Multiscale Model for Pedestrian Dynamics – Review and Implementation

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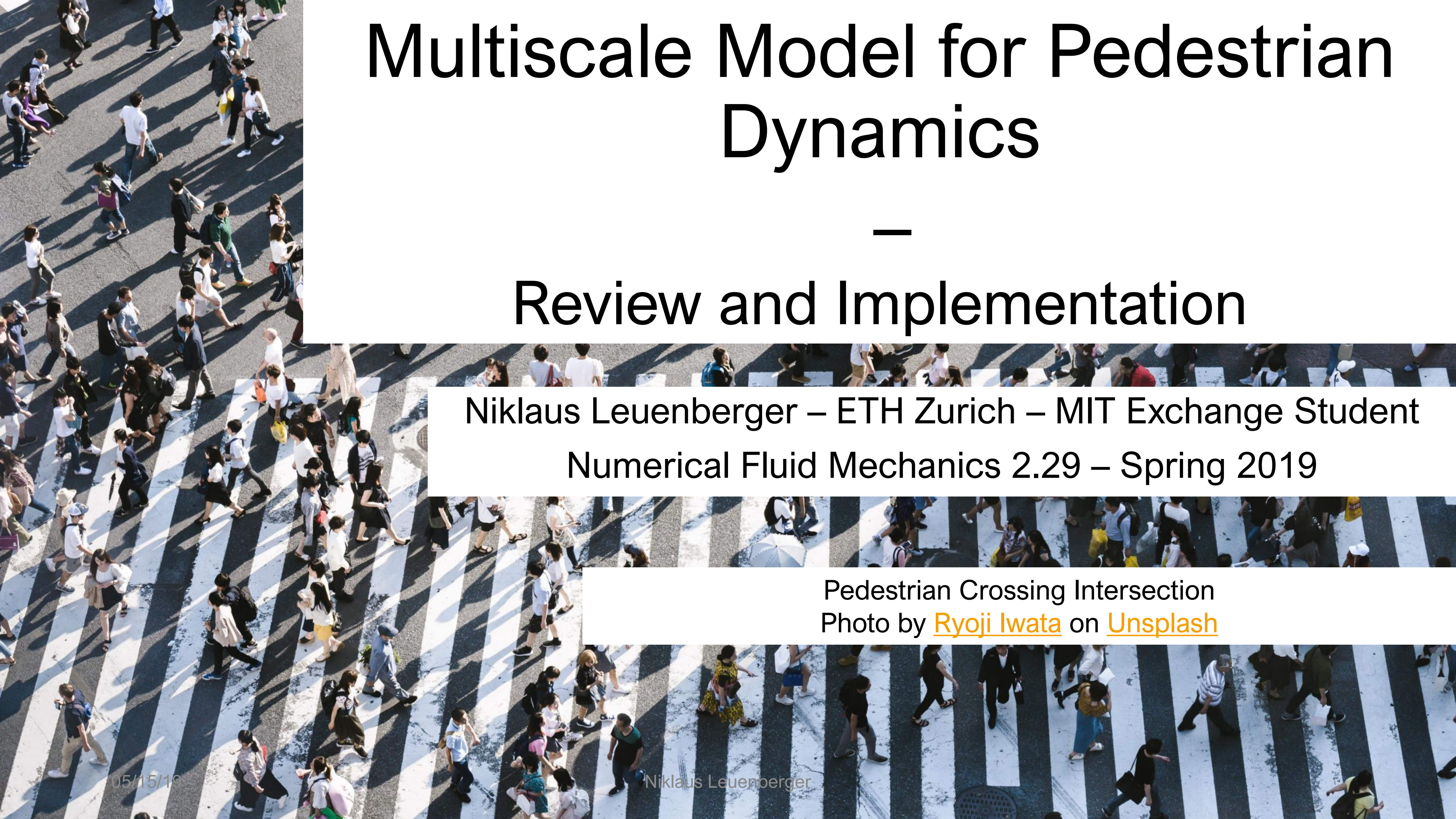
Numerical Fluid Mechanics 2.29 – Spring 2019



Pedestrian Crossing Intersection
Photo by Ryoji Iwata on Unsplash

References

- Cristiani, Emiliano, Benedetto Piccoli, and Andrea Tosin. *Multiscale modeling of pedestrian dynamics*. Vol. 12. Springer, 2014.

Connection to Fluid Dynamics

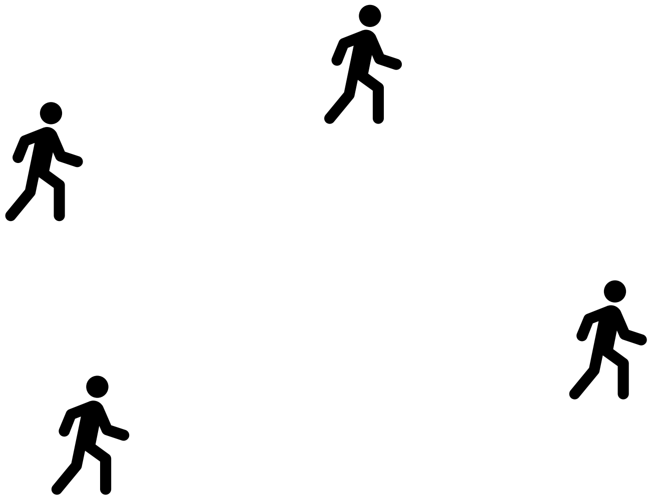
Continuum Fluid Mechanics	Rarefied Gas Dynamics	Molecular Dynamics
↕	↕	↕
Macroscopic Modeling	Multiscale Model	Microscopic Model

Governing Equations

μ_t Measure at time t.

$\mu_t(E) \geq 0$ Number of Pedstrians in *E* at time t.

