A Literature Review of Variable Fidelity Methods and their Use in Airfoil Optimization

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Outline

Aerodynamic Analysis and Optimization Methods

Surrogate Based Optimization

Application and Influence of VFO on Numerical Methods
- Numerical Example
- Other Approaches in VFM

Aerodynamic Analysis

- What are aerodynamic coefficients for a given surface?
  - C_l Lift coefficient
  - C_d Drag coefficient

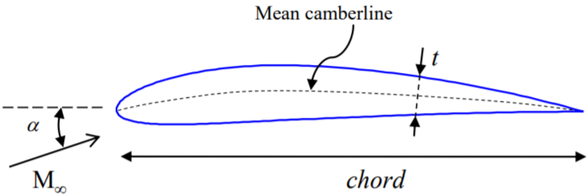


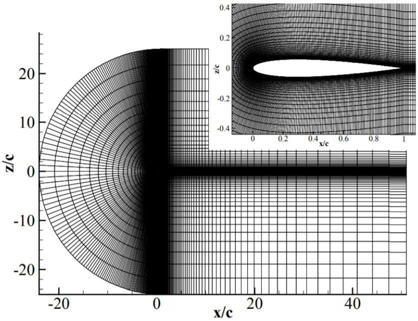
Figure: Sample NACA Airfoil

Aerodynamic Shape Optimization

Objective: use a search algorithm for the design of aerodynamic surfaces and adhere to appropriate constraints

History

- Conjugate-gradient method was first for 2D airfoil shapes (Hicks et al. 1974)
- Steepest-gradient method for 3D transonic wing design (Hicks and Henne 1978)
- Gradient-based and gradient-free approaches in use now



Gradient-Free vs Gradient-Based

Gradient-Free Approaches

- best for problems with a few design variables
- explore a search space
- exploit design as it approaches the global optimum
- successful in non-smooth design spaces
- requires large number of model evaluations (esp. in large design space)

Gradient-Free vs Gradient-Based

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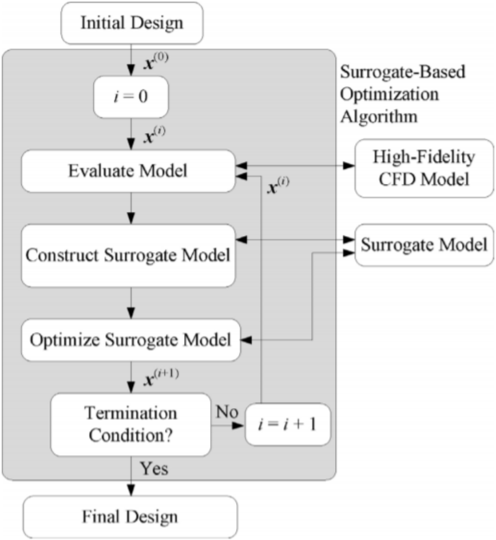
Gradient-Based Approaches

- applicable to problems with large number of design variables
- requires substantial amount of samples to ensure good accuracy
- cost of gradient calculation can be made nearly independent of number of design variables (with use of adjoint approach)
- considered current state of the art

Surrogate Modeling

- mathematical approximation that mimics the deterministic computationally expensive response or behavior of an original system
- improves global accuracy over entire domain
- approximates to the optimum to locally improve the current design



Surrogate Modeling

Challenges

- accuracy requirements
- computational efficiency
- grid deformations

Surrogate Modeling

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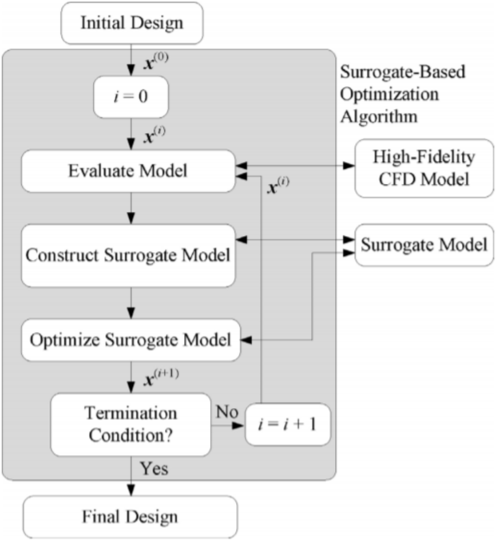
Existing Categories

