Dynamic Mode Decomposition

Various Methods of DMD Applied to POSYDON Real Time Ocean Ensemble Forecasts (Sea Surface)

Spring 2020 2.29 Project

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Dynamic Mode Decomposition (DMD)

- My goal with this project is to see how well various DMD methods can predict the evolution of features of the ocean
  - Sea Surface Temperature (SST)
  - Sea Surface Salinity
  - Sea Surface Velocity
- Future work goal is to combine Dynamically Orthogonal (DO) PDE solver with DMD
- See if DMD can predict ensemble results using one or more other ensembles

Dynamic Mode Decomposition (DMD)

- A method of extracting underlying linear dynamics from complex non-linear data
  - Extracts a global modal description of a nonlinear dynamical system
- Equation free method – Do not need equations for underlying dynamics
- Can be used for diagnostics, future state prediction, and control
- Various methods (and emerging methods)
  - Exact DMD
  - Projected DMD
  - Compressed DMD
  - Compressed Sensing DMD
  - Optimized DMD
  - DMD with Control
  - DMD of embedded data
  - …and many more

Algorithm

- Arrange data:
  - $\boldsymbol{X} = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \cdots & x_{m-1} \\ | & | & & | \end{bmatrix}$
  - $\boldsymbol{X'} = \begin{bmatrix} | & | & & | \\ x_2 & x_3 & \cdots & x_m \\ | & | & & | \end{bmatrix}$
- Perform Economy Singular Value Decomposition on **X**:
  - $\boldsymbol{X} \approx \boldsymbol{U}\,\boldsymbol{\Sigma}\,\boldsymbol{V}^*$
- Determine a rank (r) that maintains the vast majority of data (based on Singular Values in $\boldsymbol{\Sigma}$). Truncate $\boldsymbol{U}$ to r columns, $\boldsymbol{\Sigma}$ to r rows and r columns, $\boldsymbol{V}$ to r columns.

- Goal is to find matrix **A** such that $\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k$
  - Instead compute $\widetilde{\boldsymbol{A}}$, the r-by-r projection of A onto Proper Orthogonal Decomposition (POD) modes:
    - $\widetilde{\boldsymbol{A}} = \boldsymbol{U}^*\boldsymbol{X}'\boldsymbol{V}\boldsymbol{\Sigma}^{-1}$
- Eigendecomposition
  - $\tilde{A}W = W\Lambda$
- DMD Modes (Eigenvectors of A)
  - $\boldsymbol{\Phi} = \boldsymbol{X}'\boldsymbol{V}\boldsymbol{\Sigma}^{-1}\boldsymbol{W}$ (or $\boldsymbol{\Phi} = \boldsymbol{U}\boldsymbol{W}$ for projected DMD)
- Define $\omega = \ln(\lambda_k)\,/\boldsymbol{\Delta}t$ and $\boldsymbol{\Omega} = diag(\omega)$
- Then any given time can be predicited by
  - $\boldsymbol{x}(t) \approx \boldsymbol{\Phi}\exp(\boldsymbol{\Omega}t)\,\boldsymbol{b}$

