# Implementation of Overlapping Finite Element Method to 2D conduction problem

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2.29 Project

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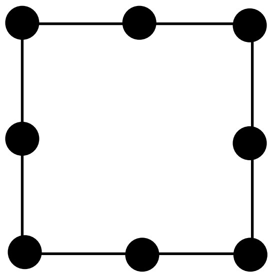
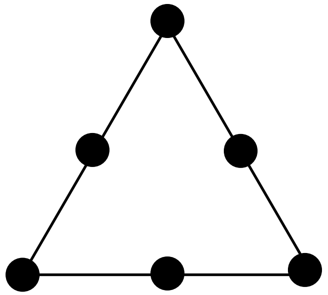
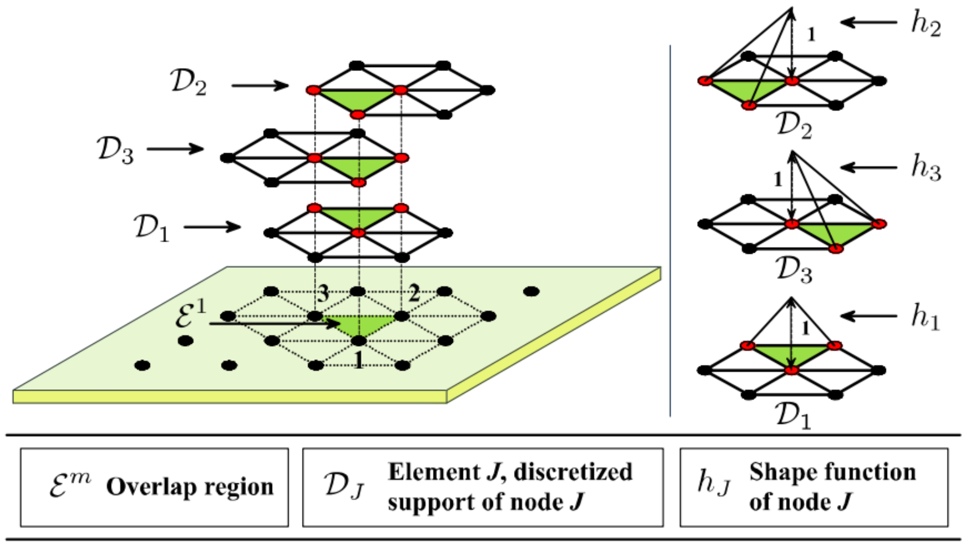
# Overlapping finite element (OFE)

1) In finite element methods, elements with no interior nodes are frequently used due to its cheap computational cost.
2) Yet, when mesh distortion is introduced, they lose accuracy of a solution field.
3) Distorted mesh is frequently observed in many finite element analyses such as fluid-solid interaction and nonlinear stress- strain problems.