- Data Fit Models
- Reduced-Order Models
- **Variable Fidelity Models**



Variable Fidelity Optimization

- replace a computationally expensive model with a cheap surrogate model
- high-fidelity model f
- low-fidelity model c
- $\#$ of evaluations of $f < \#$ of evaluations of c

Variable Fidelity Optimization

- replace a computationally expensive model with a cheap surrogate model
- high-fidelity model f
- low-fidelity model c
- $\#$ of evaluations of $f < \#$ of evaluations of c
- convergence can be guaranteed with proper local search methods
- correction methods reduce prediction error
- reduces computation effort significantly at extremes of flight envelopes

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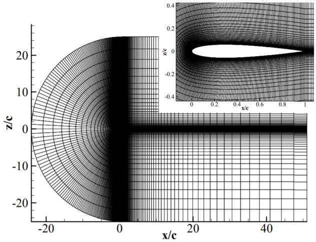
Model Setup in Numerical Example

Example VFM High-Fidelity Model Setup

- **Geometry**: NACA Airfoil
- **Flow Equations**: steady RANS equations with turbulence model by Spalart and Allmaras
- **Grid Generation**: Structured curvilinear body-fitted C-topology ($\sim$ 400,000 mesh cells and 1000 iteration limit)
- **Numerical Solver**: upwind-biased second-order Roe flux scheme performed in FLUENT; convergence by L^2 norm

Low-Fidelity Model: Coarser mesh and relaxed convergence criteria ($\sim$ 32,000 cells and 100 iteration limit)



Model Construction in Numerical Example

Correction Method: Output Space Mapping (linear transformations or mappings)

$$\vec{X_d} = \text{design variable}$$
$$\vec{S_{al}} = \text{vector of uncertain variables}$$
$$\mathbf{x} = \left[\vec{X_d}\ \vec{S_{al}}\right]^T$$
$$C_{l,f}, C_{d,f} = \text{high fidelity lift and drag}$$
$$f(x) = \left[C_{l,f}(x)\ \ C_{d,f}(x)\right]^T$$
$$C_{l,c}, C_{d,c} = \text{low fidelity lift and drag}$$
$$s(x) = A(x) \circ c(x) = \left[a_l(x)\, C_{l,c}(x) + d_1 \quad a_d(x)\, C_{d,c}(x) + d_d\right]^T$$

Model Construction in Numerical Example

Response correction parameters

$$\text{center of the design space } \mathbf{x}^0 = (\mathbf{x}^L + \mathbf{x}^U)/2$$

$$\mathbf{A}(\mathbf{x}) = [a_{l.0} + [a_{l.1}\, a_{l.2}\, \ldots\, a_{l.n}] \cdot (\mathbf{x} - \mathbf{x}^0) \quad a_{d.0} + [a_{d.1}\, a_{d.2}\, \ldots\, a_{d.n}] \cdot (\mathbf{x} - \mathbf{x}^0)]^T$$

$$\text{Response correction parameters } \mathbf{A} \text{ and } \mathbf{D}$$

$$[\mathbf{A}, \mathbf{D}] = \arg \min_{\overline{\mathbf{A}}, \overline{\mathbf{D}}} \sum_{k=1}^{K} ||f(\mathbf{x}^k) - (\overline{\mathbf{A}}(\mathbf{x}^k) \circ c(\mathbf{x}^k) + \mathbf{D})||^2,$$

Model Construction in Numerical Example

Least-square optimal solution to the linear regression

correction parameters $\mathbf{A}$ and $\mathbf{D}$

$$\begin{bmatrix} a_{l.0} \\ a_{l.1} \\ \vdots \\ a_{l.n} \\ d_l \end{bmatrix} = \left(\mathbf{C}_l^T \mathbf{C}_l\right)^{-1} \mathbf{C}_l^T \mathbf{F}_l \,, \begin{bmatrix} a_{d.0} \\ a_{d.1} \\ \vdots \\ a_{d.n} \\ d_d \end{bmatrix} = \left(\mathbf{C}_d^T \mathbf{C}_d\right)^{-1} \mathbf{C}_d^T \mathbf{F}_d$$