  where $\boldsymbol{b} = \boldsymbol{\Phi}^{+}\boldsymbol{x}_1$

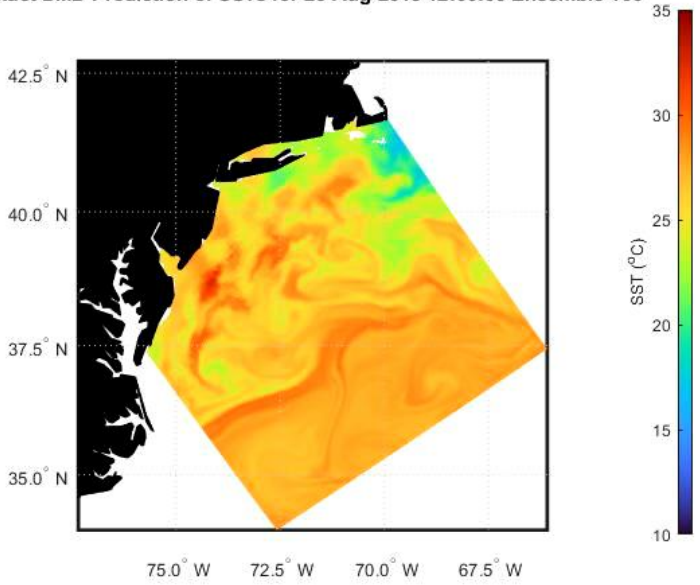
Example

- 300 ensembles of data obtained using a Monte Carlo Simulation
  - Off east coast of United States
  - 3km grid spacing
  - Arbitrarily chose ensemble 100 for DMD runs
- 85 hours of data use to batch compute DMD data (DMD modes, etc.) using several DMD methods (Exact, Total, Projected Compressed, Optimized, Koopman Embedded)
- Predictions into the future (for each hour up to 12 hours) are compared to a persistence
- Note: Some errors likely due to 3D nature of ocean
- RMSE: Root Mean Square Error
- PCC: Pattern Correlation Coefficient
  - Ref. Lermusiaux, P.F.J., 1999a. Data assimilation via Error Subspace Statistical Estimation. Part II: Middle Atlantic Bight shelfbreak front simulations and ESSE validation.
  - Perfect correlation: 1
  - No Correlation: 0
  - Perfect anti-correlation: -1

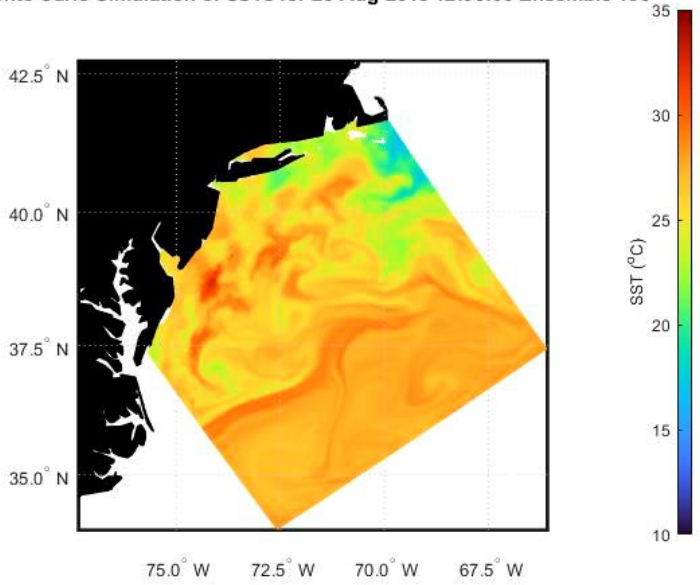
Times to Compute DMD for SST

- Exact DMD
  - Elapsed time = 0.620610 s
- Total DMD
  - Elapsed time = 0.926634 s
- Compressed DMD (1/4 of measurements)
  - Elapsed time = 0.576771 s
- Koopman Embedded DM
  - Elapsed time = 1.488855 s
- Optimized DMD
  - Elapsed time = 6.734162 s
- Projected DMD
  - Elapsed time = 0.669635 s

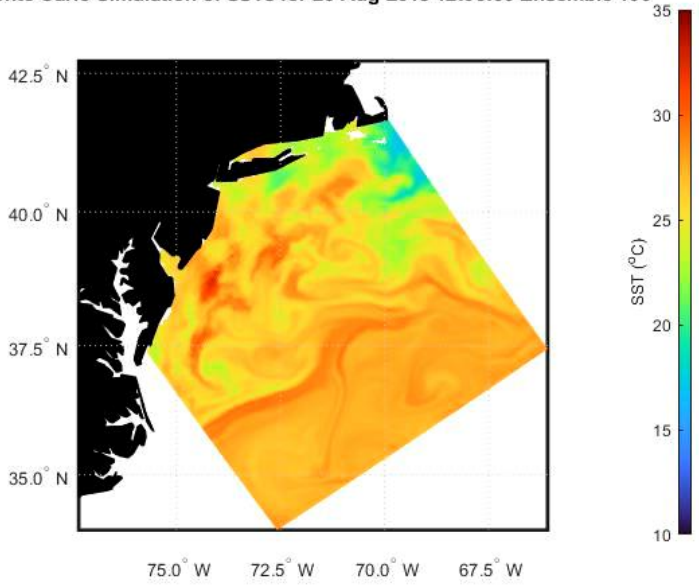
Exact DMD Prediction of SSTs for 26-Aug-2018 12:00:00 Ensemble 100



