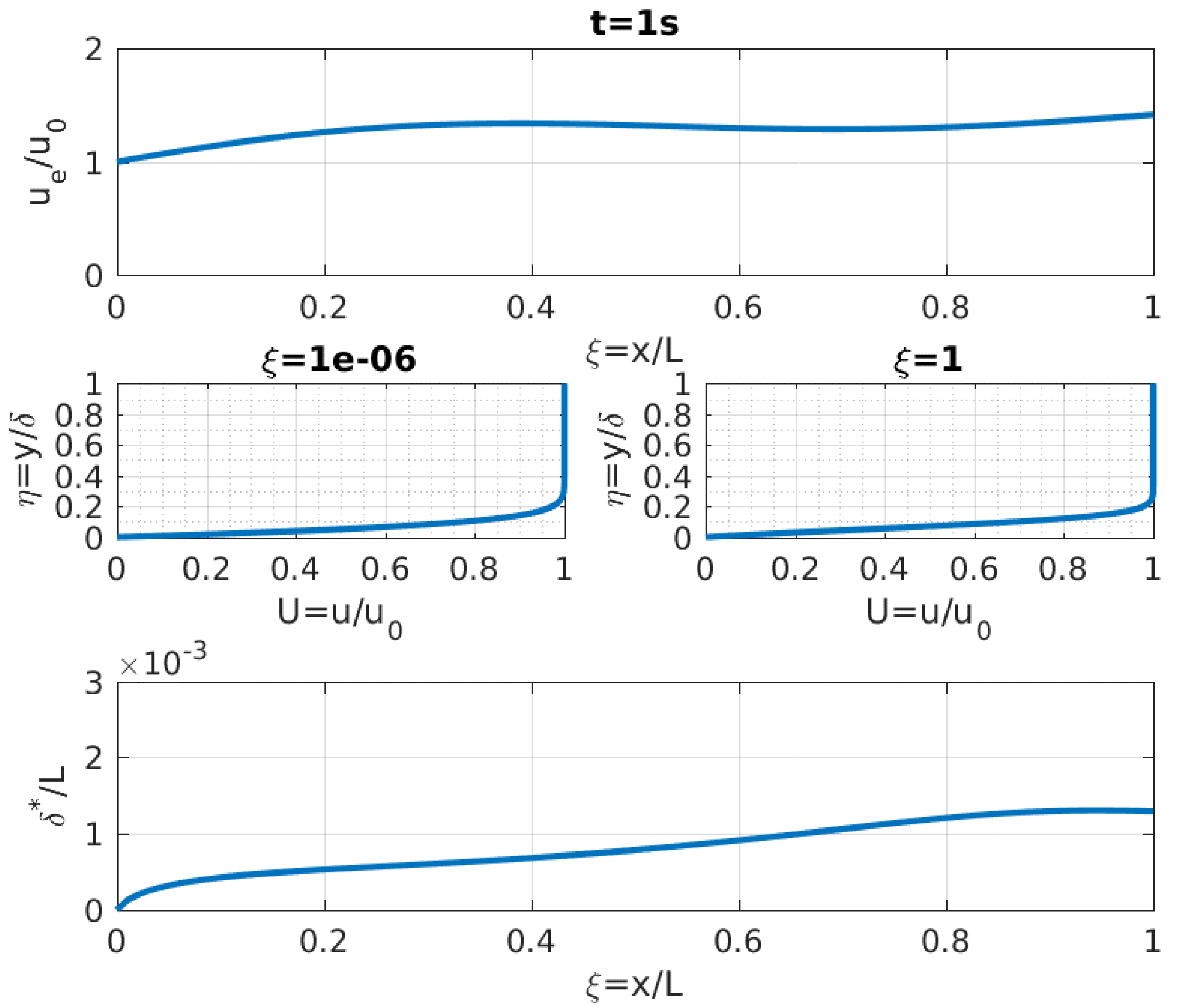
Results: Validation for Unsteady Case with Spatial Variations – Finite Difference



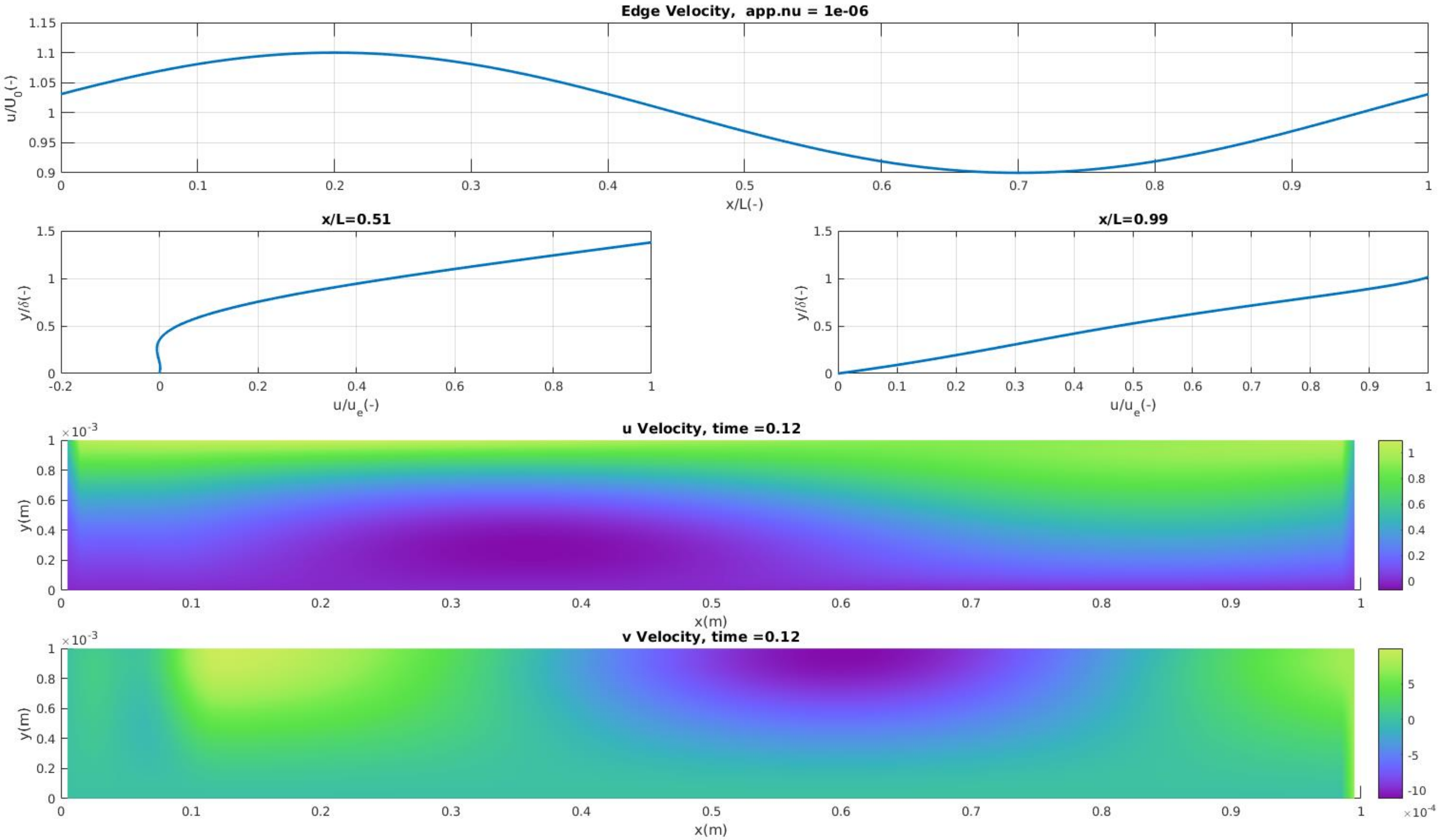
Results: Validation for Unsteady Case with Spatial Variations – Finite Difference