$$\frac{\partial \mu_t}{\partial t} + \nabla \cdot (\mu_t v) = 0$$



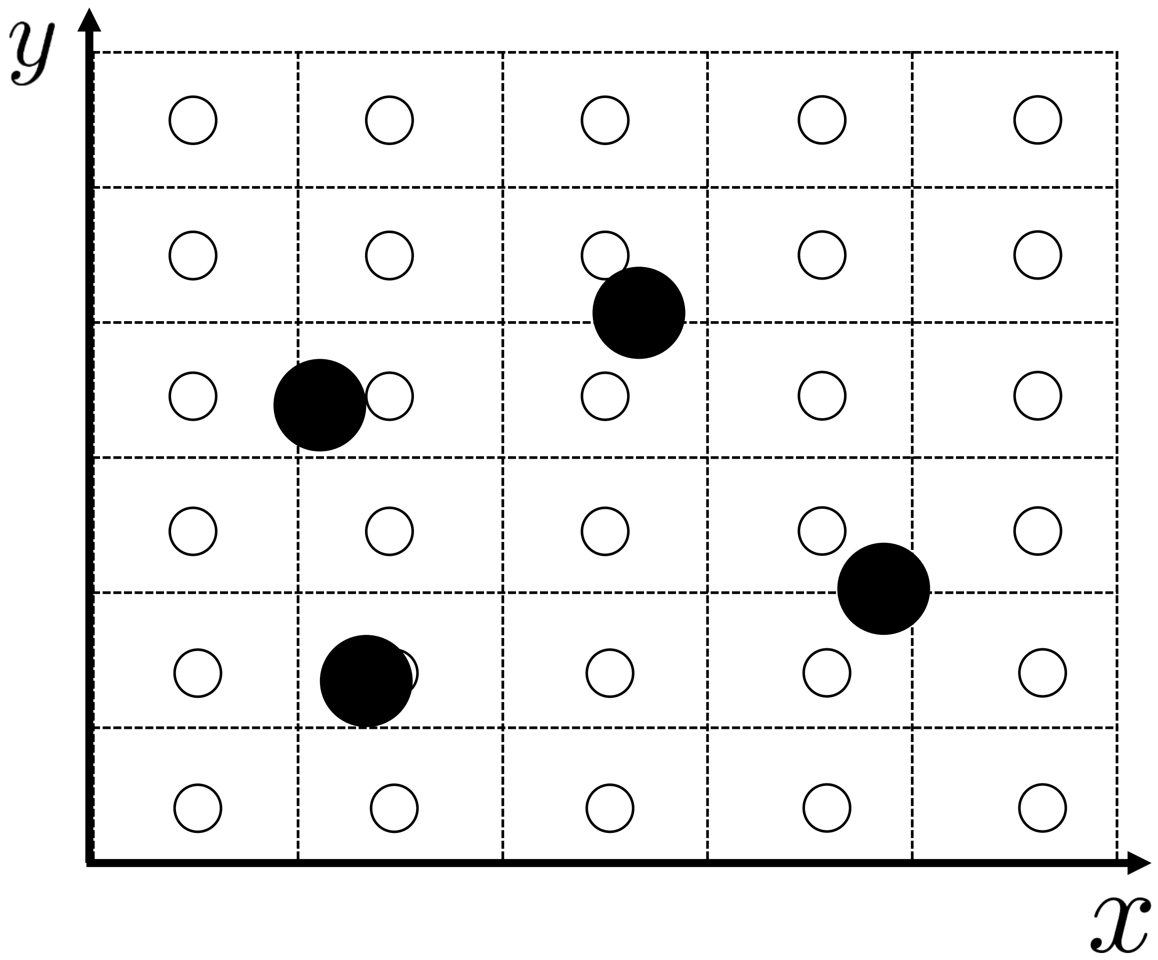
Discretization

Pedestrians, track the…
- position
- velocity

Lagrangian representation

Cells, has…
- velocity of a cell
- density in a cell

Eulerian representation



Discretization

microscopic

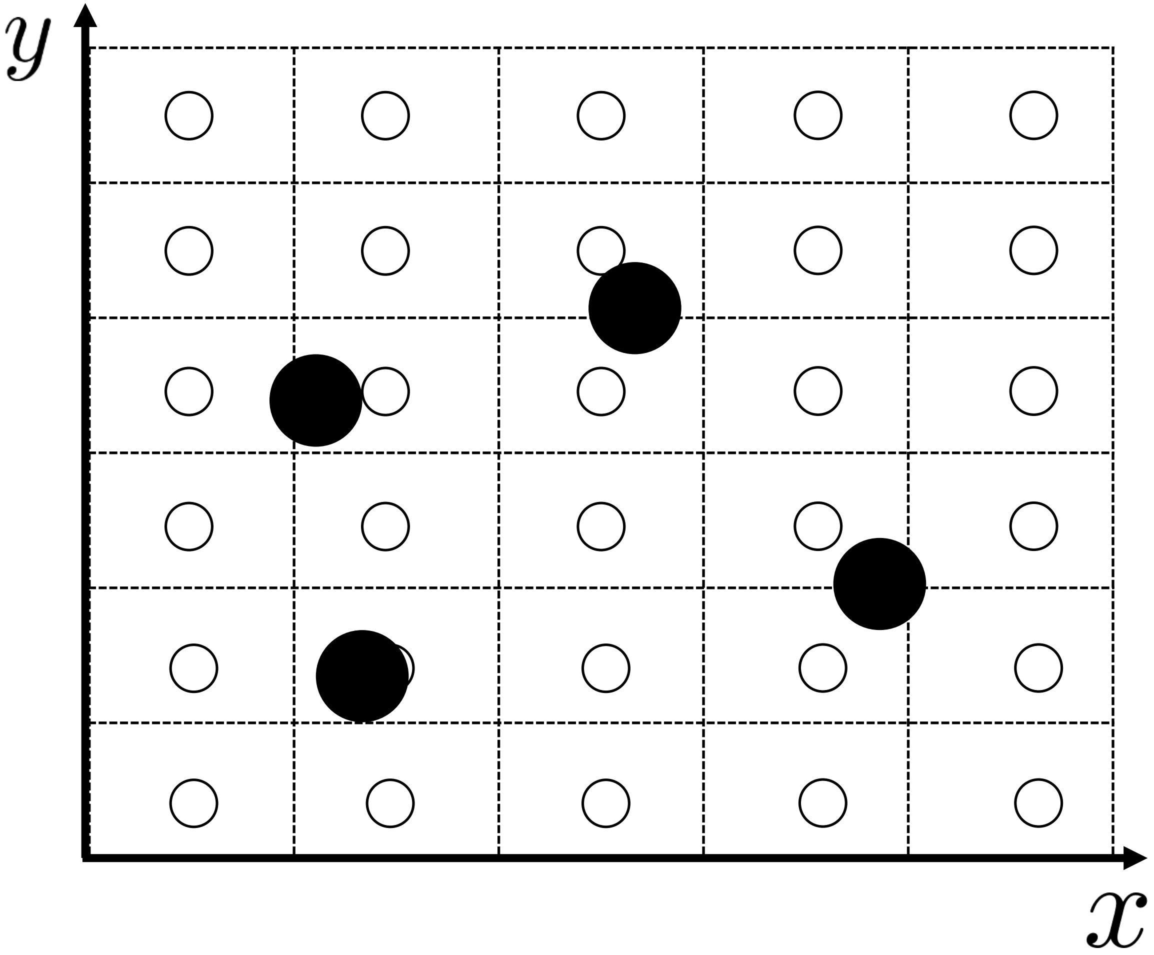
<u>Pedestrians, track the…</u>
- position
- velocity

Lagrangian representation

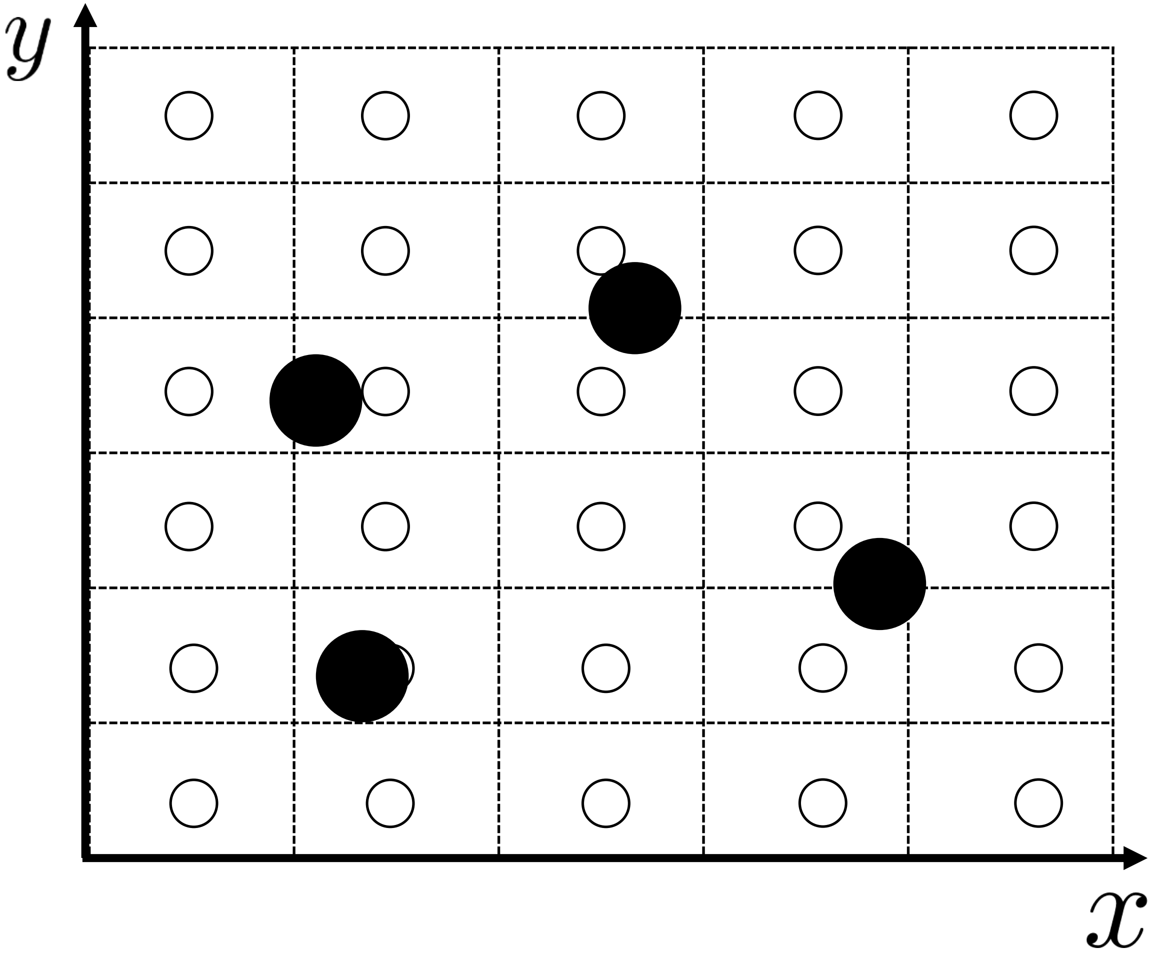
macroscopic

<u>Cells, has…</u>
- velocity of a cell
- density in a cell

Eulerian representation



Multiscale Level – Governing Equations



Multiscale level

Measure

$$\mu_t = \underbrace{\theta \sum_{k=1}^{N} \delta_{X^k(t)}}_{\text{microscopic}} + \underbrace{(1-\theta)\rho \cdot \mathscr{L}^d}_{\text{macroscopic}}$$

Equation

$$\dot{X}^k(t) = v[\mu_t](X^k(t)), \qquad k = 1 \ldots, N,$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v[\mu_t]) = 0,$$

Governing Equations

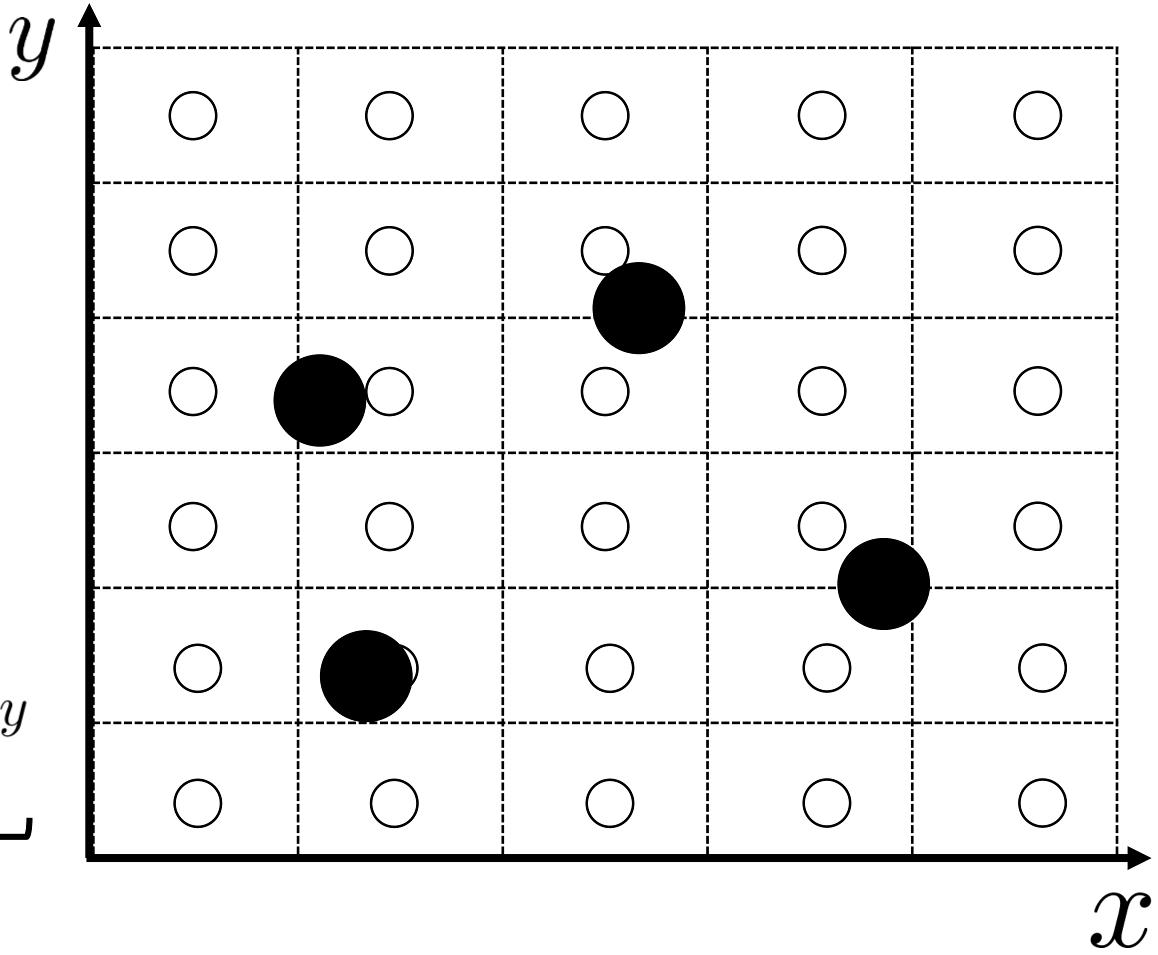
$$v[\mu_n](x) = v_d(x)$$

$$+\theta \sum_{\substack{k=1,\ldots,N \\ X_n^k \neq x}} f(|X_n^k - x|) g(\alpha_{xX_n^k}) \frac{X_n^k - x}{|X_n^k - x|}$$

microscopic

$$+(1-\theta)\Lambda \sum_{j \in Z^d} \rho_j^n \int_{E_j} f(|y-x|g(\alpha_{xy}) \frac{y-x}{|y-x|} dy$$