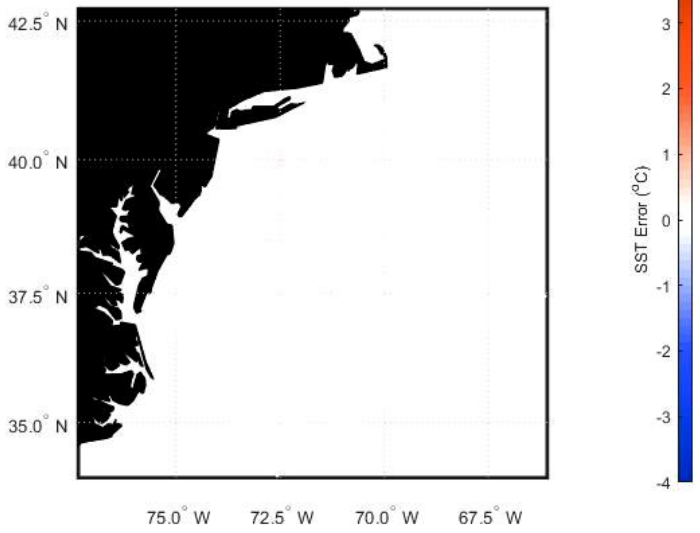
Monte Carlo Simulation of SSTs for 26-Aug-2018 12:00:00 Ensemble 100

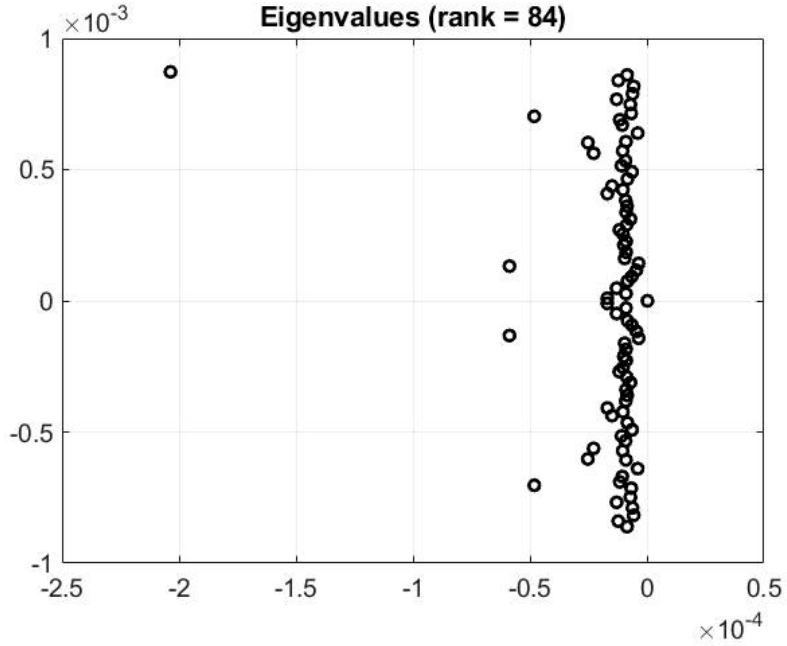


Monte Carlo Simulation of SSTs for 26-Aug-2018 12:00:00 Ensemble 100

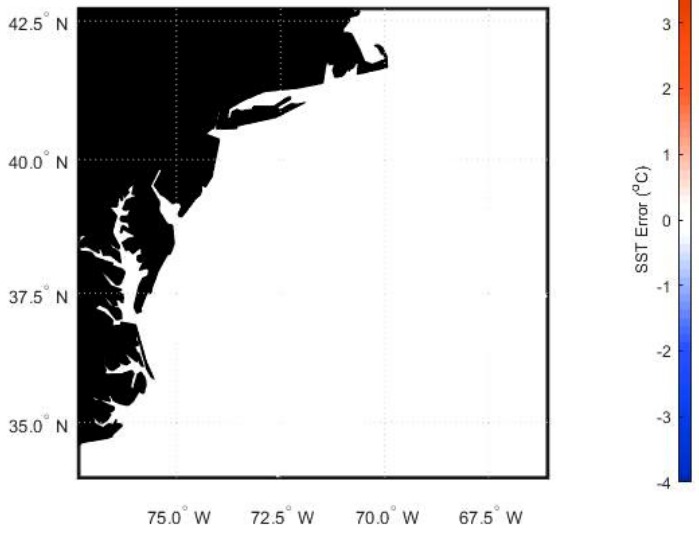


Exact DMD Error of SSTs for 26-Aug-2018 12:00:00 Ensemble 100
RMSE 7.837734e-02
PCC 9.916126e-01