Developed by Prof. KJ Bathe Group at MIT

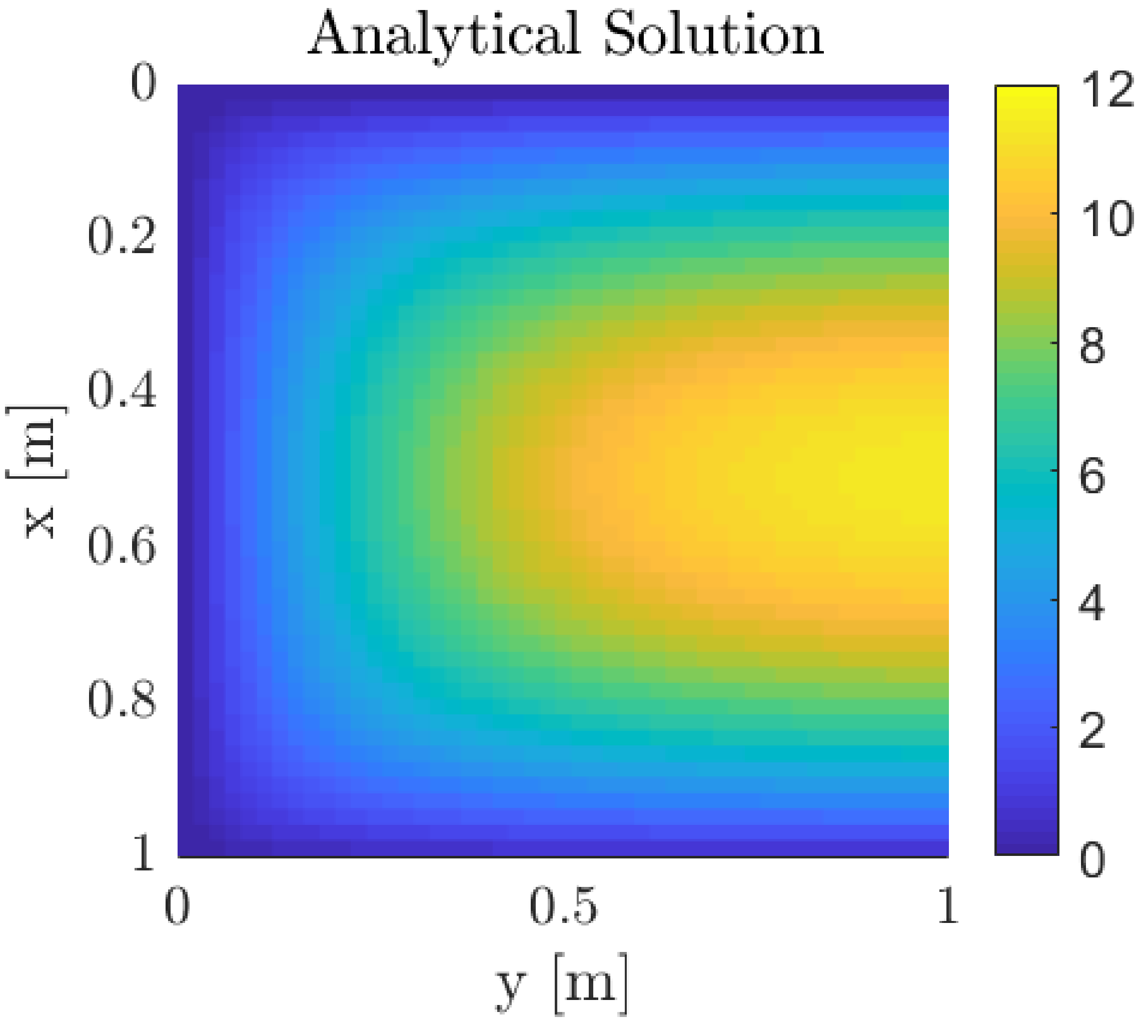


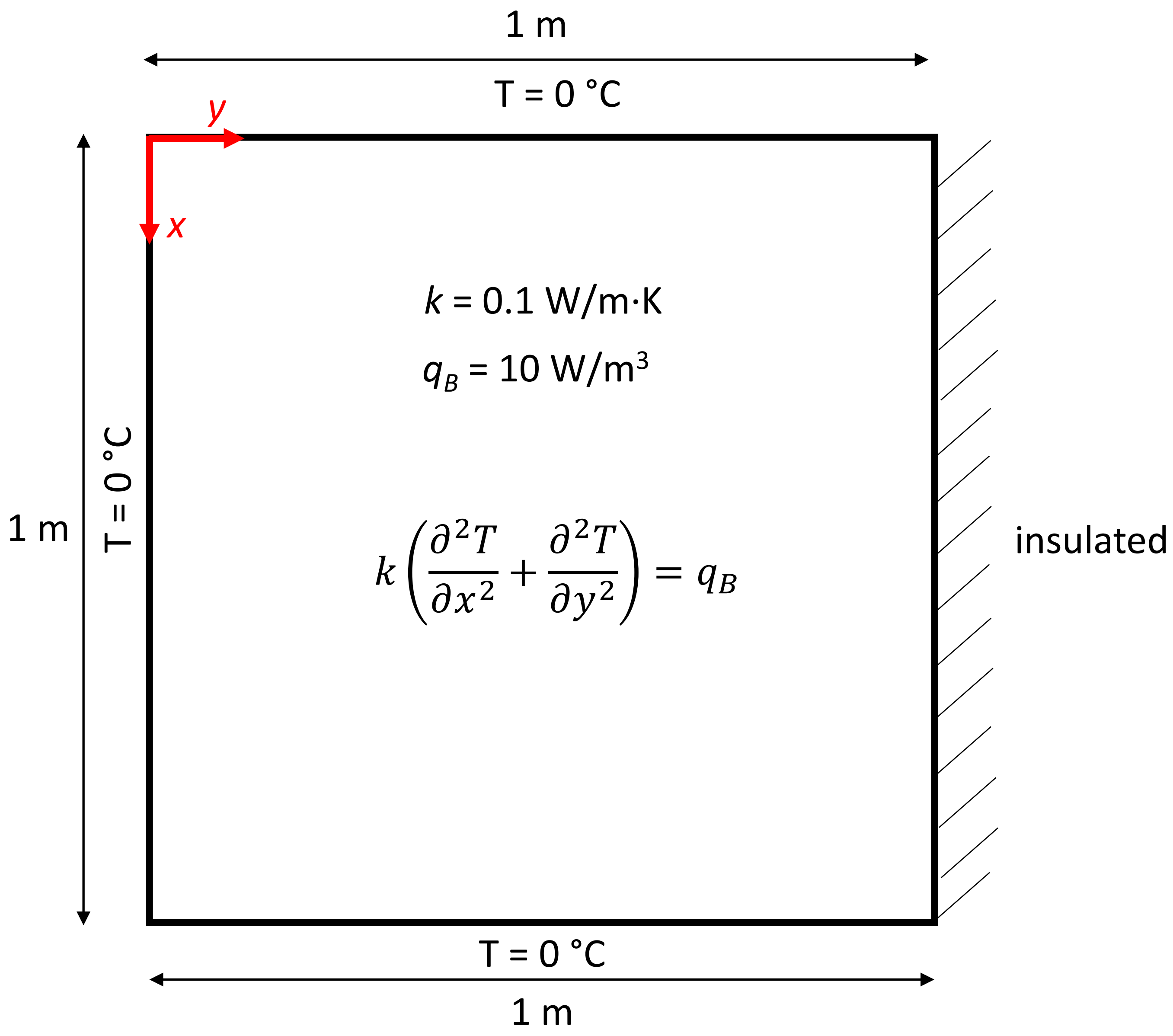

$$\mathbf{u}(\mathbf{x}) = \sum_{m=1}^{e}\sum_{I\in I_m} h_I \psi_I(\mathbf{x}) = \sum_{m=1}^{e}\sum_{I\in I_m} h_I \left( \sum_{J\in \mathcal{N}_I}\sum_{n\in \Im} \hat{\varphi}_J^I(\mathbf{x})(p_n \mathbf{a}_{Jn}) \right)$$

$$\mathbf{p}^T = [1 \quad x \quad y \quad x^2 \quad xy \quad \cdots]$$

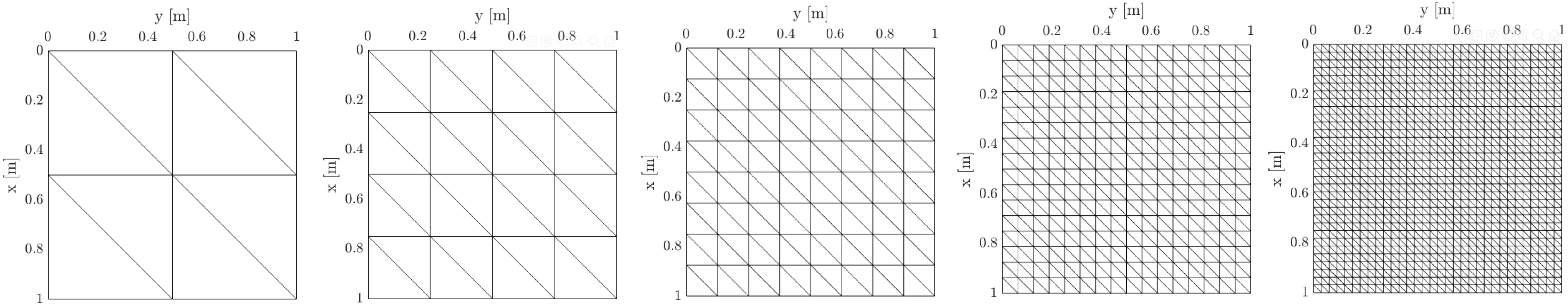