$$\mathbf{C}_l = \begin{bmatrix} C_{l.c}(\mathbf{x}^1) & C_{l.c}(\mathbf{x}^1) \cdot (\mathbf{x}_1^1 - \mathbf{x}_1^0) & \cdots & C_{l.c}(\mathbf{x}^1) \cdot (\mathbf{x}_n^1 - \mathbf{x}_n^0) & 1 \\ C_{l.c}(\mathbf{x}^2) & C_{l.c}(\mathbf{x}^2) \cdot (\mathbf{x}_1^2 - \mathbf{x}_1^0) & \cdots & C_{l.c}(\mathbf{x}^2) \cdot (\mathbf{x}_n^2 - \mathbf{x}_n^0) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ C_{l.c}(\mathbf{x}^K) & C_{l.c}(\mathbf{x}^K) \cdot (\mathbf{x}_1^K - \mathbf{x}_1^K) & \cdots & C_{l.c}(\mathbf{x}^K) \cdot (\mathbf{x}_K^2 - \mathbf{x}_K^0) & 1 \end{bmatrix}$$

$$\mathbf{F}_l = \left[C_{l.f}(\mathbf{x}^1) \quad C_{l.f}(\mathbf{x}^2) \quad \cdots \quad C_{l.f}(\mathbf{x}^K)\right]^T$$

$$\mathbf{C}_d = \begin{bmatrix} C_{d.c}(\mathbf{x}^1) & C_{d.c}(\mathbf{x}^1) \cdot (\mathbf{x}_1^1 - \mathbf{x}_1^0) & \cdots & C_{d.c}(\mathbf{x}^1) \cdot (\mathbf{x}_n^1 - \mathbf{x}_n^0) & 1 \\ C_{d.c}(\mathbf{x}^2) & C_{d.c}(\mathbf{x}^2) \cdot (\mathbf{x}_1^2 - \mathbf{x}_1^0) & \cdots & C_{d.c}(\mathbf{x}^2) \cdot (\mathbf{x}_n^2 - \mathbf{x}_n^0) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ C_{d.c}(\mathbf{x}^K) & C_{d.c}(\mathbf{x}^K) \cdot (\mathbf{x}_1^K - \mathbf{x}_1^K) & \cdots & C_{d.c}(\mathbf{x}^K) \cdot (\mathbf{x}_K^2 - \mathbf{x}_K^0) & 1 \end{bmatrix}$$

$$\mathbf{F}_d = \left[C_{d.f}(\mathbf{x}^1) \quad C_{d.f}(\mathbf{x}^2) \quad \cdots \quad C_{d.f}(\mathbf{x}^K)\right]^{T\ T}$$

Model Construction in Numerical Example

Design variable vector $\vec{X}_d$ with NACA shape parameters $m, p, t/c$

$$\vec{X}_d = [m\ p\ t/c\ \alpha]^T$$

$$0.0 \leq m \leq 0.05$$

$$0.3 \leq p \leq 0.7$$

$$0.08 \leq t/c \leq 0.14$$

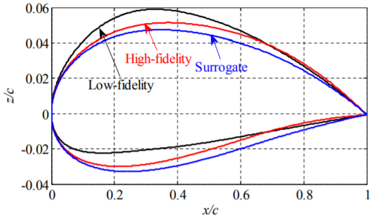
$$0^\circ \leq \alpha \leq 2^\circ$$

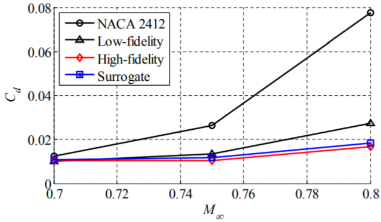
$$0.7 \leq M_\infty \leq 0.8$$

with NACA 2412

Results

Variable	Low-fidelity	High-fidelity	Surrogate
m	0.0198	0.0150	0.0100
p	0.3607	0.6287	0.6220
t/c	0.0800	0.0800	0.0800
α [deg]	1.5991	0.9232	0.9598
μ_{C_l}	0.4978	0.5186	0.5379
μ_{C_d}	0.0656	0.0348	0.03768
σ_{C_d}	0.0056	0.0040	0.0064
N_c	42	0	53
N_f	0	42	11
N	< 1	42	< 12