Persistance Error of SSTs for 26-Aug-2018 12:00:00 Ensemble 100
RMSE 0
PCC 1



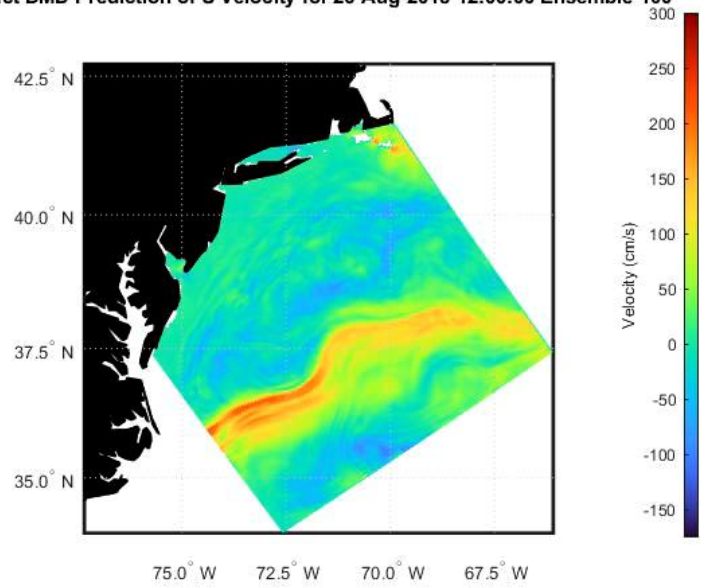
Pattern Correlation Coefficients

PCC for Sea Surface Temperatures							
	Persistence	Exact DMD	Total DMD	Projected DMD	Compressed DMD	Koopman Embedded DMD	Optimized DMD
0 hr	1	0.99161263	0.999024829	0.999024829	0.9927343	0.96577593	0.999983405
1 hr	0.984988451	0.990609304	0.991920905	0.991920905	0.991349084	0.965611432	0.996430804
2 hr	0.933986601	0.984606382	0.97284053	0.97284053	0.985241419	0.963345454	0.980268861
3 hr	0.854438751	0.967538491	0.944279184	0.944279184	0.967065725	0.954040636	0.951630882
4 hr	0.766198298	0.943716609	0.912837648	0.912837648	0.941211772	0.936199366	0.919830165
5 hr	0.690360325	0.913416787	0.884657264	0.884657264	0.910543781	0.913519223	0.892339308
6 hr	0.625767163	0.880415195	0.855269237	0.855269237	0.877820667	0.888396626	0.866312304
7 hr	0.579470587	0.850188453	0.827259682	0.827259682	0.84797846	0.865927872	0.843748832
8 hr	0.551514235	0.822899855	0.800168352	0.800168352	0.820851191	0.845286571	0.82233589
9 hr	0.534765469	0.796602438	0.774037931	0.774037931	0.795833126	0.826551946	0.801212954
10 hr	0.531252488	0.780126293	0.754914493	0.754914493	0.778833942	0.812710829	0.785516656
11 hr	0.538679731	0.758933759	0.736134566	0.736134566	0.757064495	0.792877696	0.761856084
12 hr	0.551874627	0.746710169	0.721776485	0.721776485	0.744609853	0.772218861	0.732493621

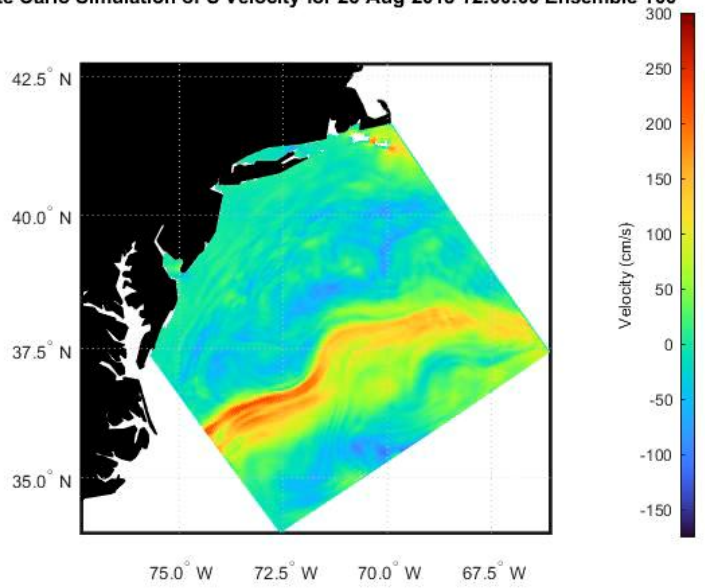
U and V Velocity

- Exact DMD
  - Elapsed time = 1.321074 s
- Total DMD
  - Elapsed time = 1.845198 s
- Compressed DMD (1/4 of measurements)
  - Elapsed time = 1.133518 s
- Koopman Embedded DM
  - Elapsed time = 2.834869 s
- Optimized DMD
  - Elapsed time = 6.807211 s
- Projected DMD
  - Elapsed time = 1.179153 s