Adapted from L. Zhang *et al.*, Computers and Structures, 2018

# 2D conduction problem



# Convergence test: exponential mesh refinement




*Element size h* = $1/2^1$ $1/2^2$ $1/2^3$ $1/2^4$ $1/2^5$

**Case 1: Traditional finite element method**

1) 6-node quadratic triangular element which allows second order temperature field inside an element

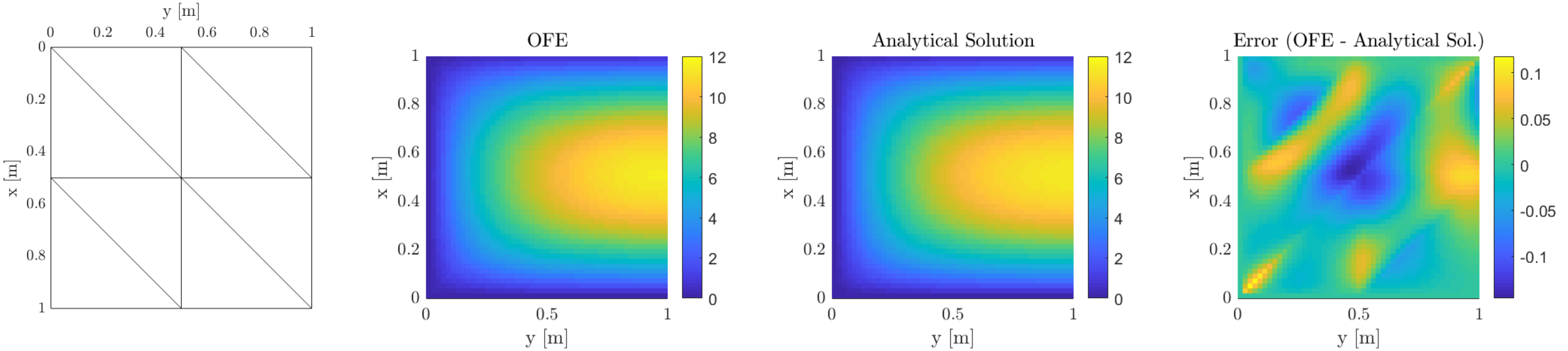
**Case 2: Overlapping finite element method**