macroscopic

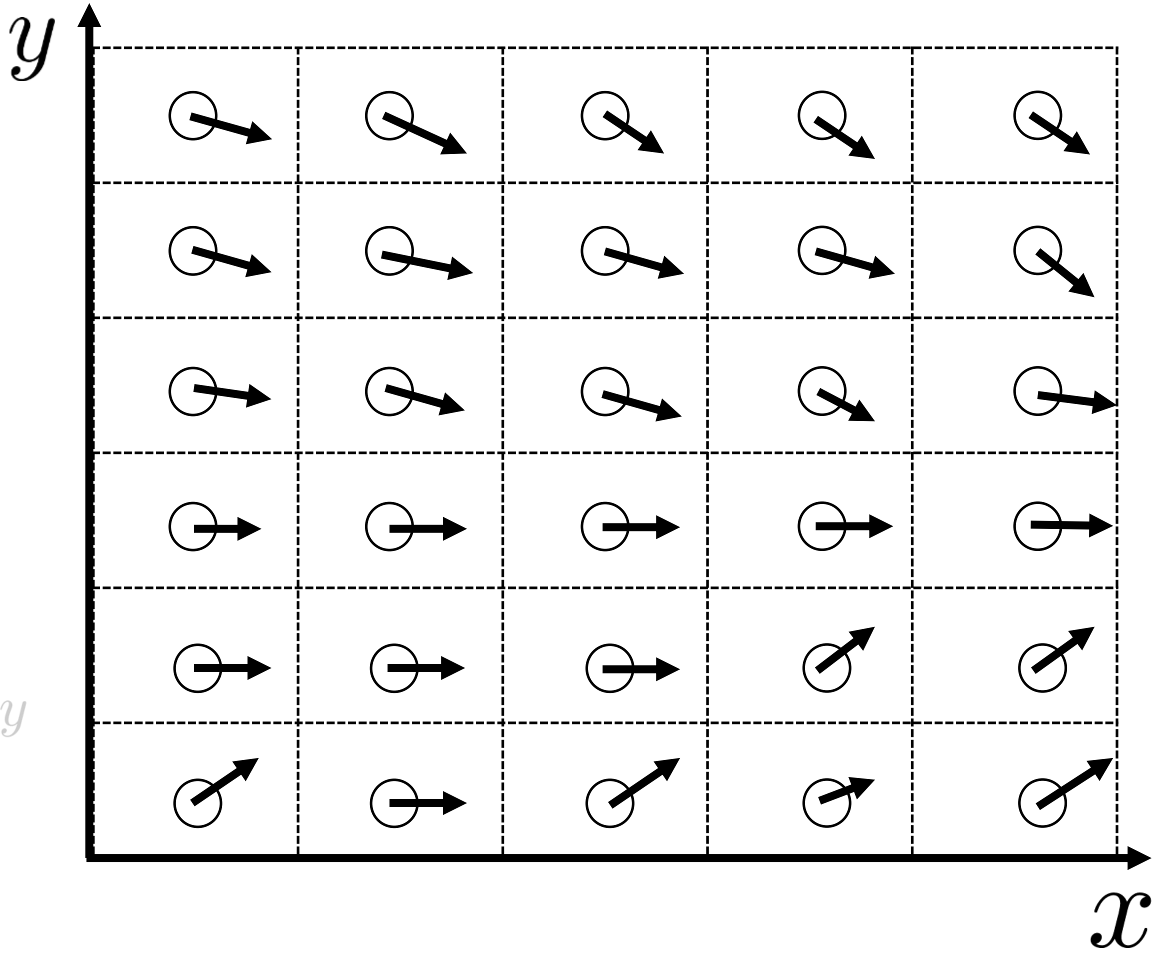


Desired Velocity

$$v[\mu_n](x) = v_d(x)$$

$$+\theta \sum_{k=1,\ldots,N} f(|X_n^k - x|)g(\alpha_{xX_n^k})\frac{X_n^k - x}{|X_n^k - x|}$$

$$+(1-\theta)\Lambda \sum_{j \in Z^d} \rho_j^n \int_{E_j} f(|y - x|g(\alpha_{xy})\frac{y - x}{|y - x|}dy$$



Governing Equations

$$v[\mu_n](x) = v_d(x)$$

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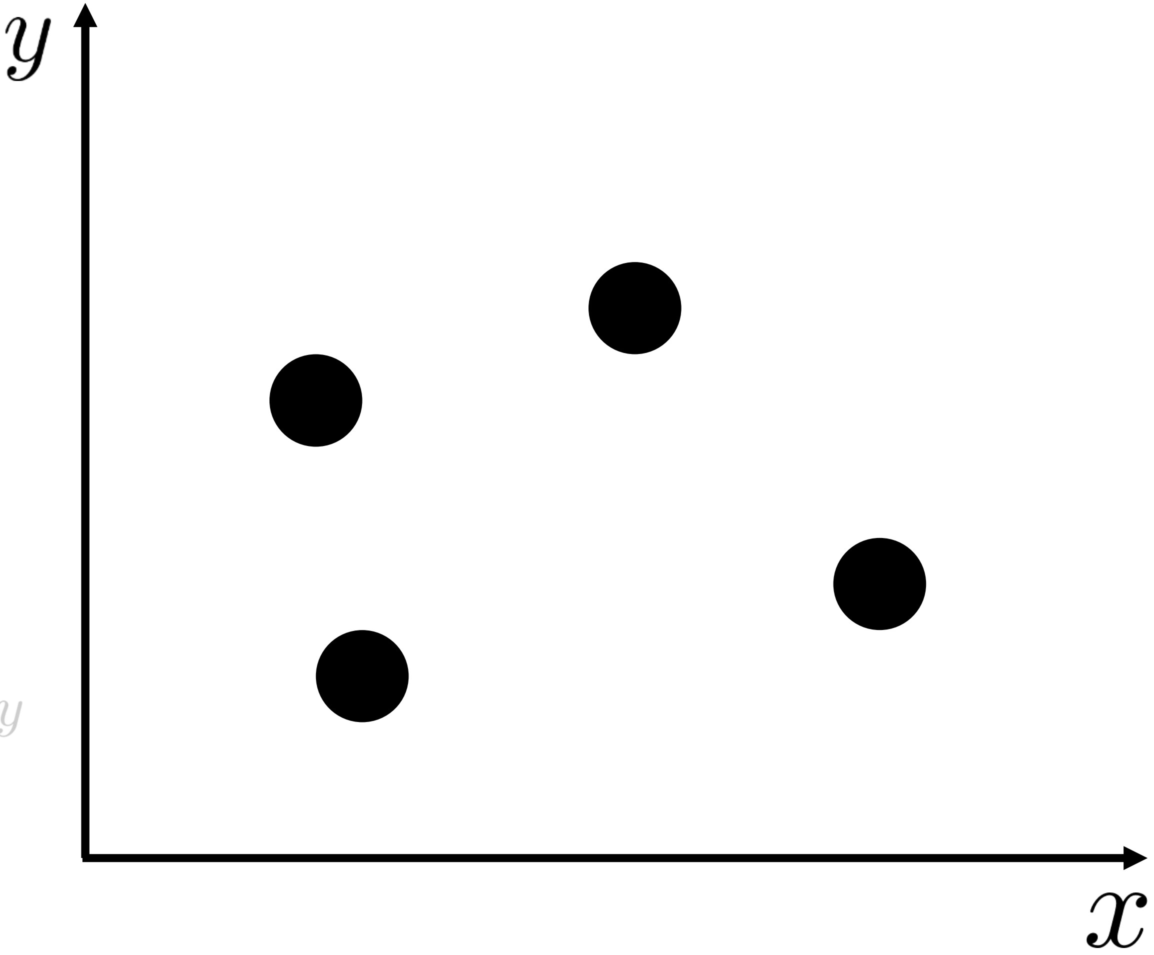
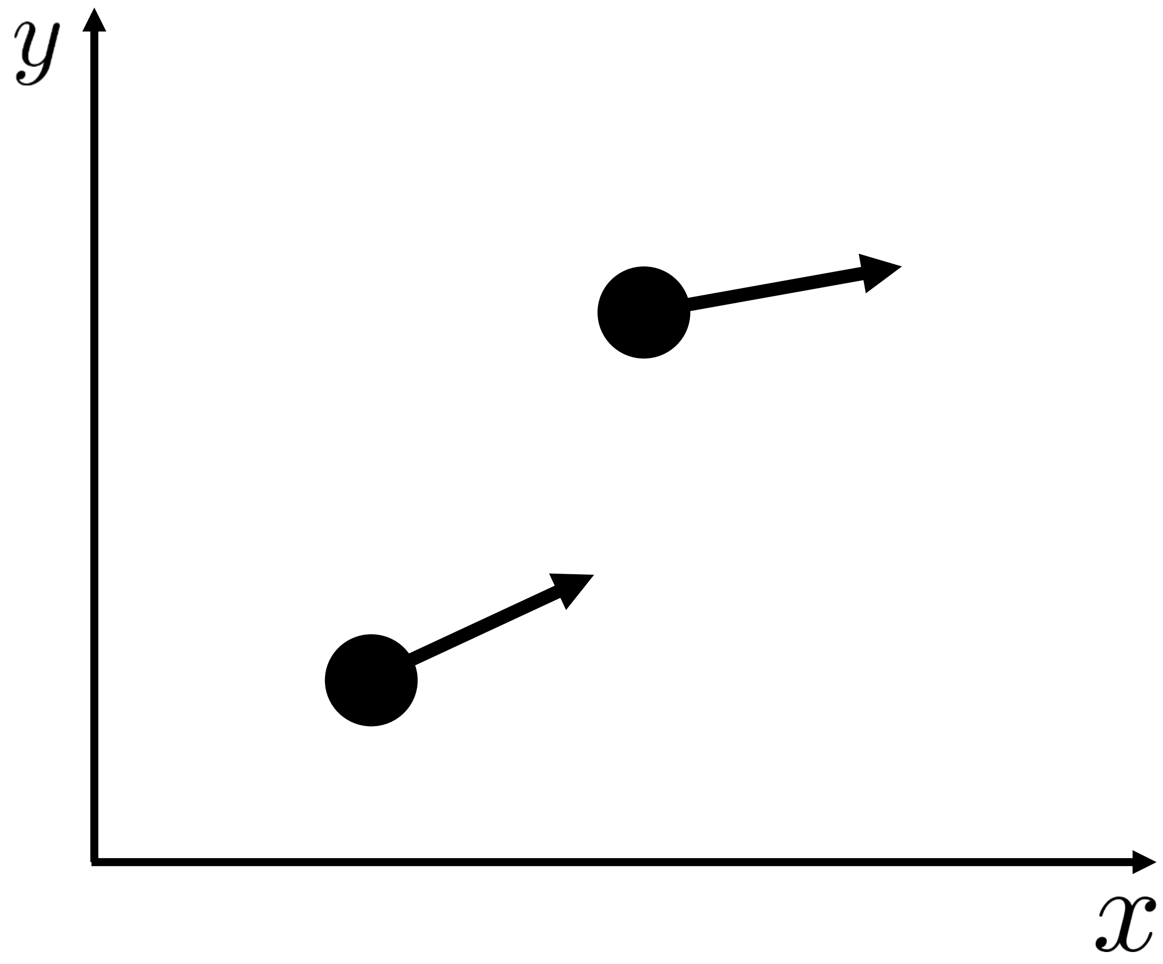


Governing Equations

$$+\theta \sum_{\substack{k=1,\dots,N \\ X_n^k \neq x}} f(|X_n^k - x|) g(\alpha_{x X_n^k}) \frac{X_n^k - x}{|X_n^k - x|}$$