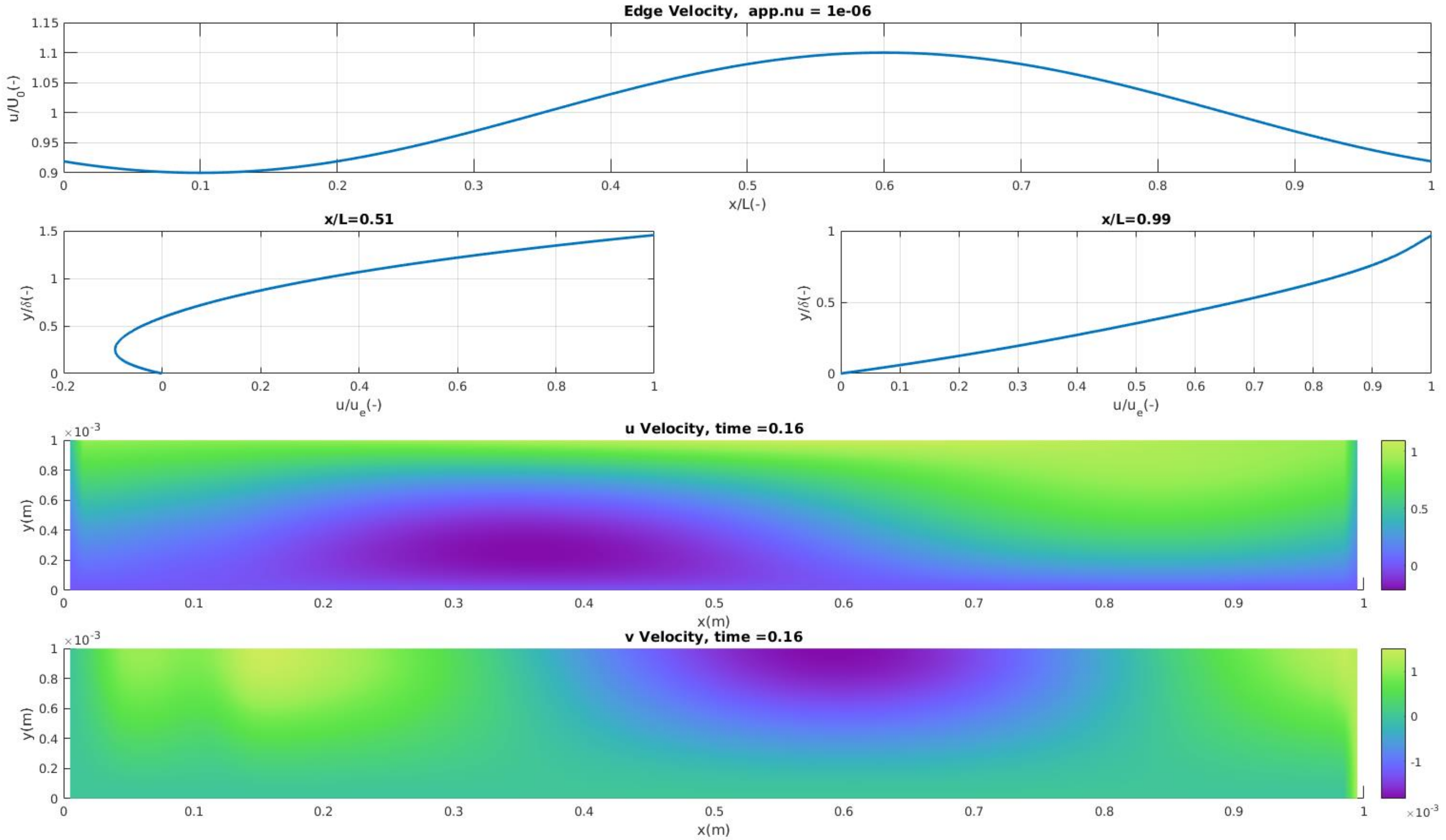
Results: Validation for Unsteady Case with Spatial Variations – Finite Difference



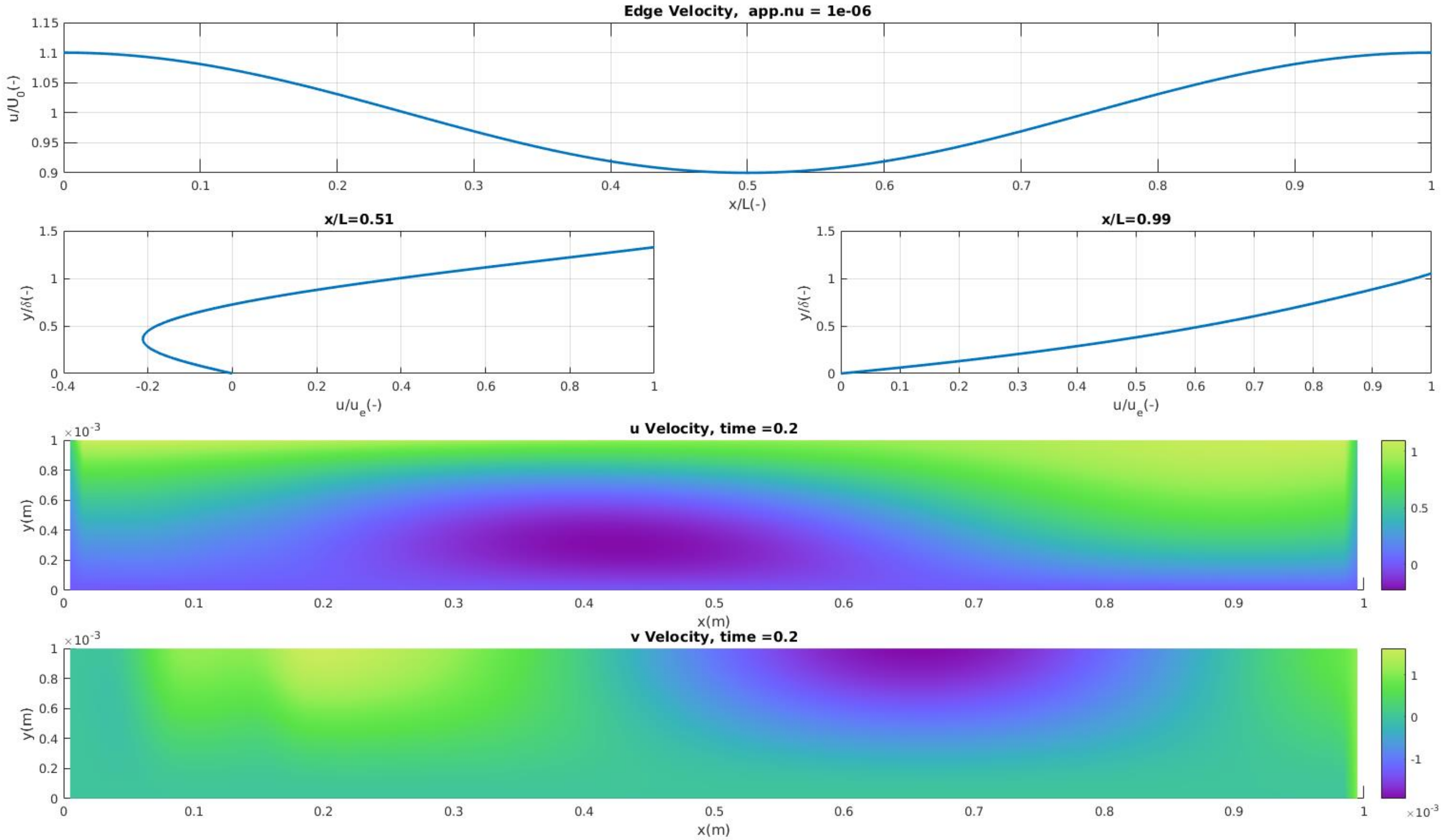
Results: Validation for Unsteady Case with Spatial Variations – Finite Volume