1) Linear basis per node $\mathbf{p}^T = [\boxed{1 \quad x \quad y} \quad x^2 \quad xy \quad \cdots]$
2) 3-node linear interpolation function
3) Hence, second order temperature field allowed inside an element

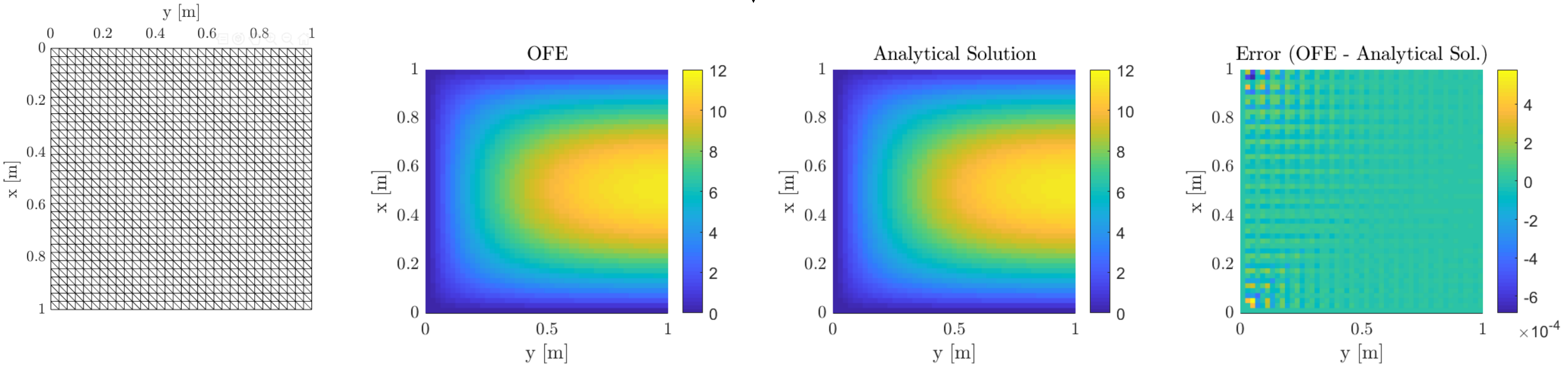
*Therefore, both methods are expected to have 2+1 = 3rd order convergence of temperature field (L2 norm)*