Governing Equations

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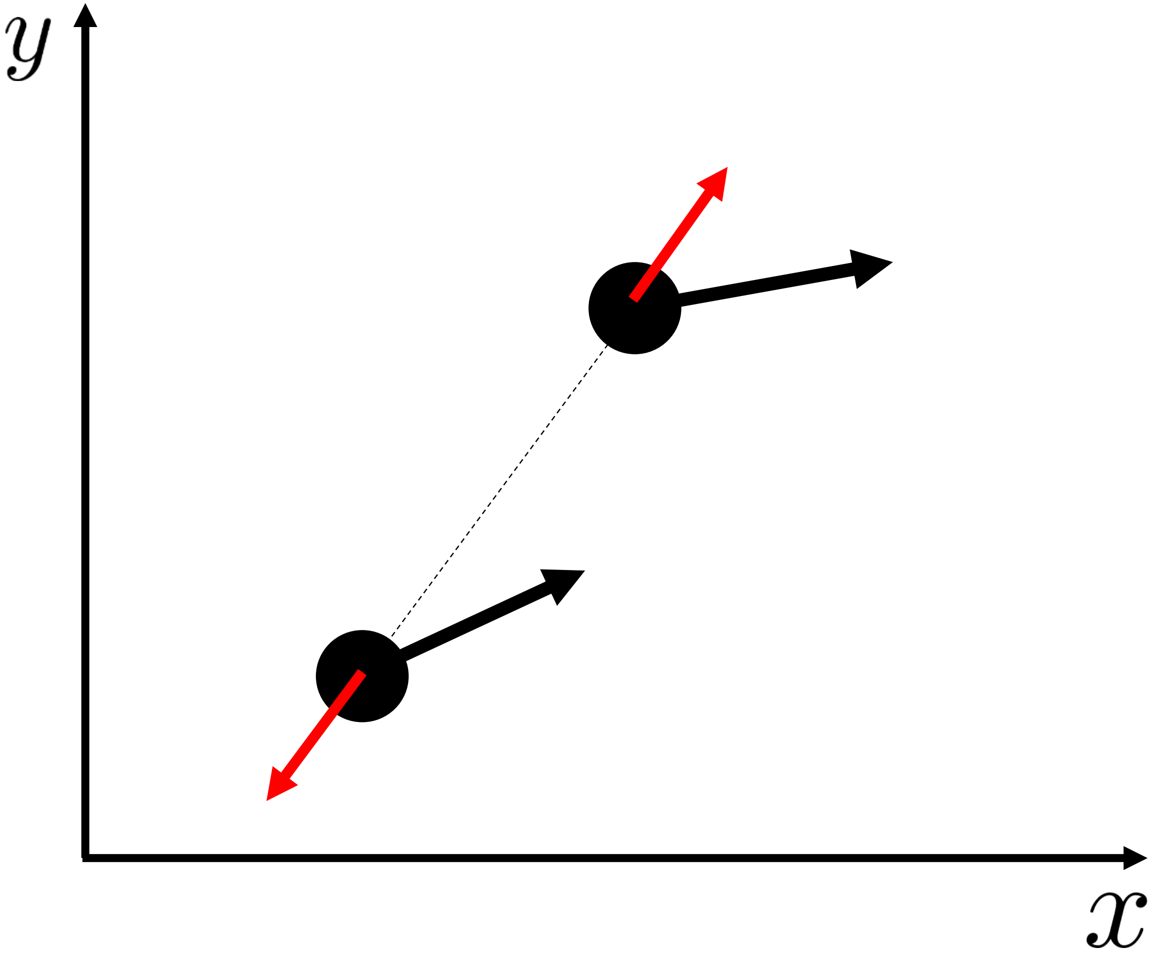
$$f(x) = \begin{cases} \frac{1}{|X_n^k - x|} & |X_k^n - x| \leq R \\ 0 & \text{otherwise} \end{cases}$$

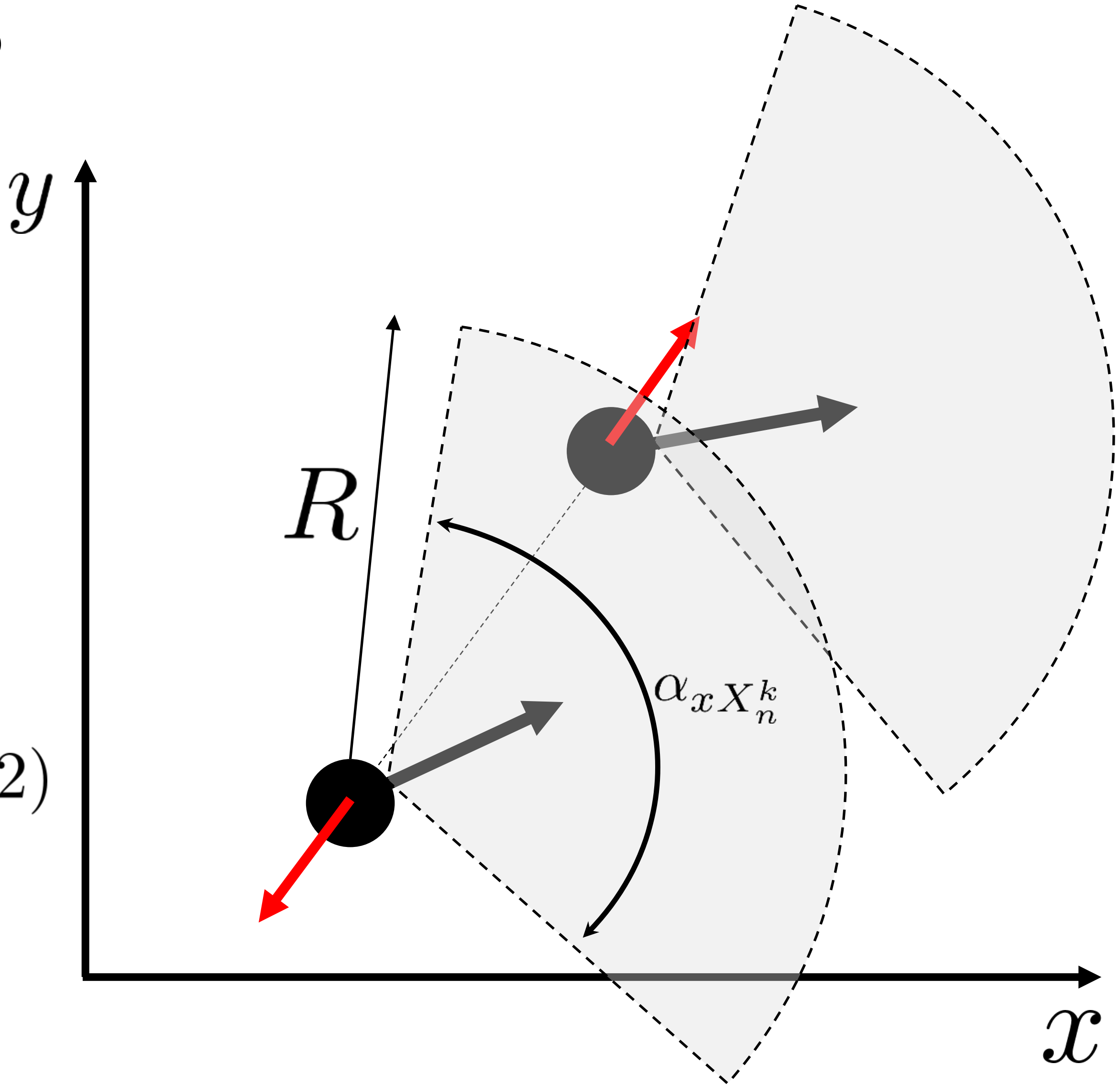


Governing Equations

$$+\theta \sum_{\substack{k=1,\dots,N \\ X_n^k \neq x}} f(|X_n^k - x|)\, g(\alpha_{xX_n^k}) \frac{X_n^k - x}{|X_n^k - x|}$$

$$f(x) = \begin{cases} \frac{1}{|X_n^k - x|} & |X_k^n - x| \leq R \\ 0 & \text{otherwise} \end{cases}$$

$$g(\alpha_{xX_n^k}) = \begin{cases} 1 & \frac{v(x)\cdot \dot{X}_n^k}{|v(x)||\dot{X}_n^k|} \geq \cos(\alpha_{xX_n^k}/2) \\ 0 & \text{otherwise} \end{cases}$$