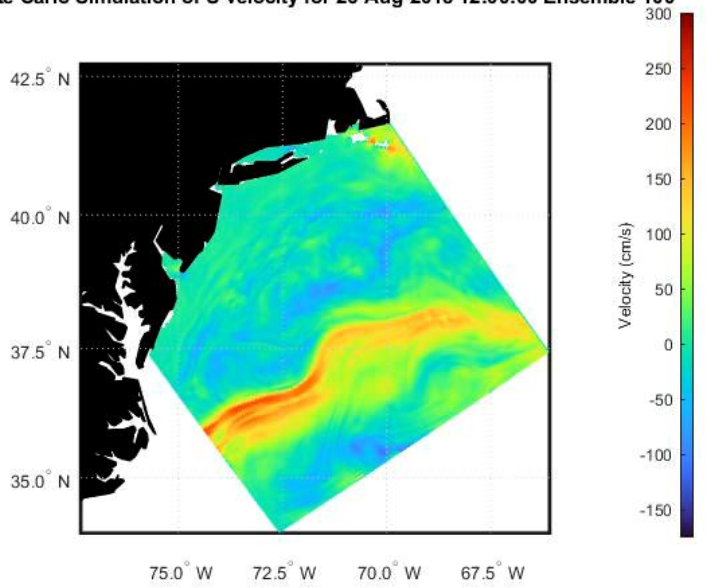
Exact DMD Prediction of U Velocity for 26-Aug-2018 12:00:00 Ensemble 100



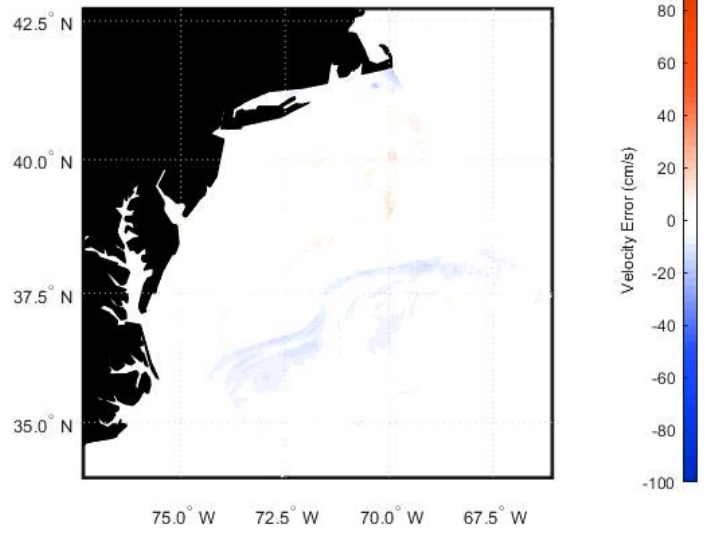
Monte Carlo Simulation of U Velocity for 26-Aug-2018 12:00:00 Ensemble 100