# OFE Convergence test: graphical representation

$h = 1/2^1$

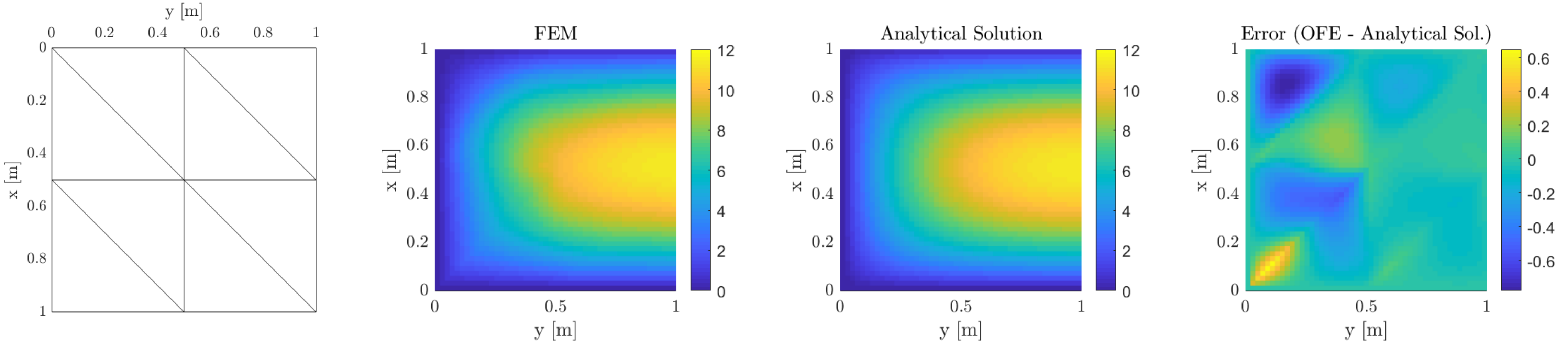



$h = 1/2^5$

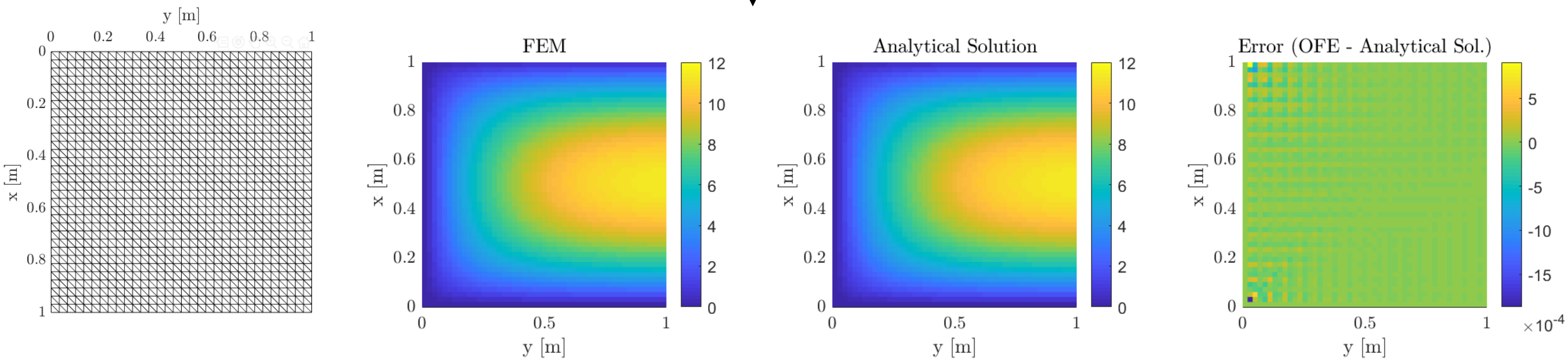


# FEM Convergence test: graphical representation

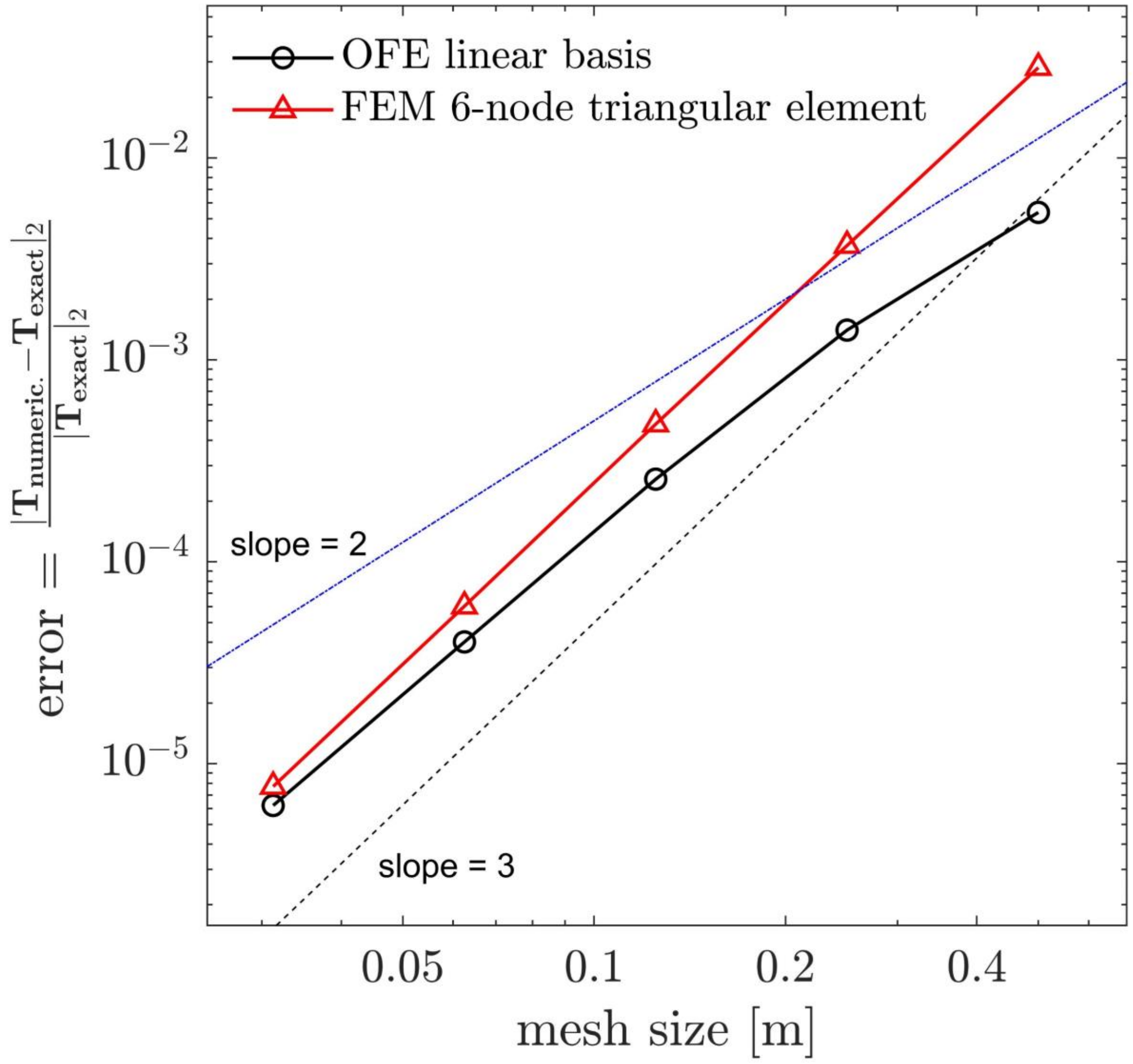
$h = 1/2^1$




$h = 1/2^5$