Governing Equations

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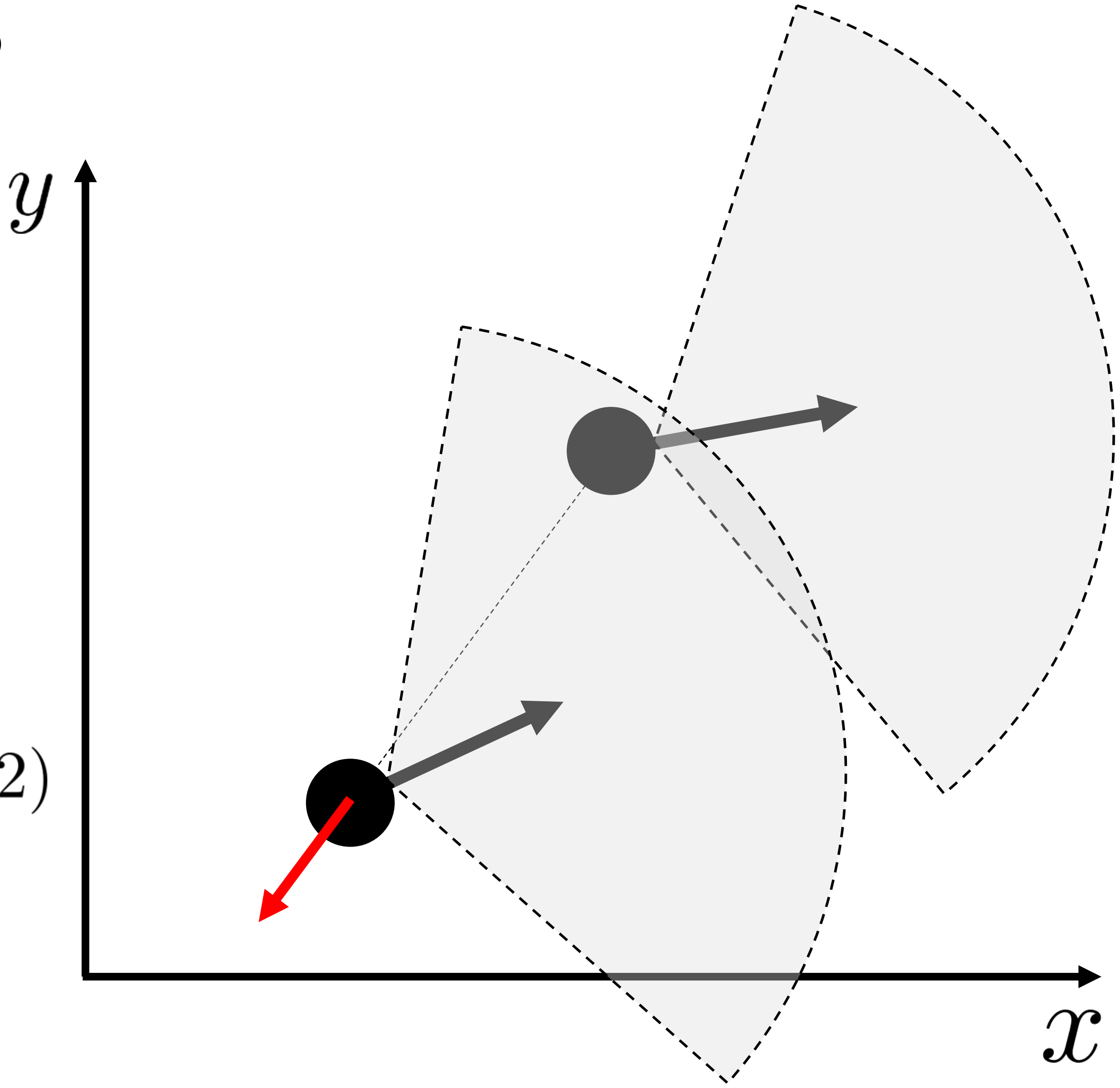
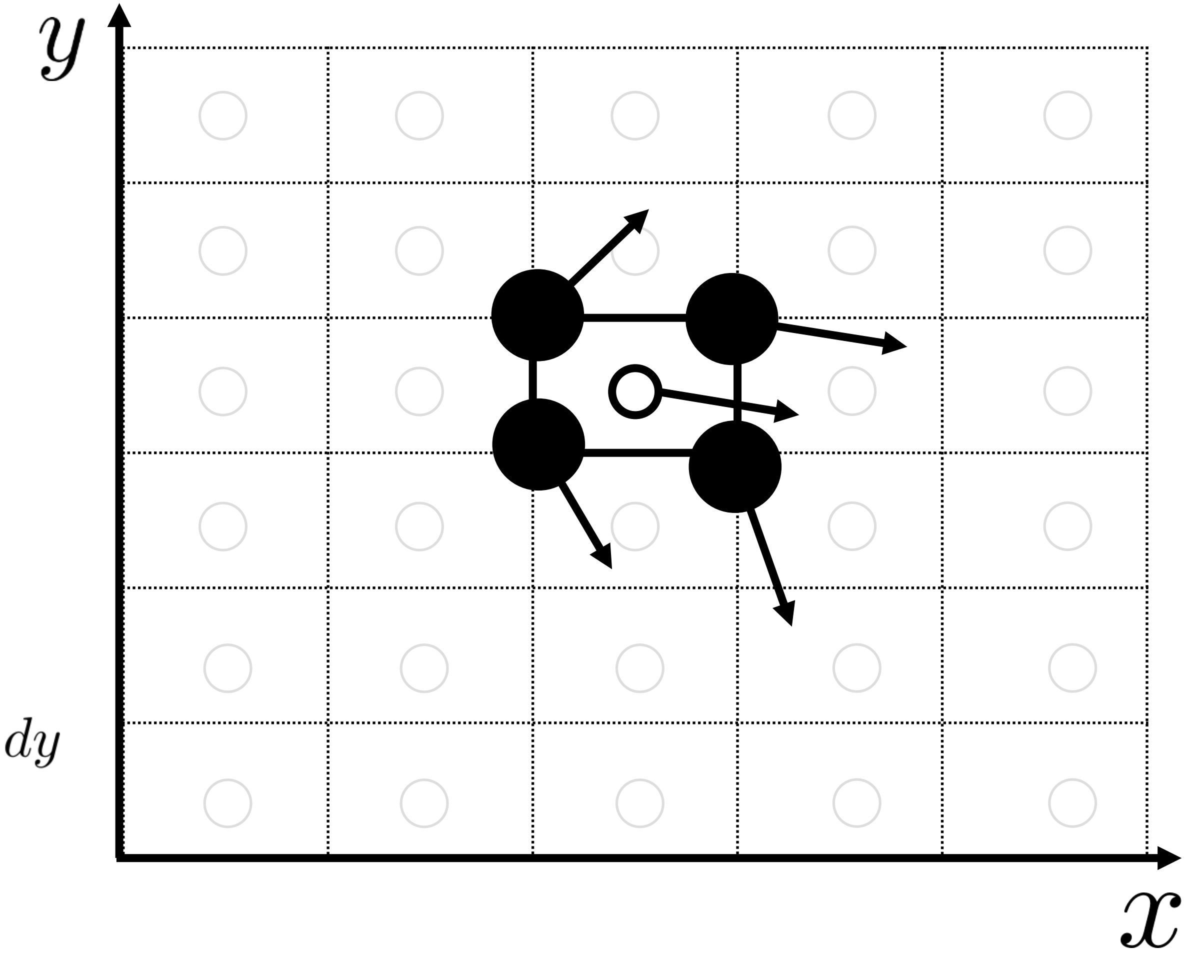


Governing Equations

$$v[\mu_n](x) = v_d(x)$$

$$+\theta \sum_{k=1,\dots,N} f(|X_n^k - x|) g(\alpha_{xX_n^k}) \frac{X_n^k - x}{|X_n^k - x|}$$

$$+(1-\theta)\Lambda \sum_{j \in Z^d} \rho_j^n \int_{E_j} f(|y-x| g(\alpha_{xy}) \frac{y-x}{|y-x|} dy$$



Multiscale Level – Governing Equations

<u>Multiscale level</u>

Measure

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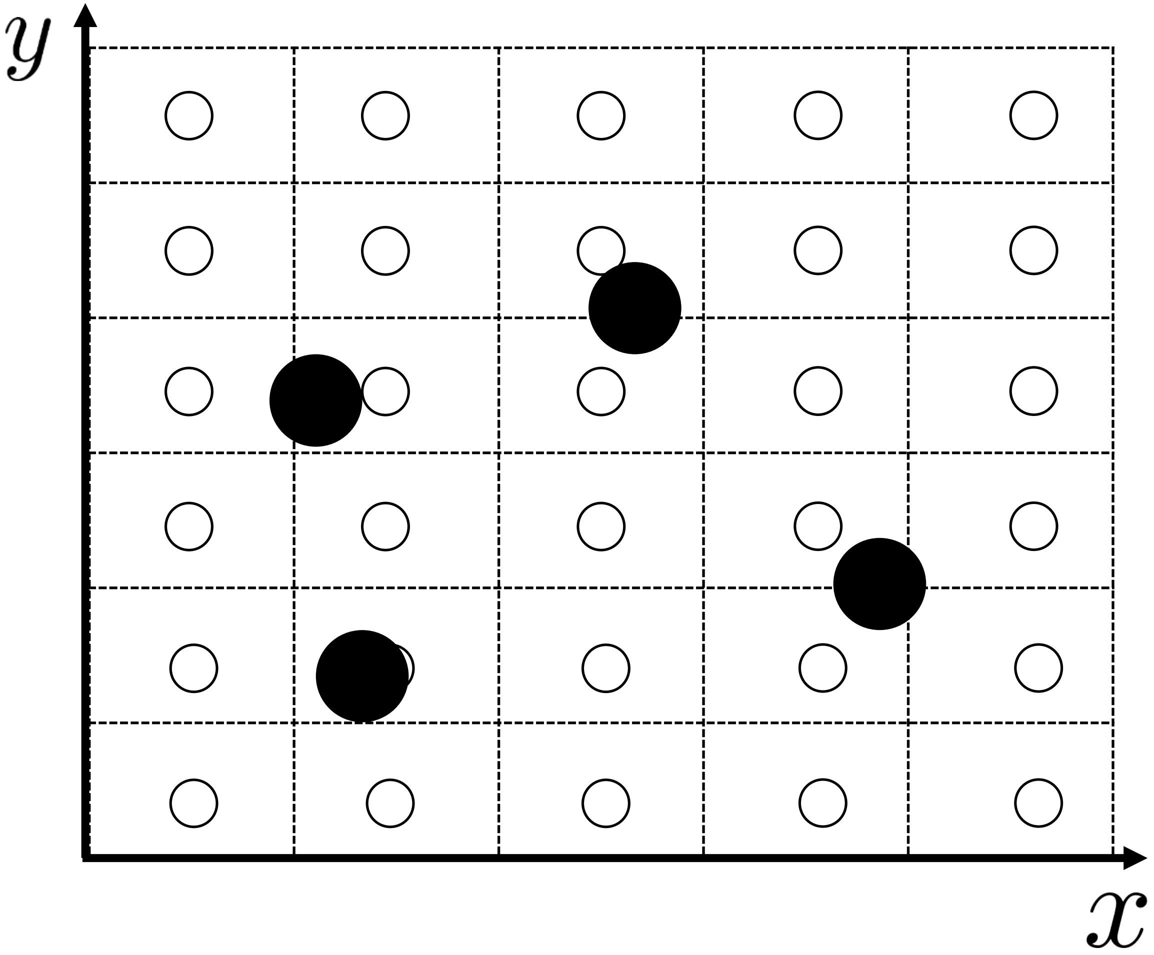
Equation

$$\dot{X}^k(t) = v[\mu_t](X^k(t)), \qquad k = 1 \ldots, N,$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v[\mu_t]) = 0,$$



Update Position and Density

Microscopic

we update the position of each pedestrian

$$X = X + \Delta t v[\mu_t](X_n^k)$$

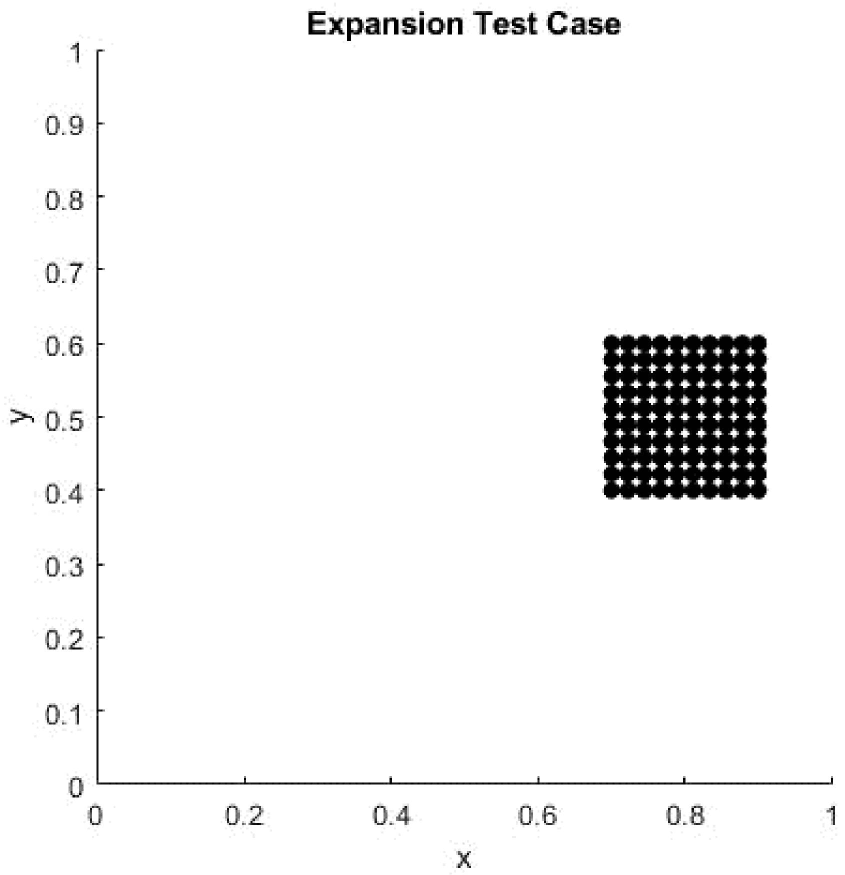
$$\dot{X}^k(t) = v[\mu_t](X^k(t)),$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v[\mu_t]) = 0,$$