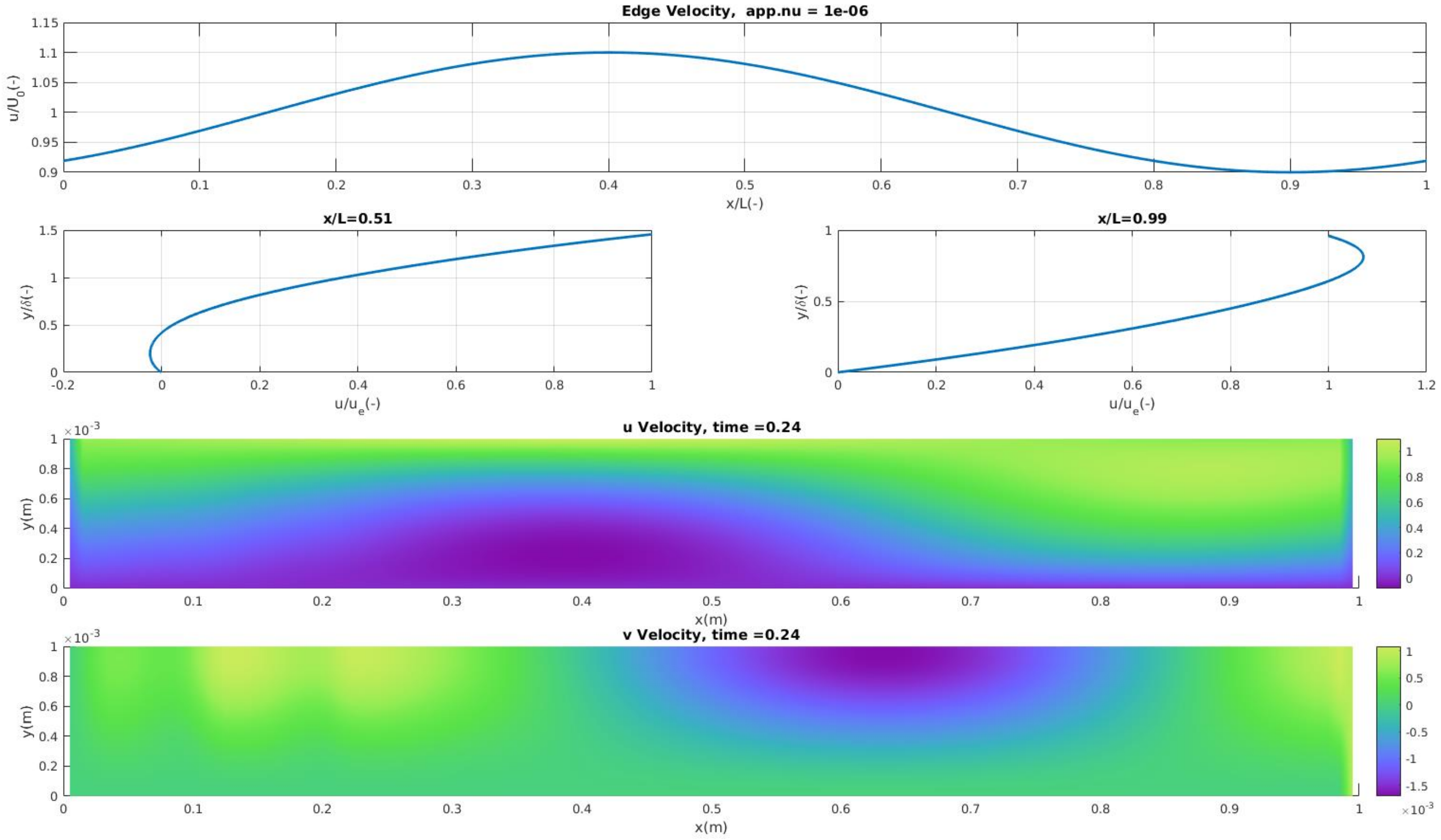
Results: Validation for Unsteady Case with Spatial Variations – Finite Volume



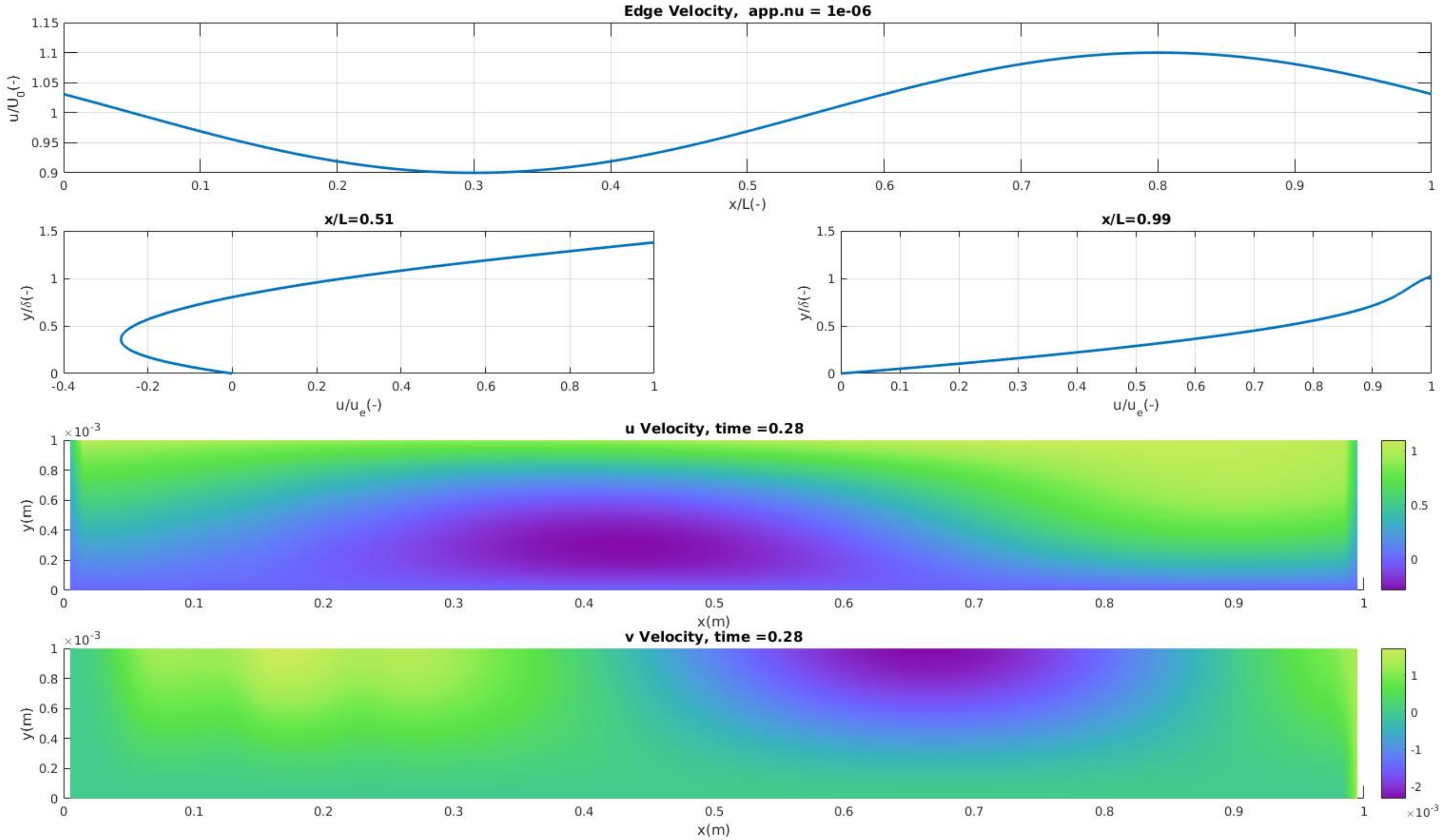
Results: Validation for Unsteady Case with Spatial Variations – Finite Volume