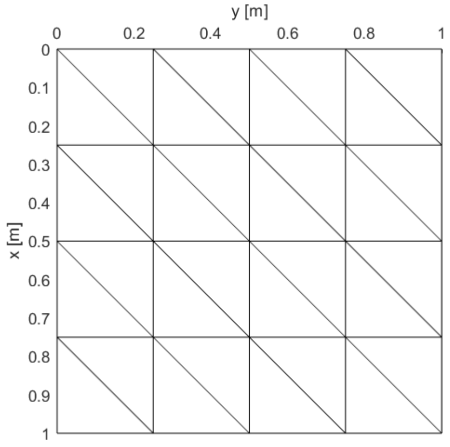


# Identifying the order of convergence

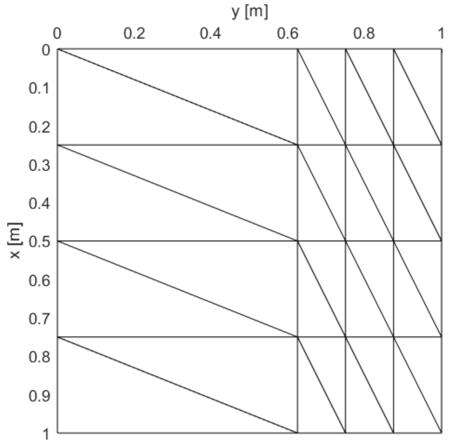



- The order of convergence of both methods is theoretically 3.
- FEM exactly follows the predicted order, while asymptotically matches against the value.