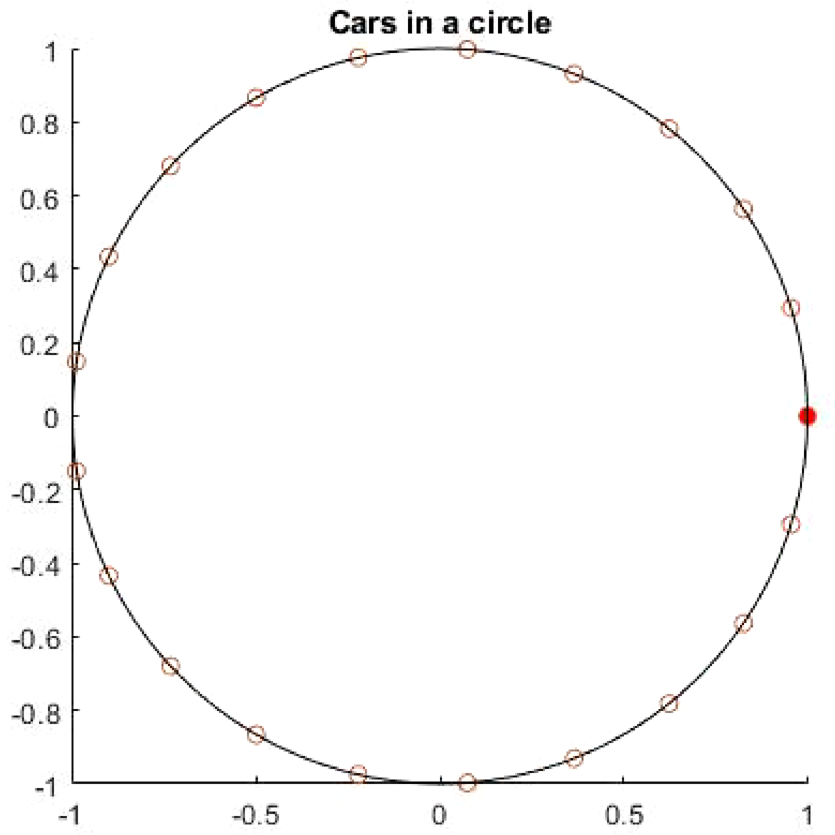
Macroscopic

we update the density value at each cell center

Result



Expansion Test Case
1
0.9
0.8
0.7
0.6
y 0.5
0.4
0.3
0.2
0.1
0
0 0.2 0.4 0.6 0.8 1
x



Cars in a circle
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1 -0.5 0 0.5 1

Future Work

- Fix the bugs in the code.

- Run test cases to test the model qualitatively and quantitatively.
- Efficient implementation of the algorithm

- Extend the code to multiple groups of pedestrians
  - Crossing Flows
  - Flow of a small group of pedestrians through a large crowd

Backup Slides

Algorithm

```
%% PSEUDOCODE
initialize position of pedestrians
define desired velocityfield
initialize density
% timeloop
for t=1:Nt
    % Calculate the velocity couplings
    v_micro_for_micro
    v_micro_for_macro
    v_macro_for_macro
    v_macro_for_micro
    % Aggregate the velocities
    v_micro = theta* v_micro_for_micro + (1-theta)*lambda*v_macro_for_micro+v_desired
    v_macro = theta* v_micro_for_macro + (1-theta)*lambda*v_macro_for_macro+v_desired
    % Update Positions
    X = X + dt*v_micro
    % Update Density
    update rho
end
```

Macroscopic Level

<u>Macroscopic level</u>

Density

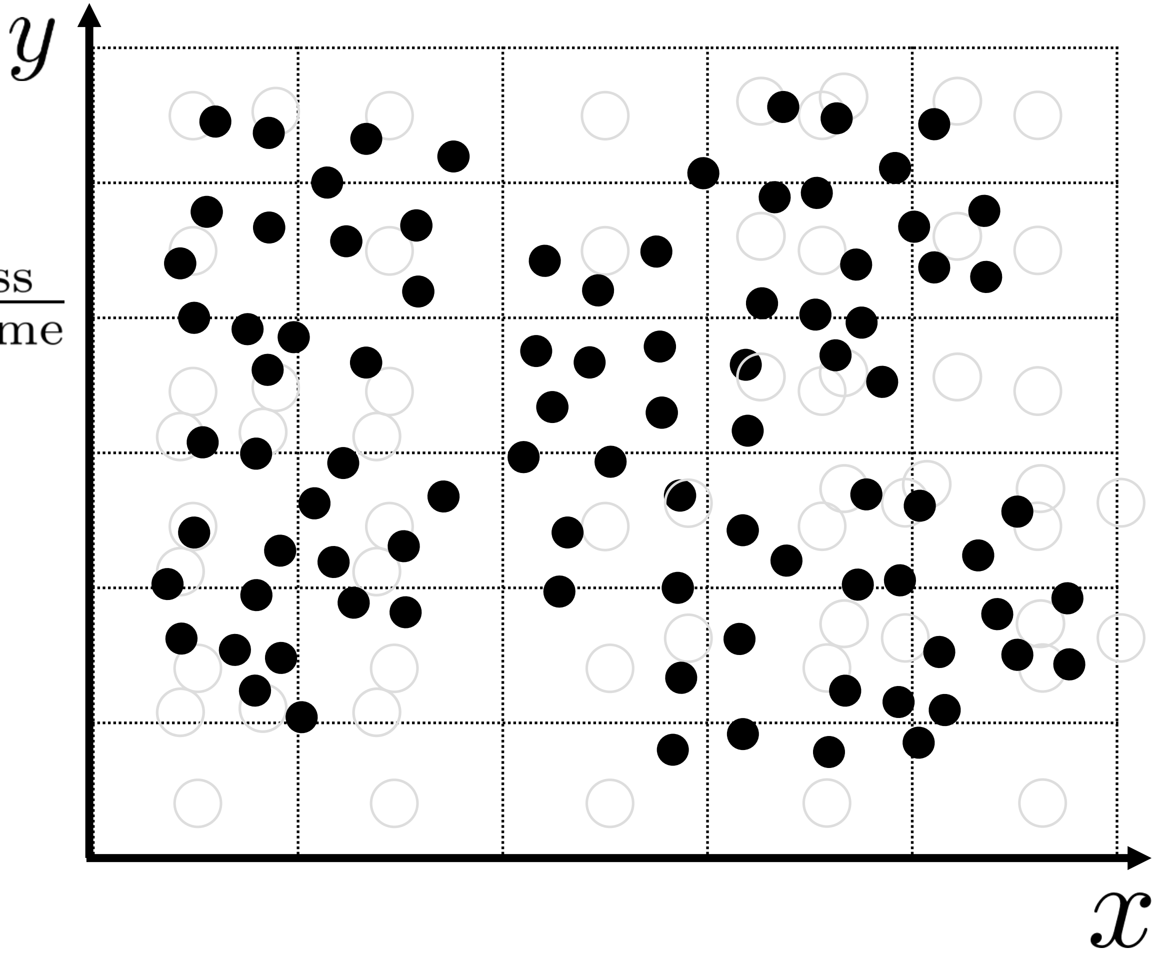
$$\rho(t,x) = \lim_{r \to 0^+} \frac{\mu_t(\mathscr{B}_r(x))}{\mathscr{L}^d(\mathscr{B}_r(x))} = \frac{\text{Mass}}{\text{Volume}}$$

Measure

$$\mu_t = \rho \cdot \mathscr{L}^d$$

Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v[\rho]) = 0,$$



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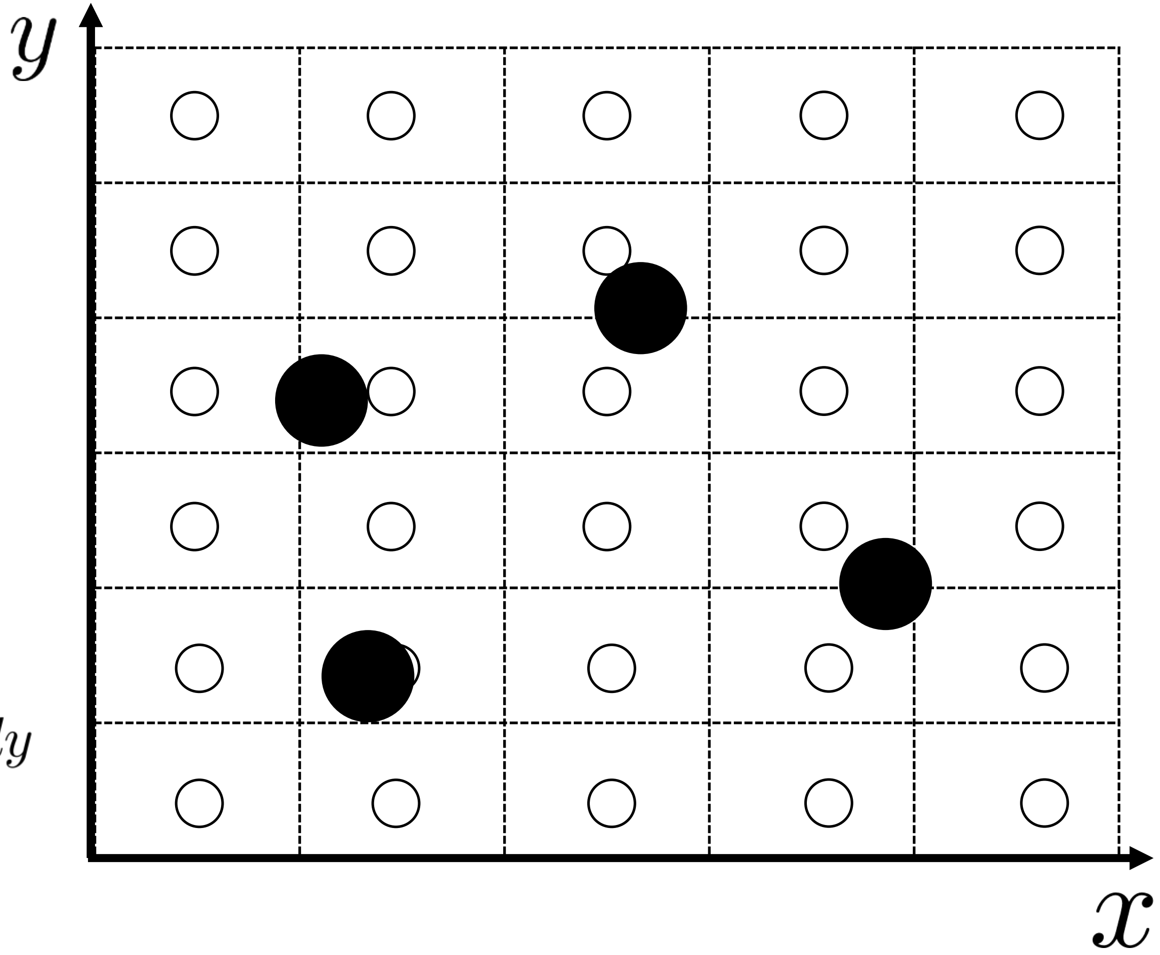