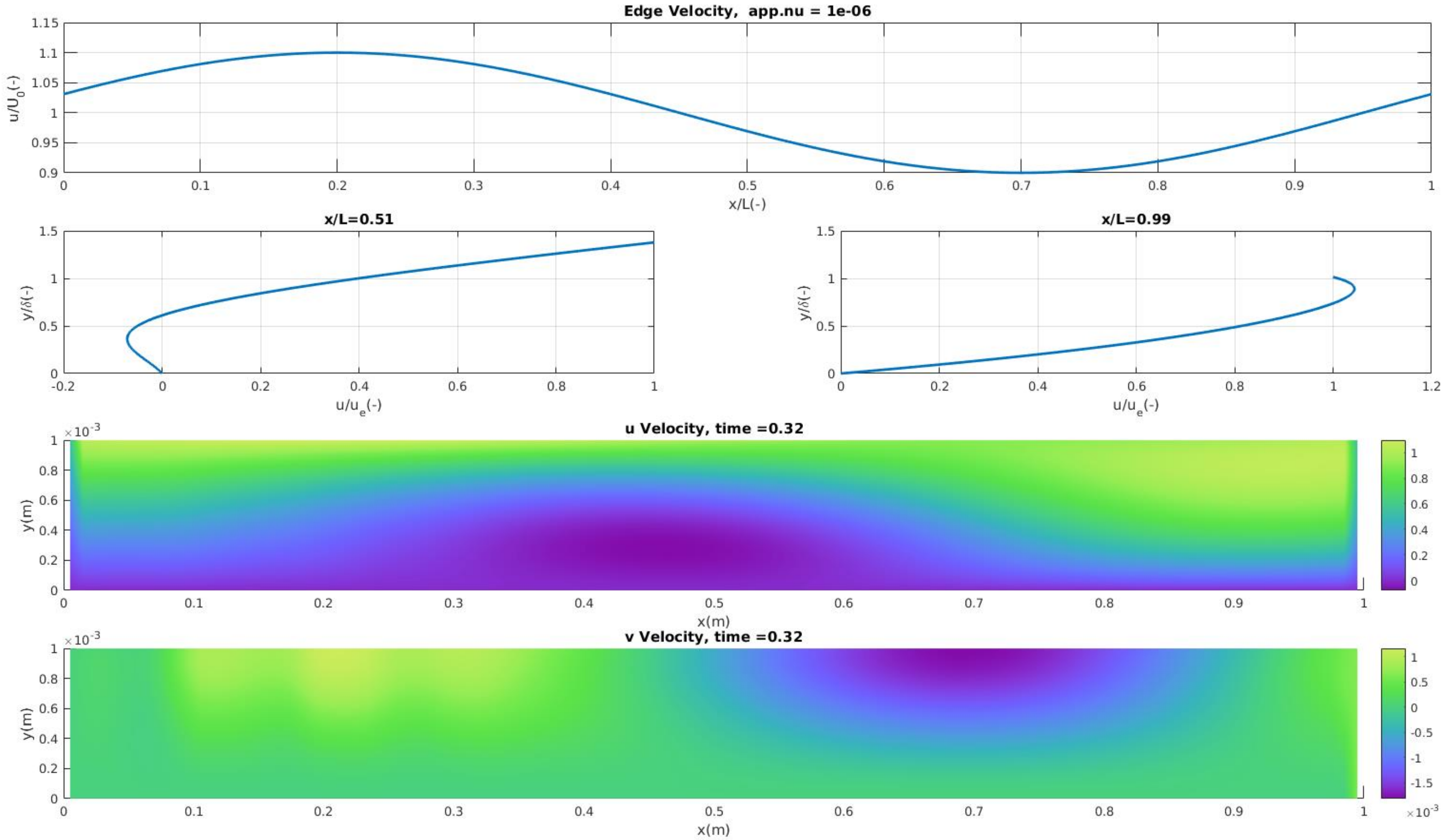
Results: Validation for Unsteady Case with Spatial Variations – Finite Volume