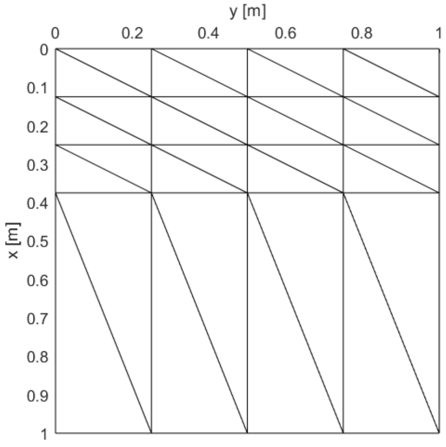
# Mesh distortion test



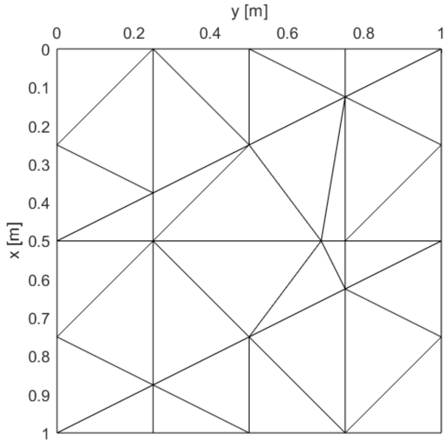

Mesh 1



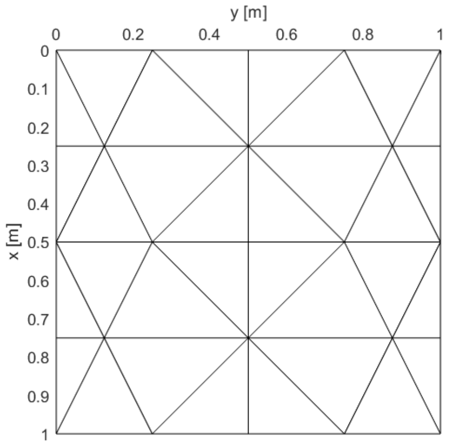

Mesh 2




Mesh 3




Mesh 4



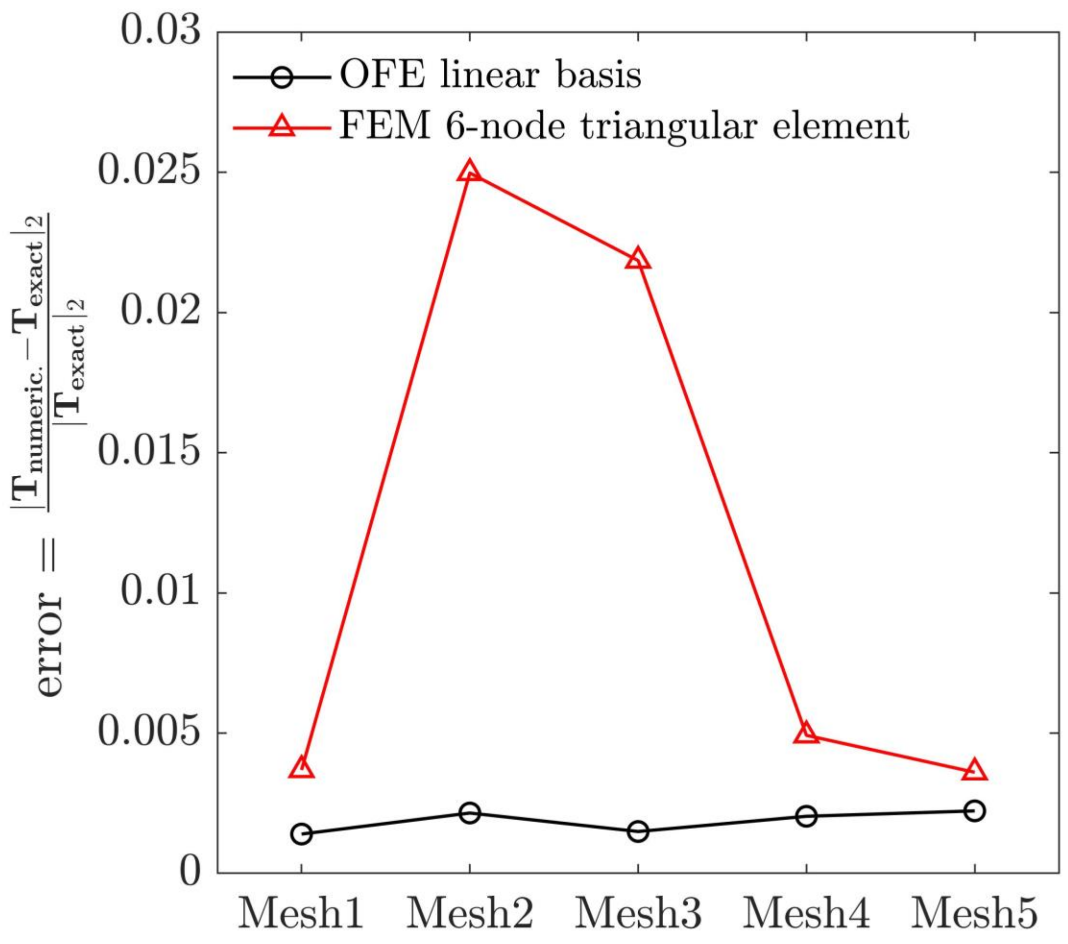

Mesh 5

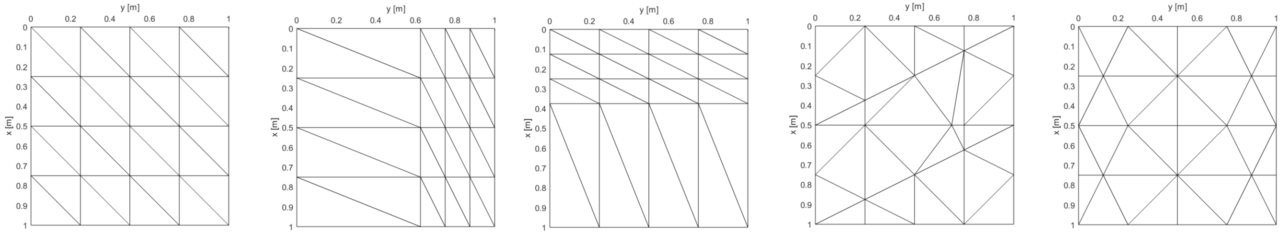
**Traditional finite element method**
- 6-node quadratic triangular element
- Degree of freedom = 81