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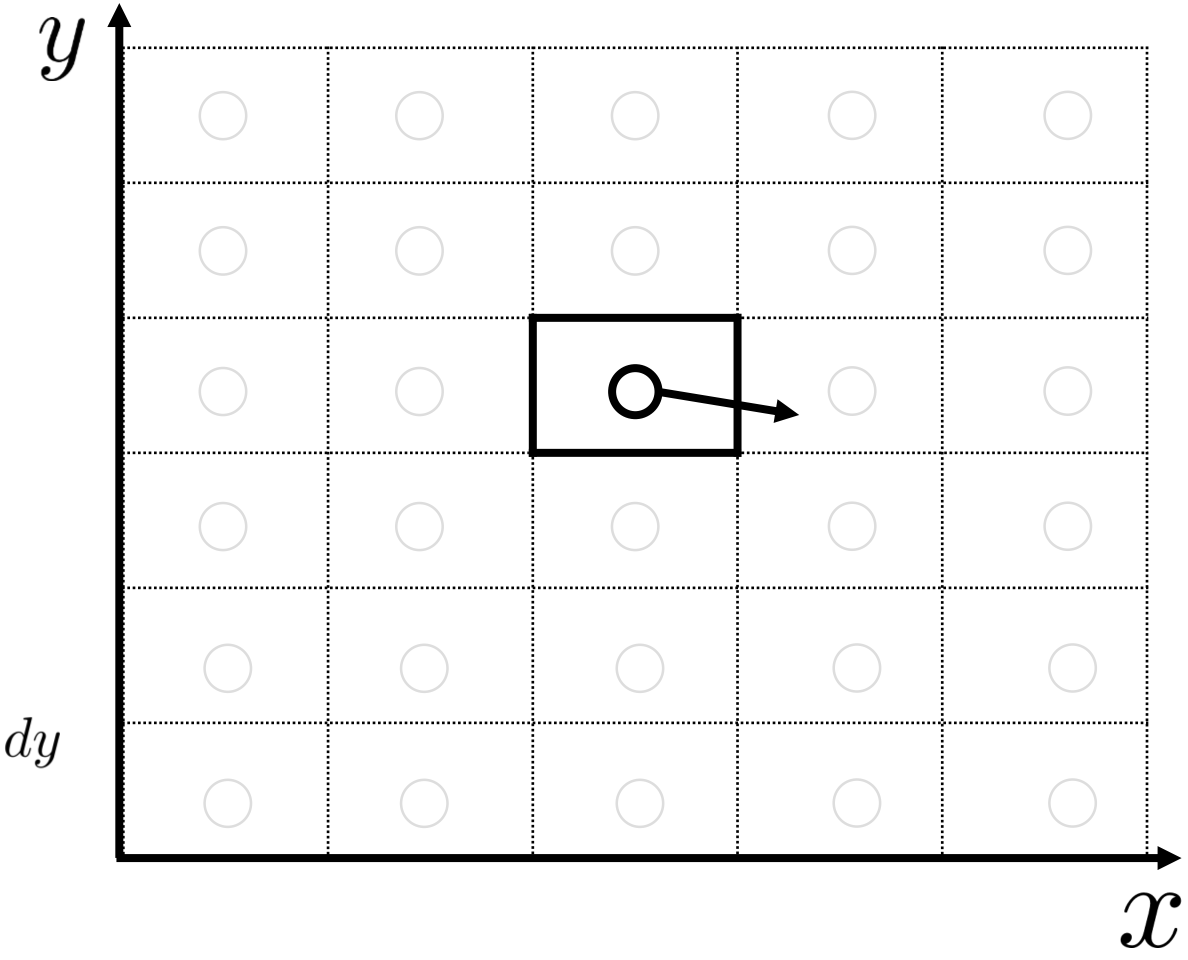

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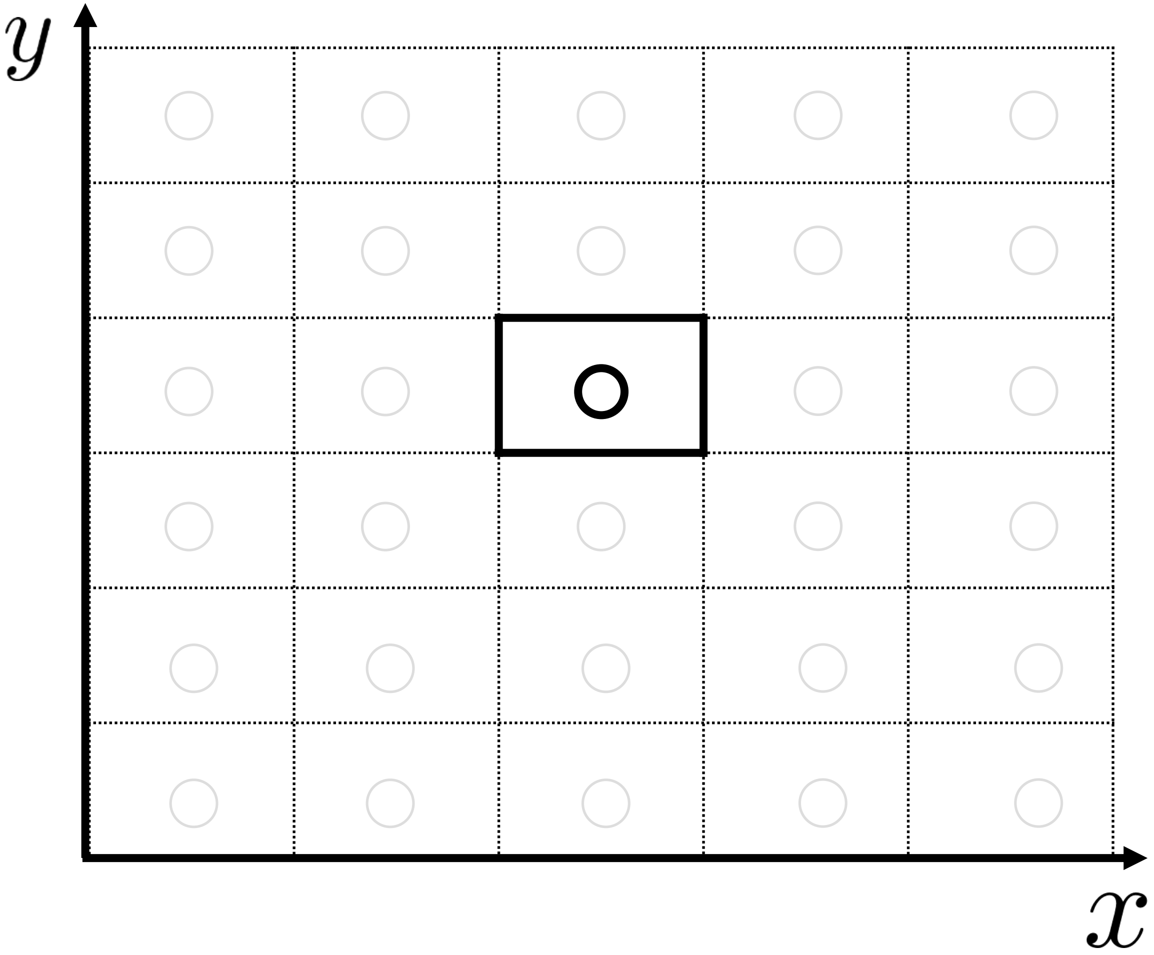
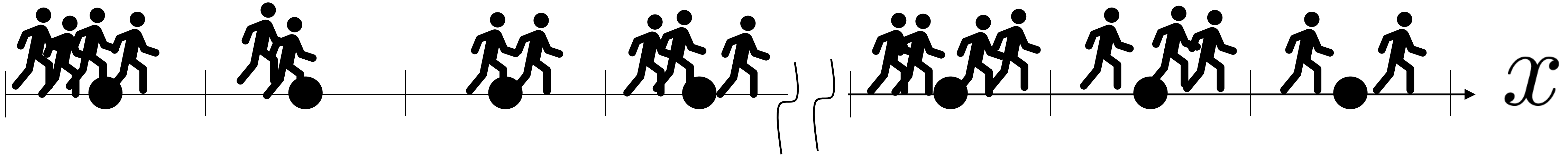


Update Density – 1D Model



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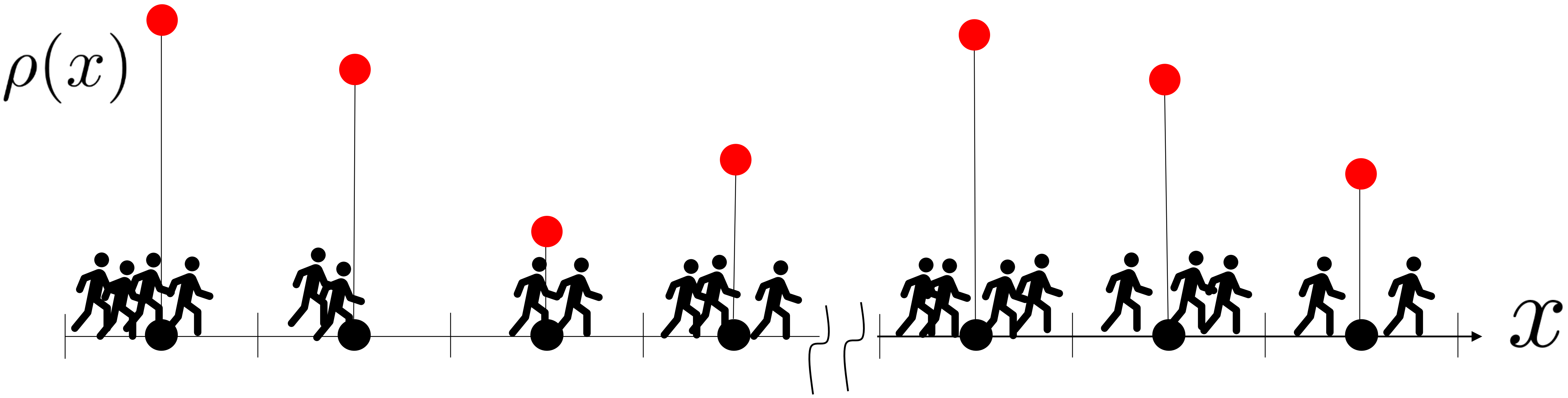