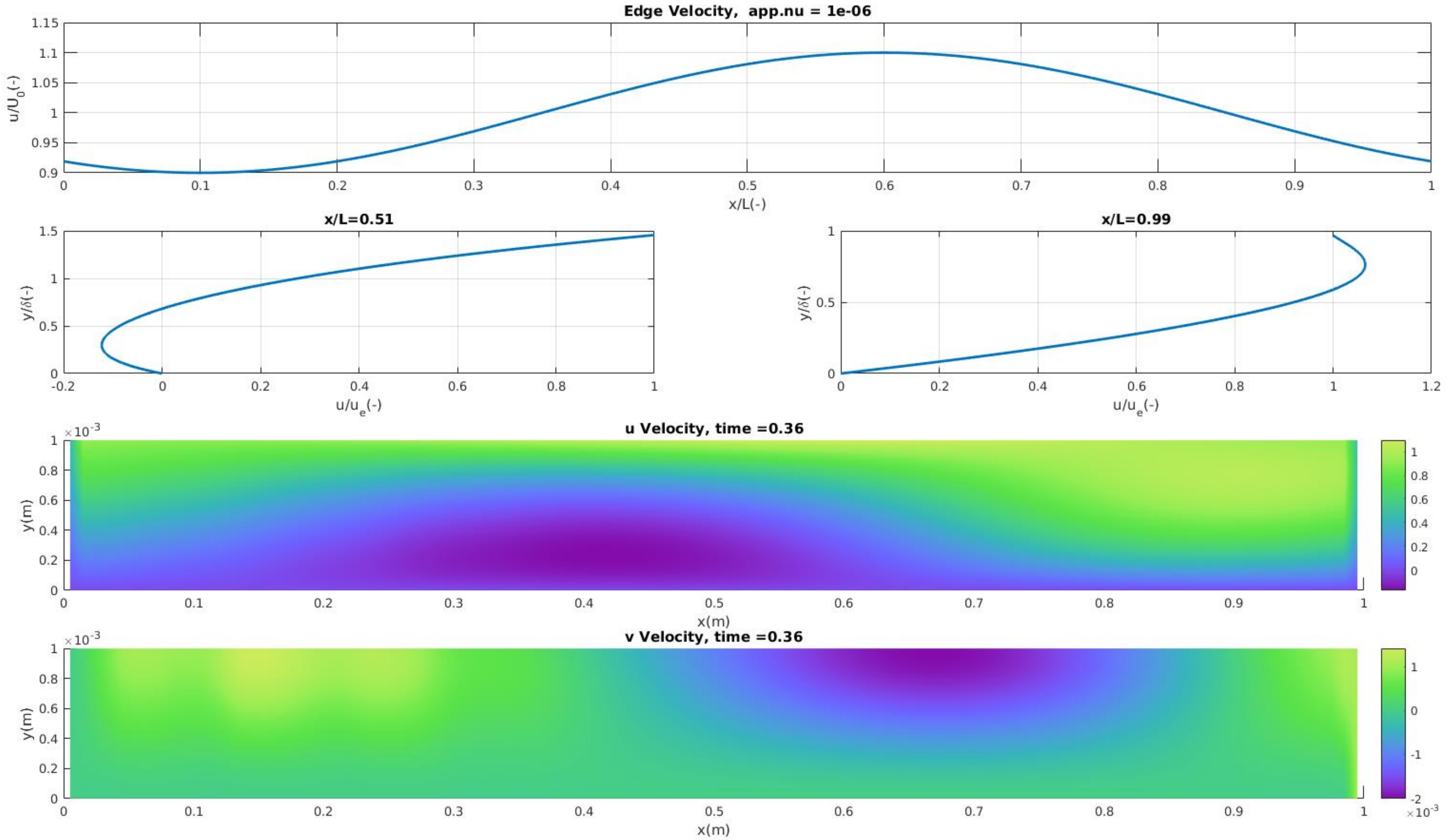
Results: Validation for Unsteady Case with Spatial Variations – Finite Volume



Results: Validation for Unsteady Case with Spatial Variations – Finite Volume



Results: Validation for Unsteady Case with Spatial Variations – Finite Volume



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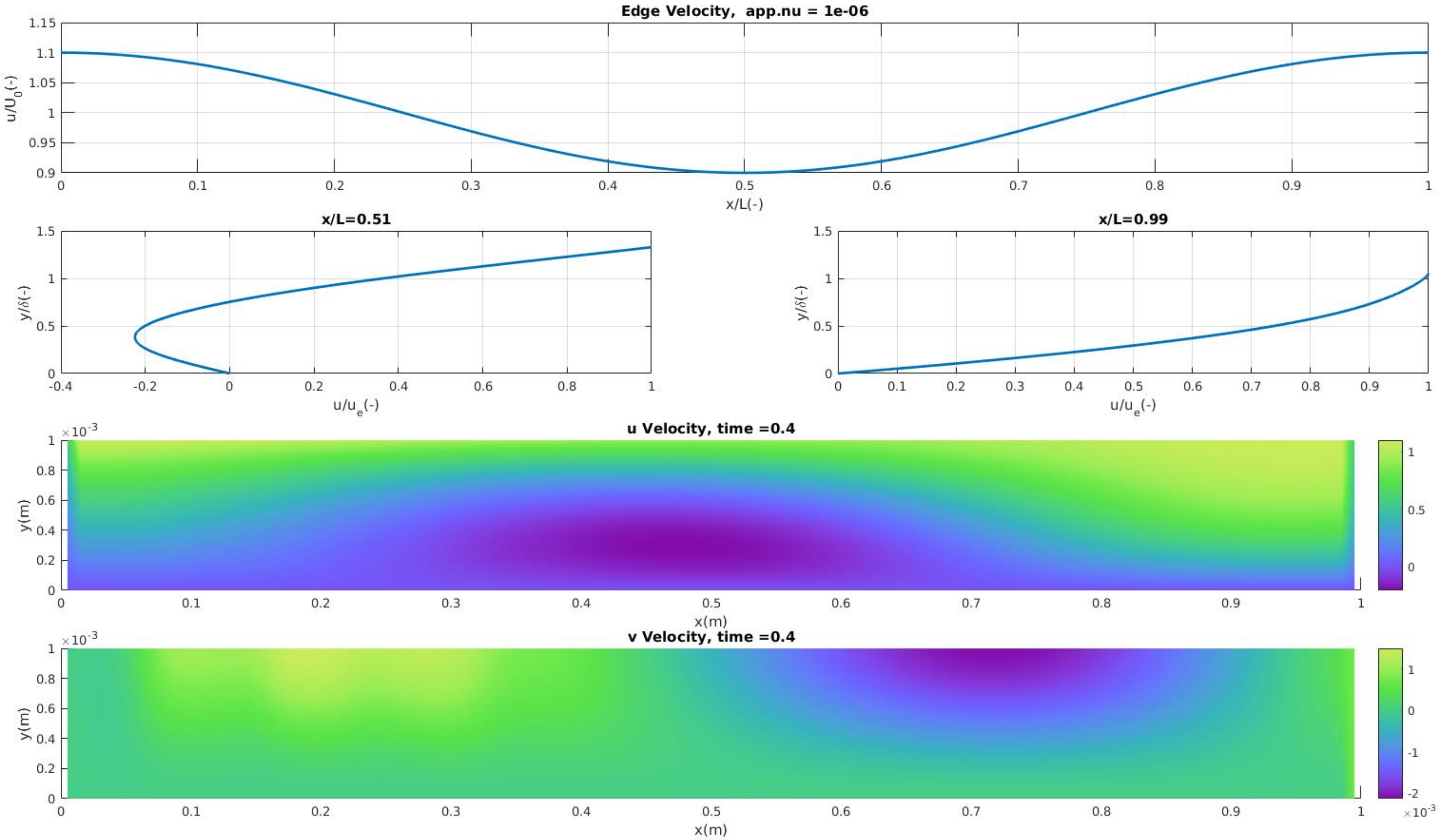
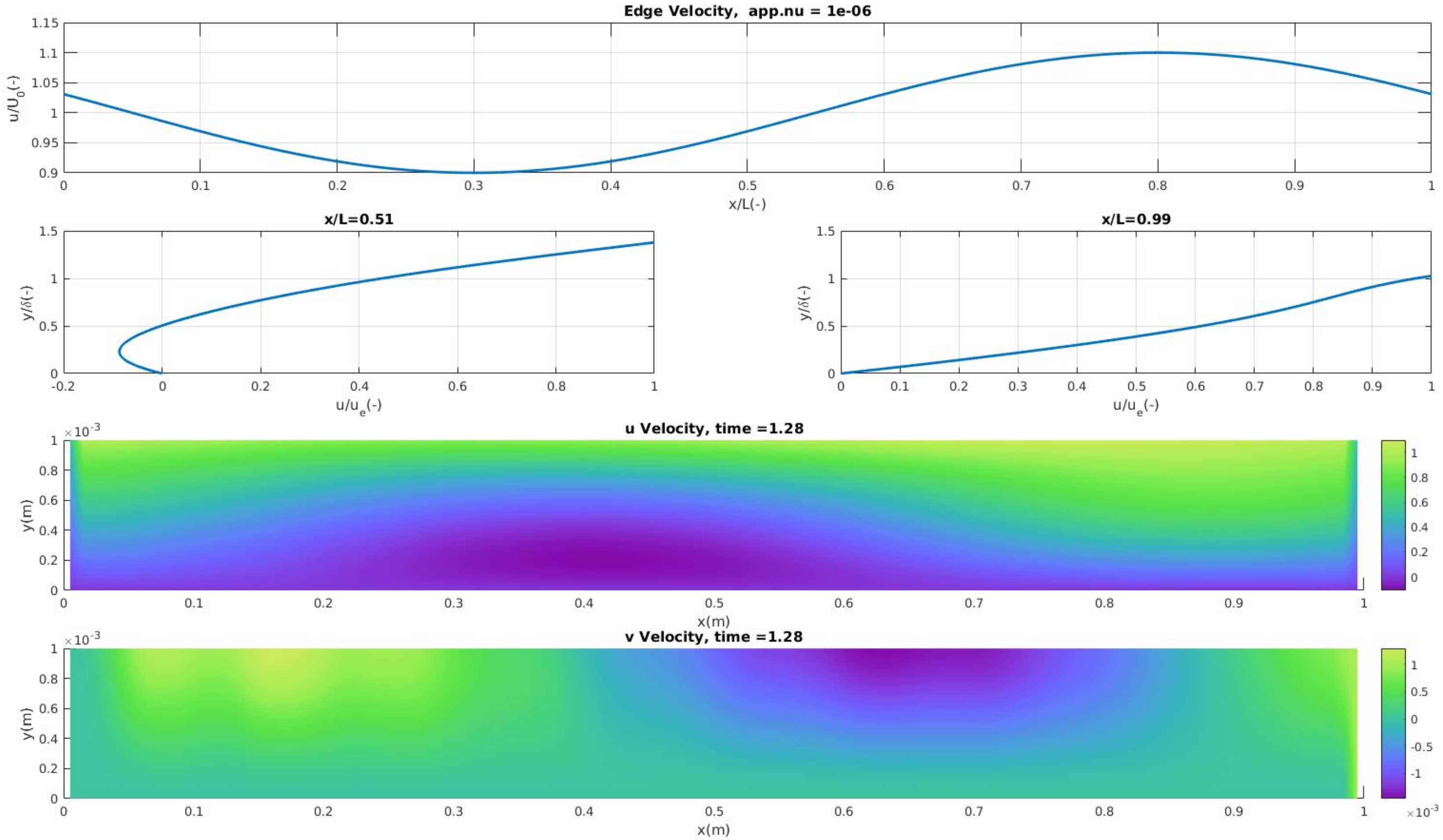