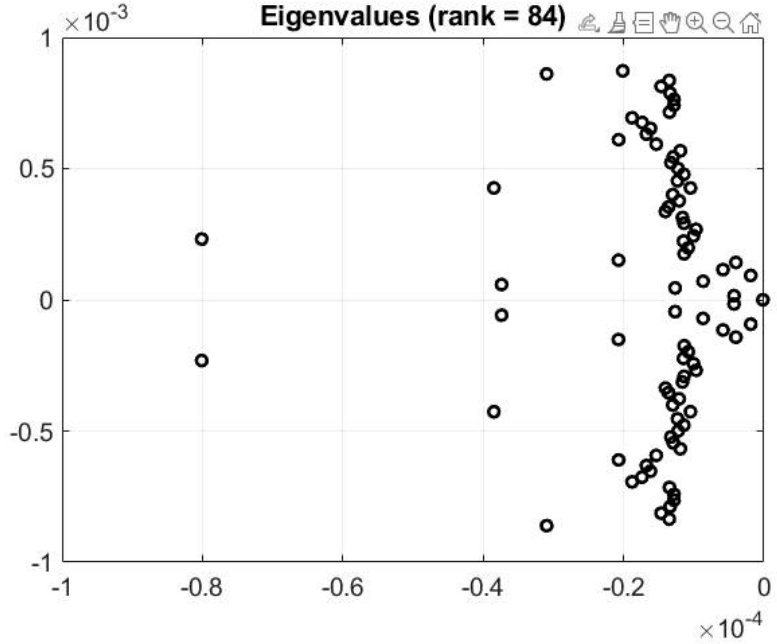


Monte Carlo Simulation of U velocity for 26-Aug-2018 12:00:00 Ensemble 100

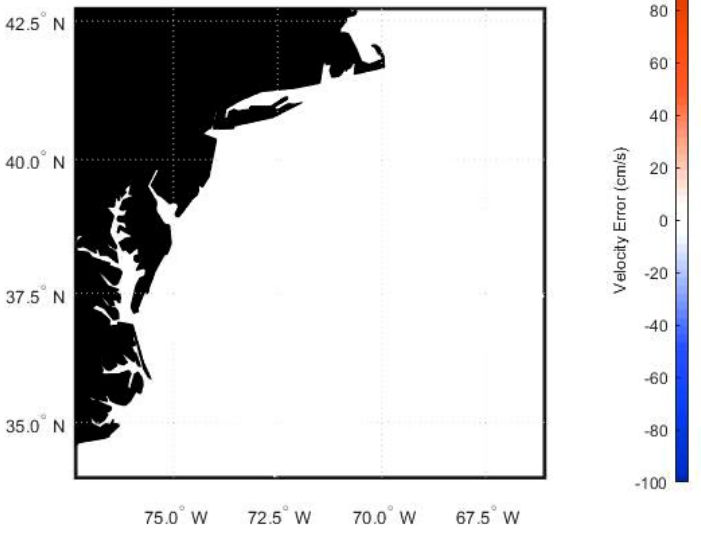


Exact DMD of U Velocity Error for 26-Aug-2018 12:00:00 Ensemble 100
RMSE 3.668814e+00
PCC 9.866745e-01





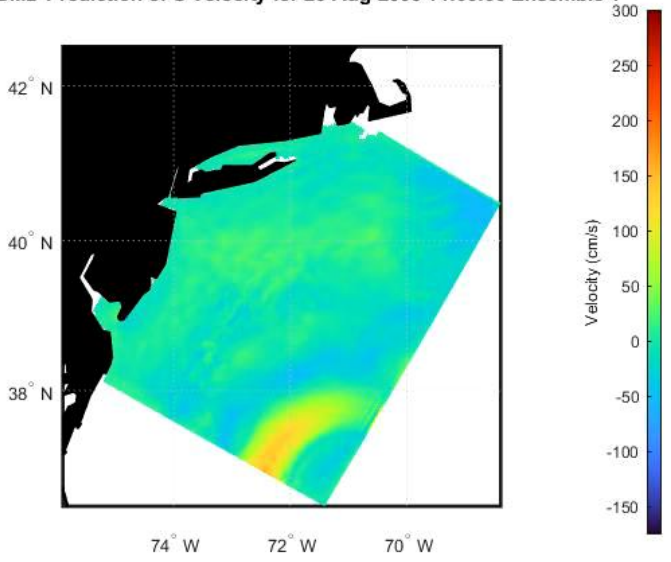
Persistance of U Velocity Error for 26-Aug-2018 12:00:00 Ensemble 100
RMSE 0
PCC 1