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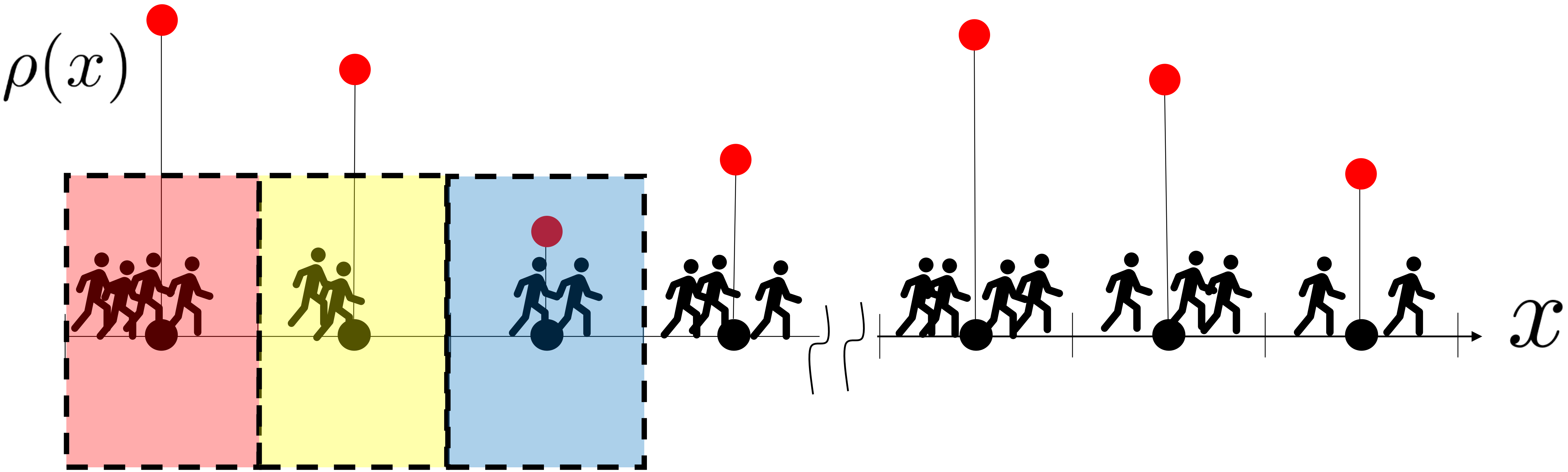

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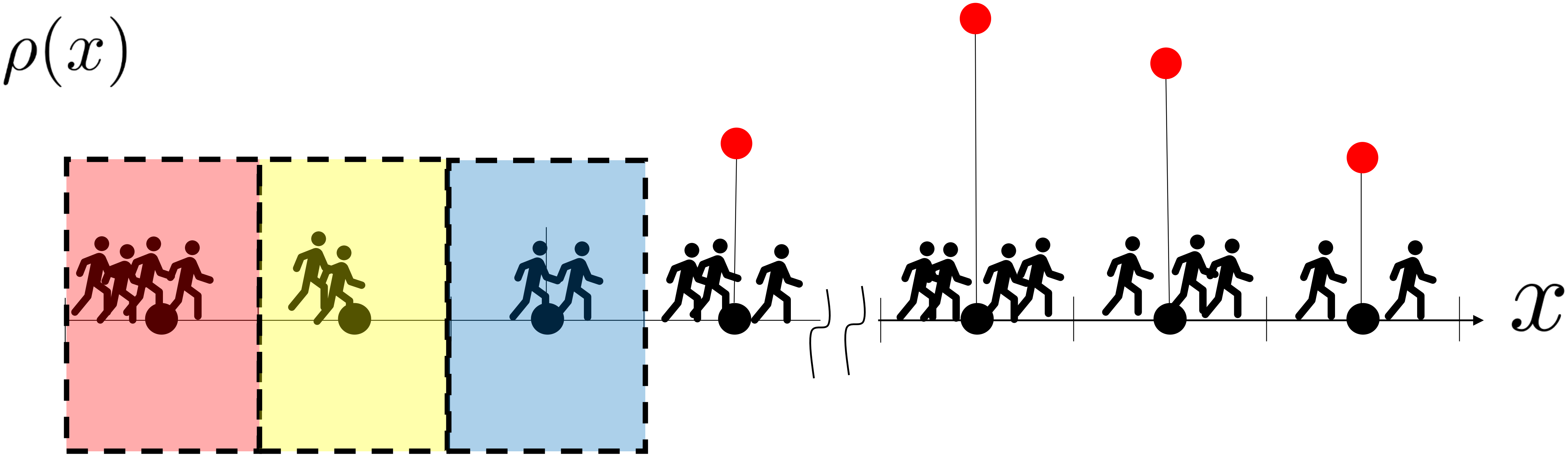

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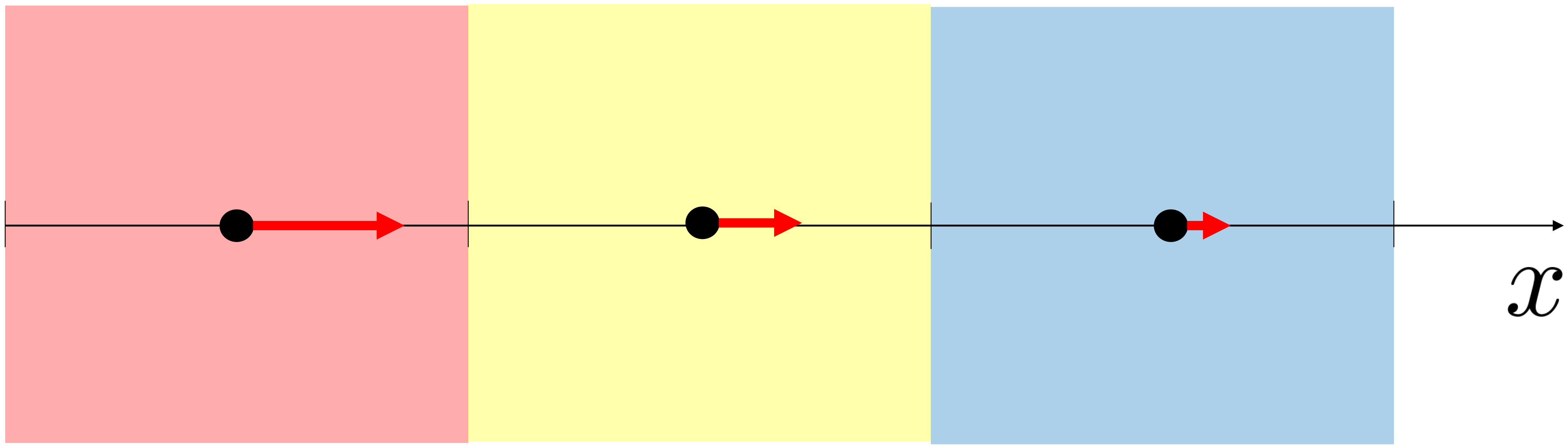

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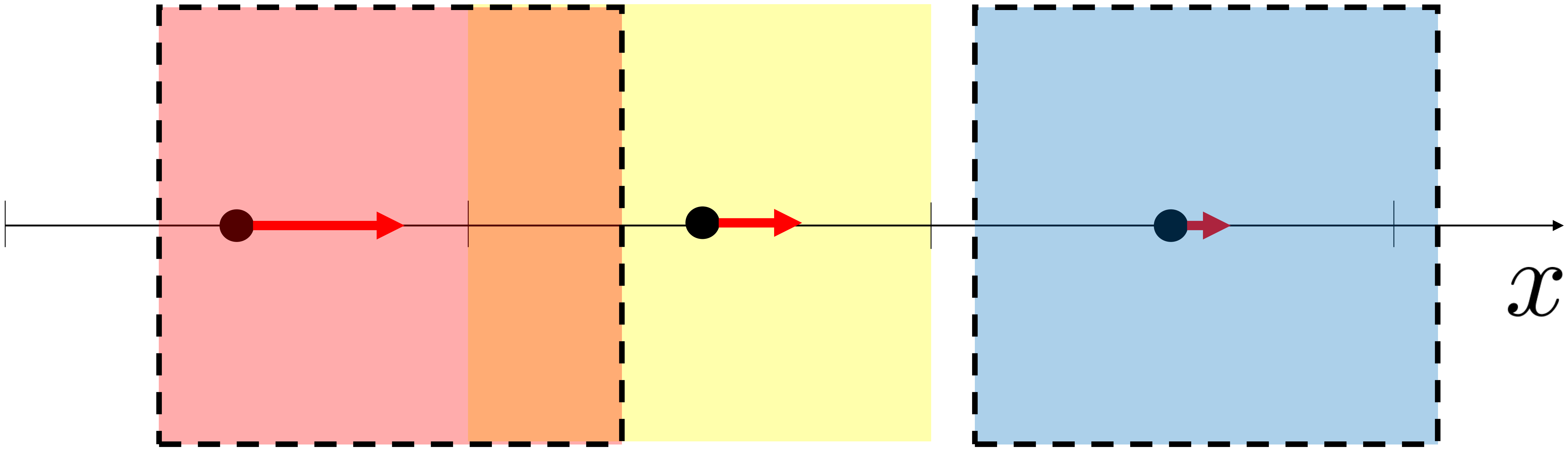

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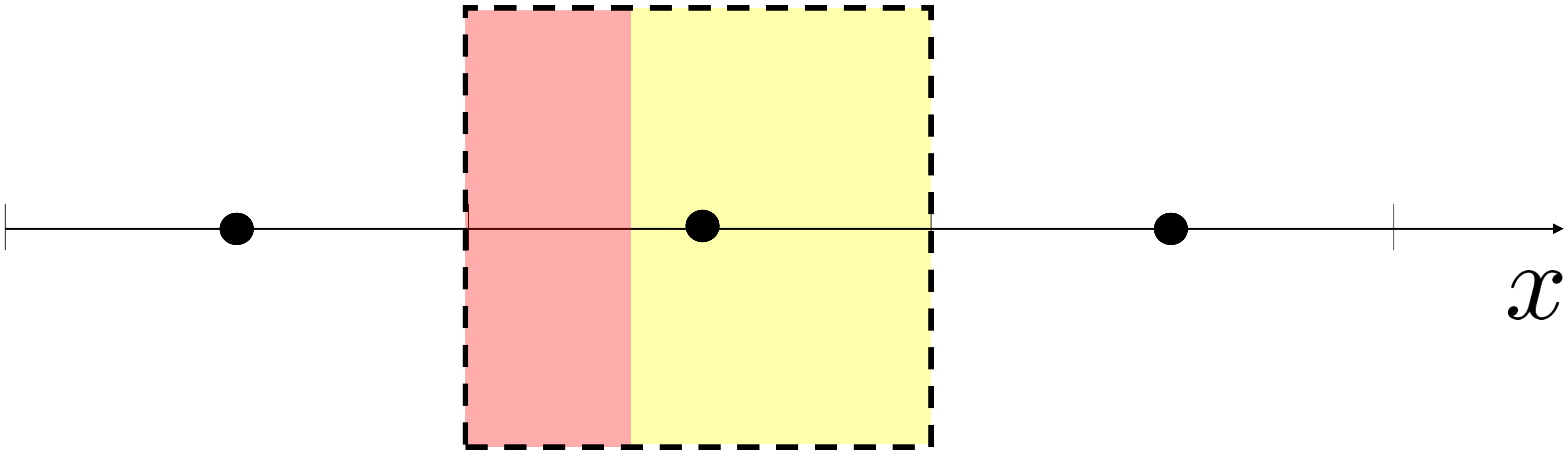

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