Results: Validation for Unsteady Case with Spatial Variations – Finite Volume



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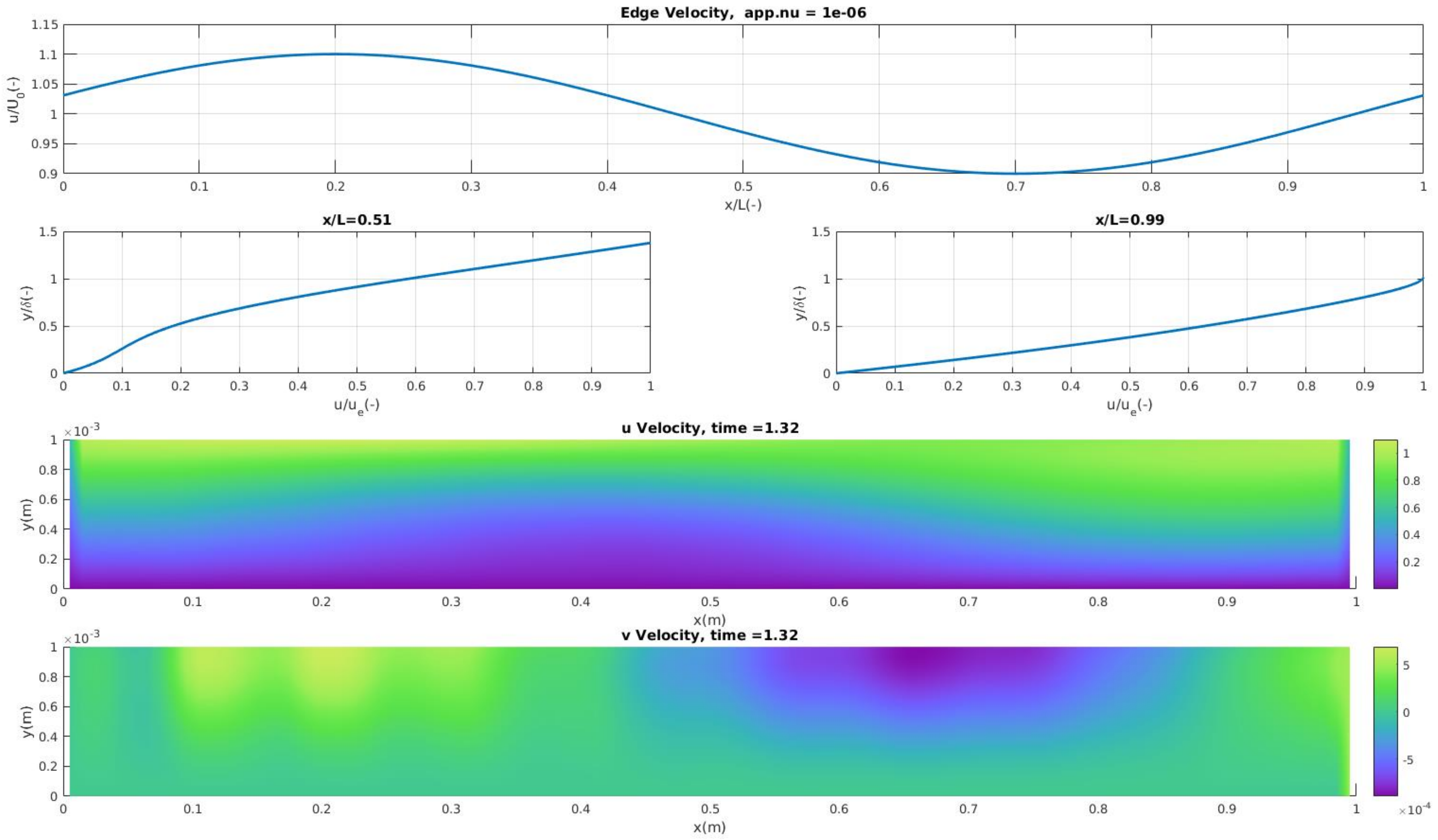
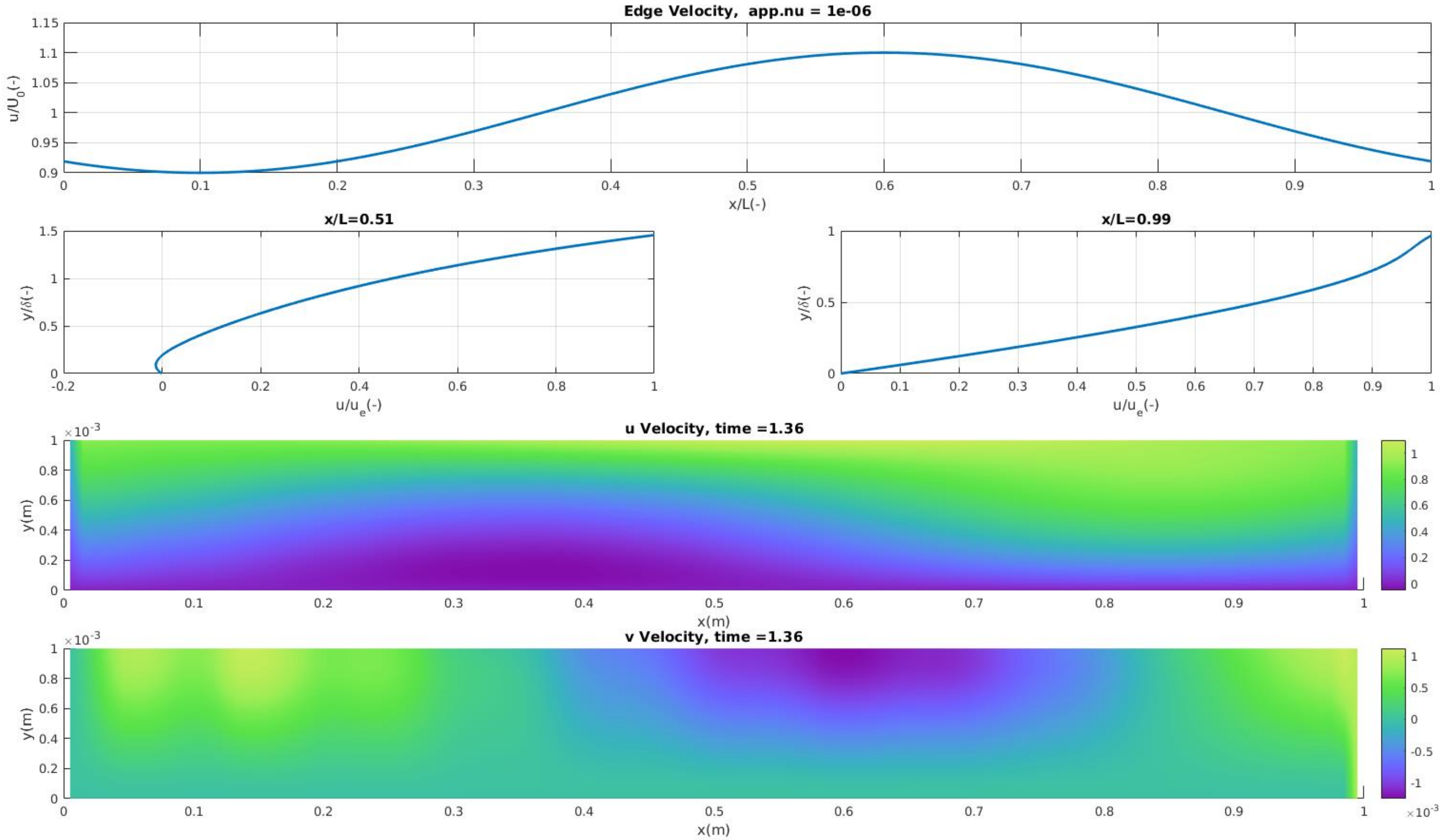
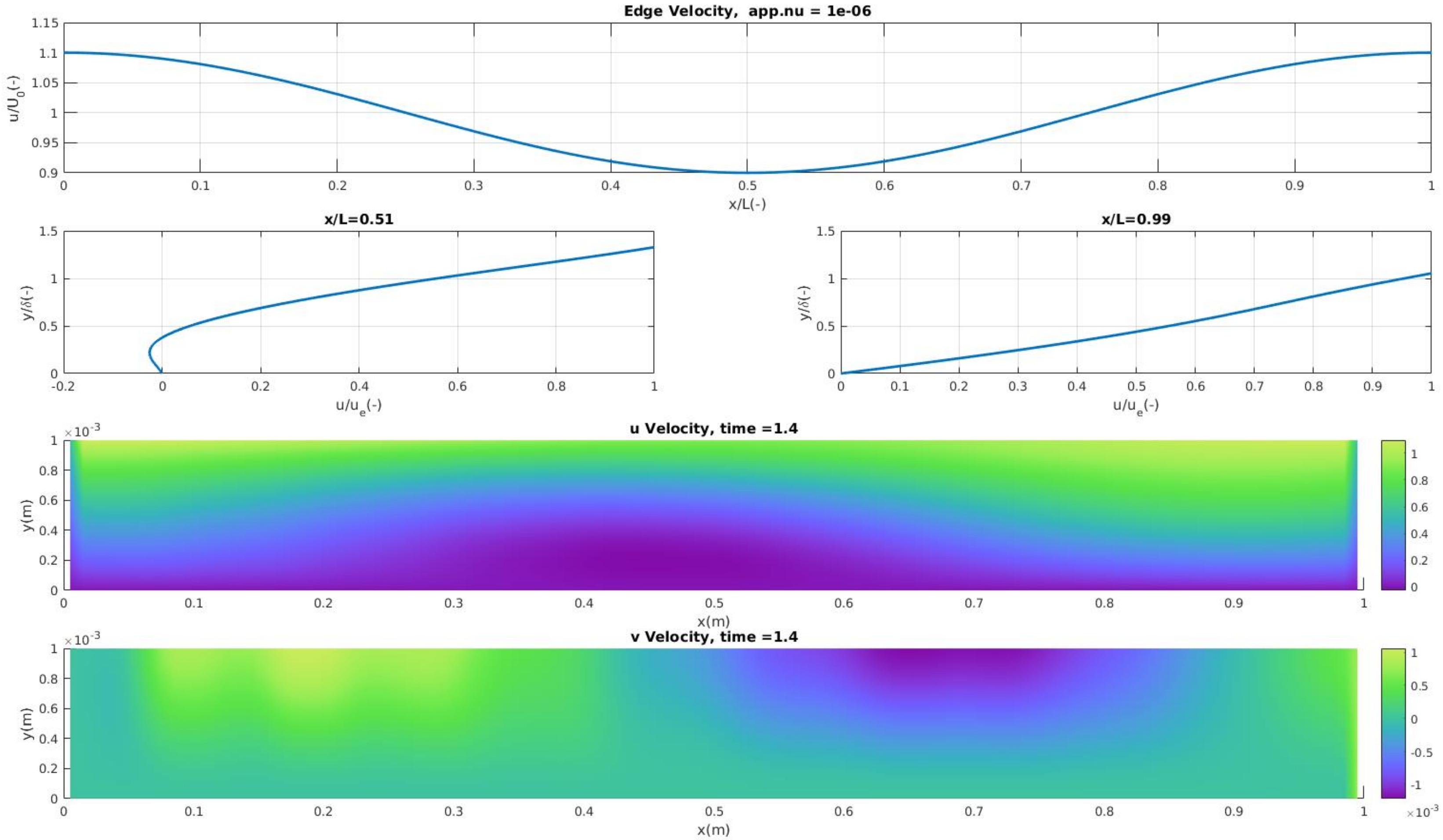


Results: Validation for Unsteady Case with Spatial Variations – Finite Volume



Results: Validation for Unsteady Case with Spatial Variations – Finite Volume



Conclusion: Explaining Discrepancies & Potential Issues

- Finite-difference code doesn’t have an interactive boundary-layer component → if pressure gradient is not favorable, detects **numerical flow separation** and doesn’t produce physical results → code needs more work