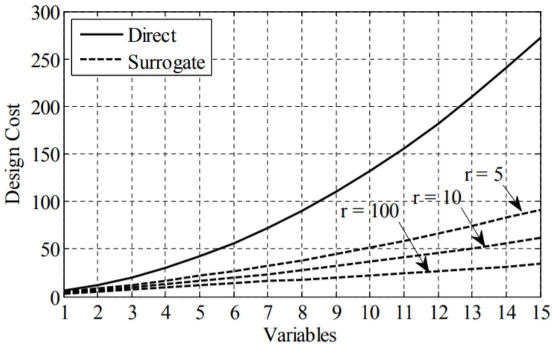


Results

Total cost $\propto$ # (design variables)^2

$$N = n^2 + 3n + 2$$
$$N = N_f + N_c/r$$

where r is ratio of high- to low-fidelity simulation times



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Correction Methods

- Space mapping (used in example)
  - simple to implement
- Multi-level optimization
  - uses multiple models so that each iteration of the algorithm requires a smaller number of model evaluations
  - considered more efficient than SM by Leifsson
- Shape-preserving response prediction
  - works at pressure distribution level (rather than aerodynamic forces directly)
- Weight gradients
  - adjust influence of linear and multiplicative corrections

Case 1 (lift maximization)
$M_\infty = 0.75$, $\alpha = 0^\circ$, $C_{dw,max} = 0.005$, $A_{min} = 0.075$

Variable	Initial	MLO	SM	SPRP
m	0.0200	0.0148	0.0150	0.0145
p	0.4000	0.7743	0.7463	0.7723
t/c	0.1200	0.1114	0.1140	0.1135
C_l	0.4745	0.5933	0.5650	0.5576
C_{dw}	0.0115	0.0050	0.0050	0.0050
A	0.0808	0.0750	0.0767	0.0767
N_c [a,b]	–	60/47	210	180
N_f [a]	–	2	4	6
Cost	–	~5	~7	~10

Other Approaches in VFM

Data Fusion Techniques

- Kriging
  - method of interpolating values with a Gaussian process
- Co-Kriging
  - uses information from other variables
  - predicts 2500×2 cases in 0.023 seconds
  - picks up viscous phenomena from high fidelity samples
- Co-Kriging POD
  - data: orthonormal set of basis functions to linear subspace
- Direct Gradient Enhanced Kriging (GEK)
  - incorporates gradients into Kriging
- Generalized Hybrid Bridge Function (GHBF)
  - exploits prediction value in low fidelity data
- Upgrade key points from low to high fidelity

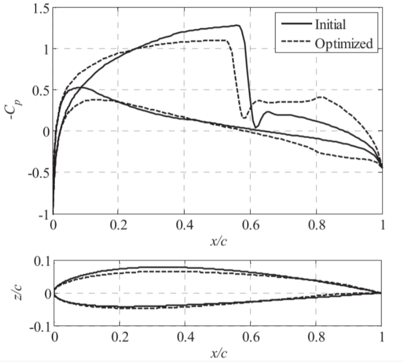
Summary

Aerodynamic opt $\rightarrow$ gradient-based $\rightarrow$ surrogate $\rightarrow$ variable fidelity

- Relatively low computational cost (less than 30% in provided example)
- Similar results to high-fidelity
- Effective correction and data fusion techniques

Future Efforts

- Development of tool boxes that minimize hand coding
- Identification of best practices for data fusion and correction methods



For Further Reading I

Yondo, et al.

A Review of Surrogate Modeling Techniques for Aerodynamic Analysis and Optimization: Current Limitations and Future Challenges in Industry.

Advances in Evolutionary and Deterministic Methods for Design, Optimization and Control in Engineering and Sciences, Computational Methods

Springer International Publishing AG 2019

Leisson, L and Koziel, S

Aerodynamic shape optimization by variable-fidelity computational fluid dynamics models: A review of recent progress

Journal of Computational Science, 10 (2015) 45-54.

Martins, J and Kennedy, G

Enabling Large-scale Multidisciplinary Design Optimization through Adjoint Sensitivity Analysis

57th AIAA Aerospace Sciences Meeting, AIAA SciTech Forum, 2019

For Further Reading II

Likeng, et al.

Research on multi-fidelity aerodynamic optimization methods

Chinese Journal of Aeronautics, 2013, 26(2): 279-286

Zhang, et al.

Variable Fidelity Methods and Surrogate Modeling of Critical Loads on X-31 Aircraft

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Low-Cost Robust Airfoil Optimization by Variable-Fidelity Models and Stochastic Expansions

51st AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition, 2013

Han, et all.

Improving variable-fidelity surrogate modeling via gradient-enhanced kriging and a generalized hybrid bridge function

Aerospace Science and Technology 25 (2013) 177-189