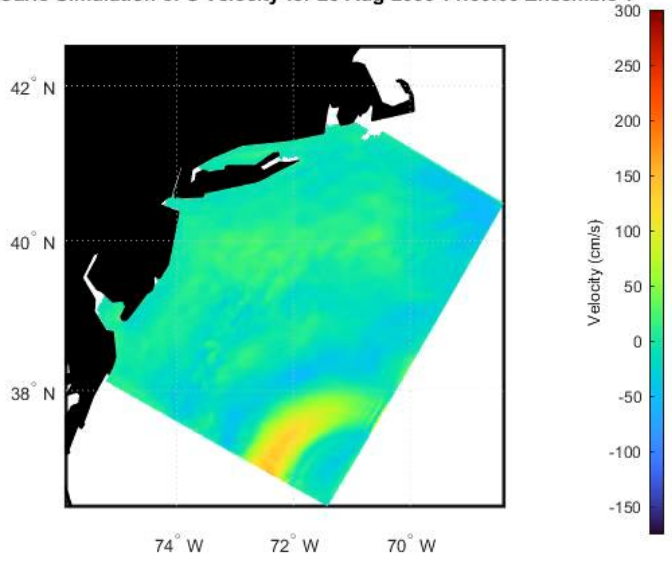
Pattern Correlation Coefficients

PCC for Sea Surface U Velocity							
	Persistence	Exact DMD	Total DMD	Projected DMD	Compressed DMD	Koopman Embedded DMD	Optimized DMD
0 hr	1	0.986674544	0.998256683	0.998256683	0.98436954	0.924693653	0.99998542
1 hr	0.965512608	0.98766877	0.984792393	0.984792393	0.985426721	0.931044996	0.994764309
2 hr	0.880209793	0.975052334	0.948320206	0.948320206	0.971977256	0.906259833	0.966508703
3 hr	0.748072758	0.932665125	0.882957801	0.882957801	0.928414799	0.824839403	0.901024287
4 hr	0.555299159	0.85645313	0.790136821	0.790136821	0.851924128	0.677824461	0.796764398
5 hr	0.31827518	0.767513355	0.699336548	0.699336548	0.763431227	0.485791104	0.672101669
6 hr	0.111726911	0.697989672	0.648527985	0.648527985	0.692056912	0.308919427	0.570369378
7 hr	-0.00859244	0.64526926	0.623238285	0.623238285	0.636432798	0.190719506	0.520505342
8 hr	-0.0388563	0.589931305	0.59013689	0.59013689	0.580349597	0.138129354	0.524311557
9 hr	0.008134132	0.538205393	0.545642524	0.545642524	0.528767098	0.158298479	0.574315769
10 hr	0.101960601	0.526091711	0.519911703	0.519911703	0.5146364	0.238141364	0.640265879
11 hr	0.196432436	0.568750583	0.543261798	0.543261798	0.553960476	0.32624663	0.682922857
12 hr	0.262778658	0.626367943	0.595229081	0.595229081	0.611134527	0.379598571	0.6878184

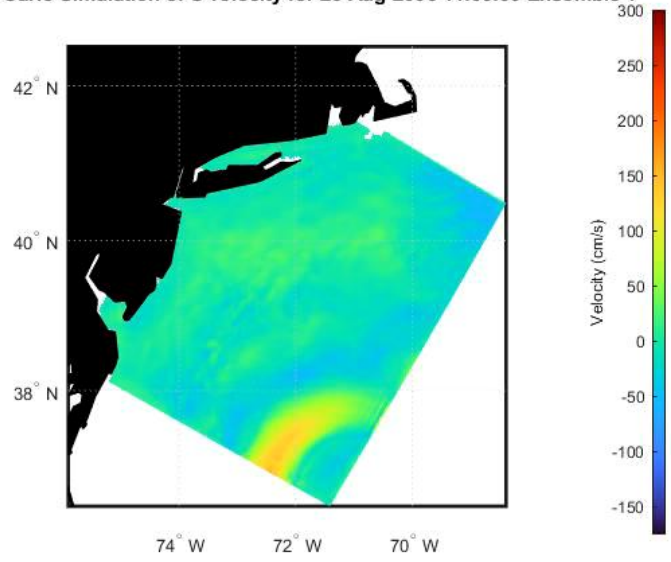
Total DMD Prediction of U Velocity for 26-Aug-2006 11:00:00 Ensemble 1



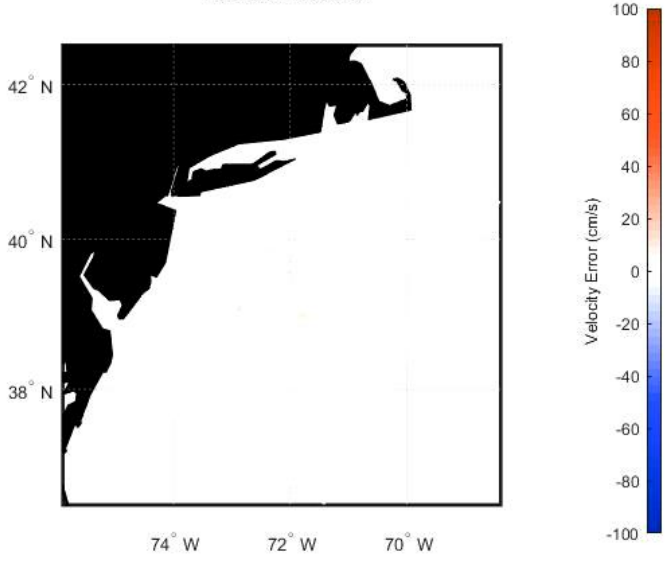
Monte Carlo Simulation of U Velocity for 26-Aug-2006 11:00:00 Ensemble 1