- **Rate of unsteady response** of BL to fluctuating BCs does not match up between finite-difference and finite volume → comparative analysis needs further investigation

- **Velocity profiles** from two codes agree for certain time stamps and disagree for others → analytical & numerical assessment needs further work to understand discrepancies

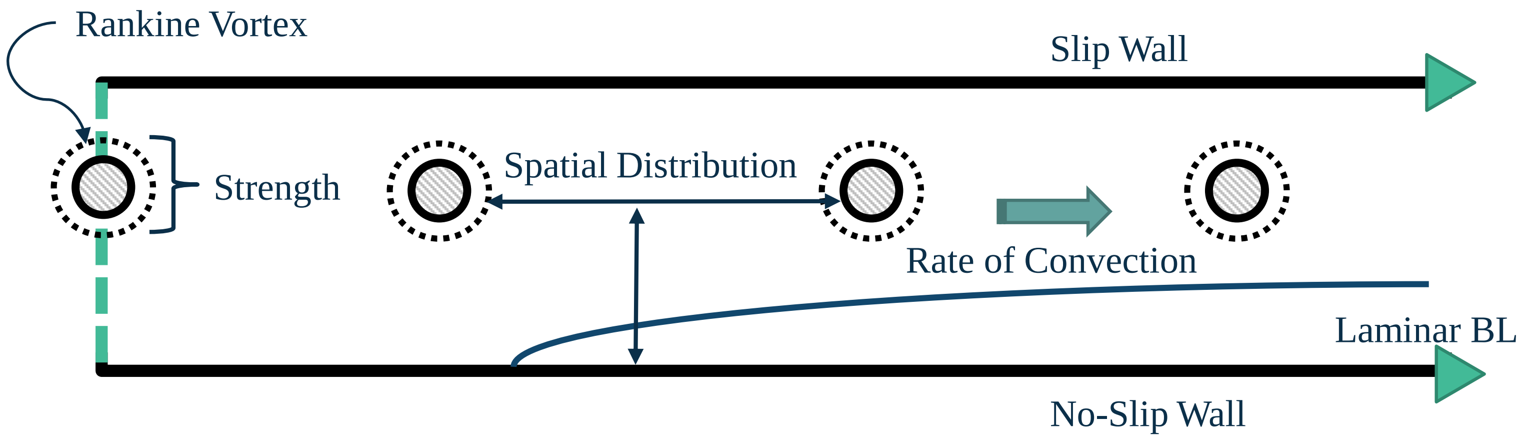
- **BCs** imposed in two different codes don’t always physically represent the same BCs → test cases need more exploration for BCs

Conclusion: Advantages of Finite-Difference Method

Integral Boundary Layer Methods	2.29 Finite Volume Code	Thin-Shear Layer Finite Difference Method
Relies on closure relations & curve fit functions → results **not valid** for **large** free-stream **variations** & fully **unsteady** response	Makes no assumptions and doesn't rely on closure relations (laminar) → results **valid for any range** of space- and time- variations in free-stream velocity	Only makes thin-shear layer assumption → results **valid for any range** of space- and time- variations in free-stream velocity
Requires only **one grid point** in y-direction → solves for integrated BL characteristics	Requires grid for entire domain → solves for **entire domain**	Scales grid based on BL thickness → only solves for **flow inside BL** Flow **separation issues** if interacting BL algorithm not implemented
Uses integrated values + closure relations → computational **cost & time** extremely low	Uses uniform grid → computational cost increases as mesh needs to be refined near wall	Uses scale transformed grid → computational **cost & time** significantly lower
-	CFL condition triggered → need **extremely small time step** to match small mesh size	Fully-implicit scheme → stable even for **large time steps**

Conclusion: Future Work

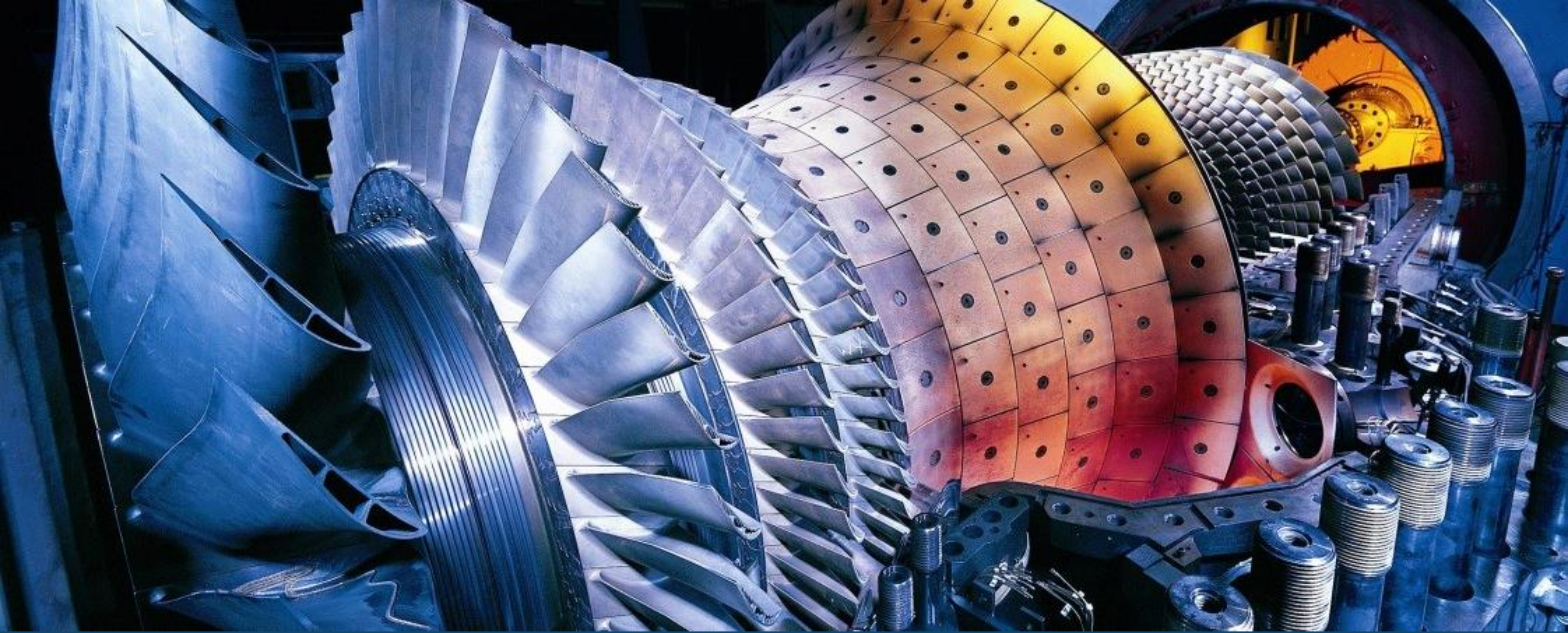
- Debug issues and assess finite difference results as compared to finite volume results
- Introduce vertical disturbances that are more characteristic of free-stream turbulence



- Translate analysis to turbulent boundary layers
- Translate analysis to high-pressure turbine blade geometry
- Incorporate combustor turbulence data from LES models as input condition for unsteady BL code

Thank You!

Prof. Lermusiaux and Wael for all the help with my project and for answering my incessant questions



Questions?

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