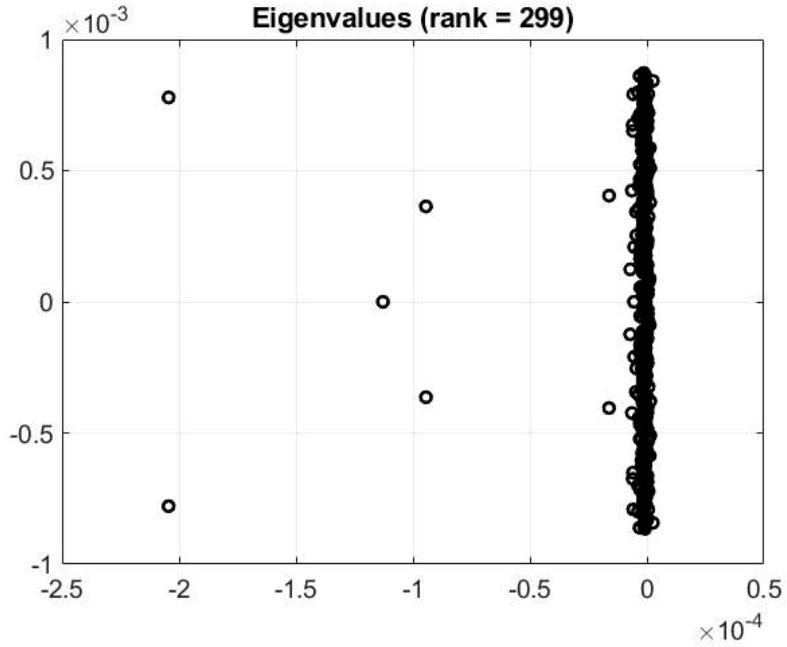


Monte Carlo Simulation of U velocity for 26-Aug-2006 11:00:00 Ensemble 1

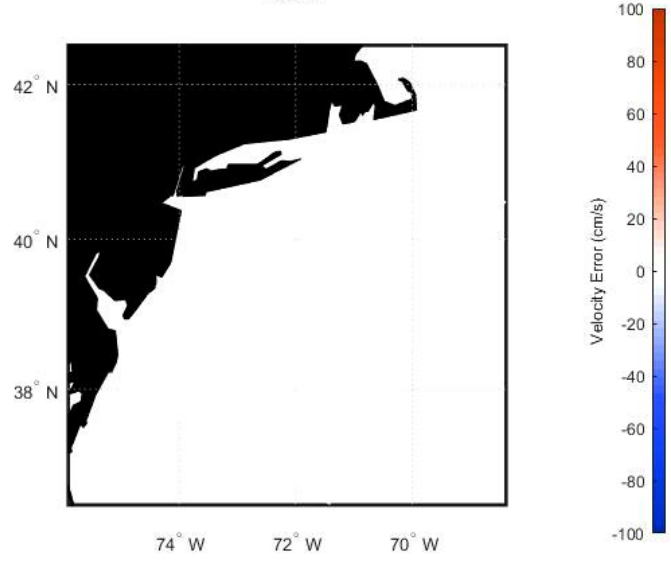


Total DMD of U Velocity Error for 26-Aug-2006 11:00:00 Ensemble 1
RMSE 1.207565e+00
PCC 9.977659e-01





Persistance of U Velocity Error for 26-Aug-2006 11:00:00 Ensemble 1
RMSE 0
PCC 1



PCC for AWACS U Velocity

PCC for U Velocity from AWACS			
	Persistence	Exact DMD	Total DMD
0 hr	1	0.90125417	0.997765853
1 hr	0.902341941	0.807869751	0.984728918
2 hr	0.638245877	0.751166924	0.960117646
3 hr	0.373117108	0.614292723	0.927034197
4 hr	0.199656773	0.687217053	0.885531436
5 hr	0.119257646	0.62055875	0.834481231
6 hr	0.129605212	0.629291118	0.752451382
7 hr	0.216213072	0.668714399	0.727725374
8 hr	0.312924836	0.753043868	0.788873631
9 hr	0.38568305	0.836583774	0.848767888
10 hr	0.443585145	0.874475009	0.876112336
11 hr	0.49767492	0.885060026	0.879733064
12 hr	0.545745018	0.870983144	0.858604361

Questions?