**vs.**

**Overlapping finite element method**
- Linear basis per node
- Degree of freedom = 75

# Mesh distortion test



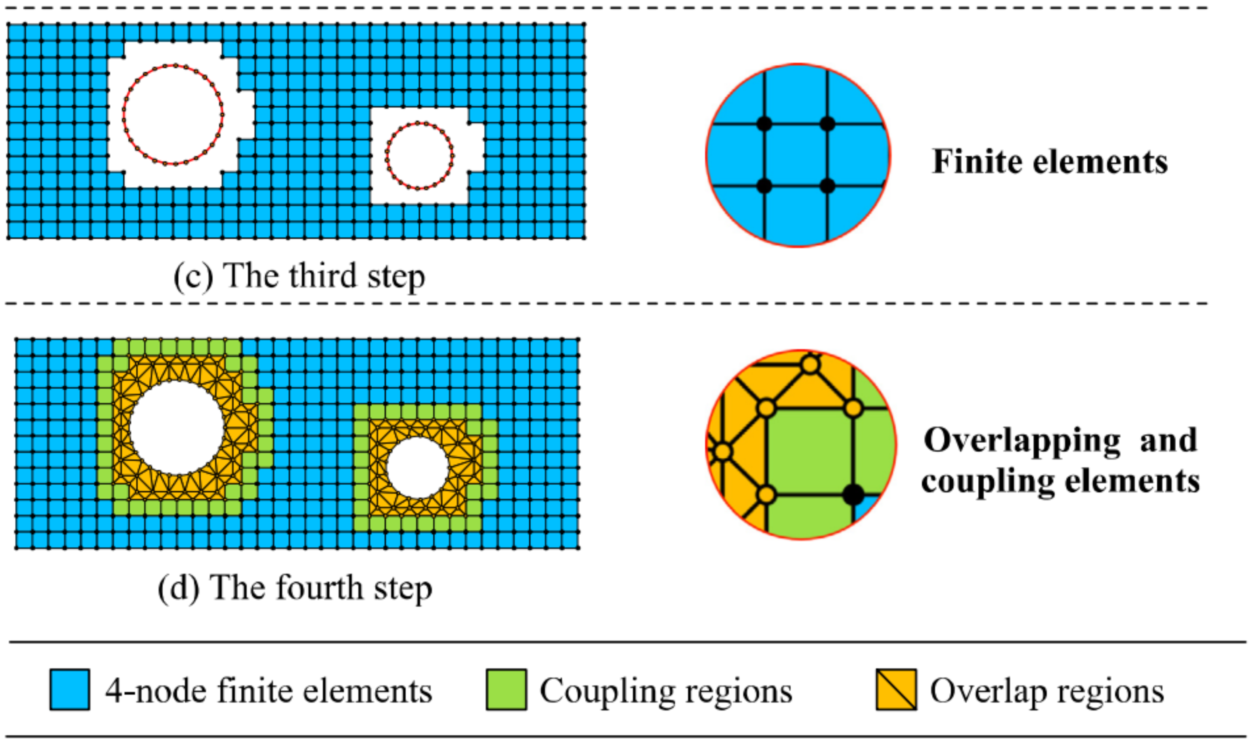
$$\left.\frac{\sigma}{E}\right|_{OFE} = 0.2$$

$$\left.\frac{\sigma}{E}\right|_{FEM} = 0.9$$

Overlapping finite elements exhibit distortion insensitivity, while the FEM based on 6-node triangular element shows the sensitivity

# Uses of OFE method

- AMORE paradigm
  - ✓ Automatic Meshing with Overlapping and Regular Elements (Prof. KJ Bathe)




Adapted from L. Zhang *et al.*, Computers and Structures, 2018

- Dynamic problems
  - ✓ Overlapping Finite Element Method enriched by polynomial and trigonometric functions.

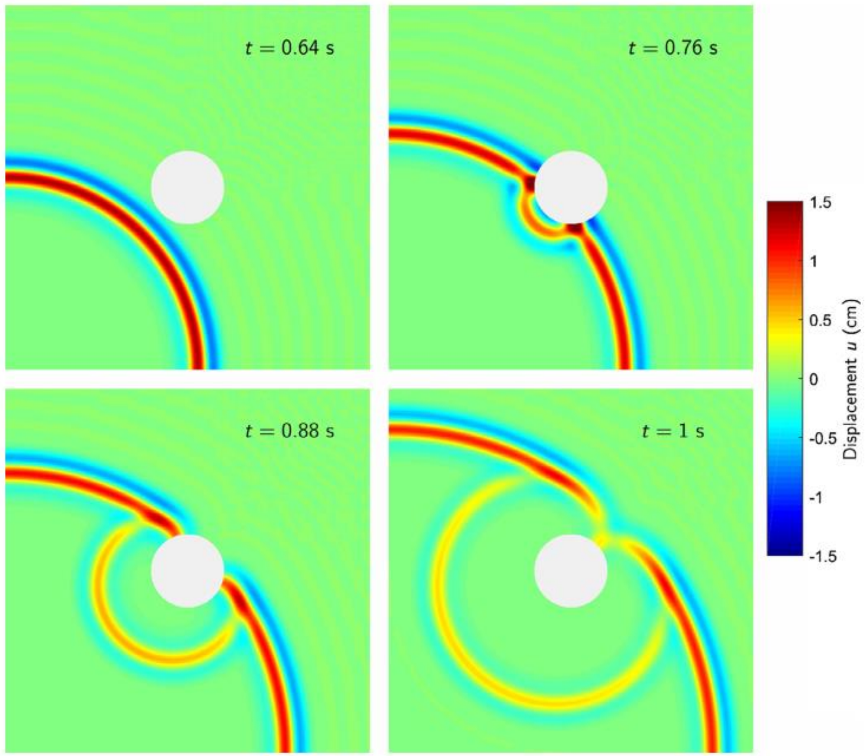



**Fig. 22.** Snapshots of displacement distributions of the membrane with circular holes at various observation times calculated using OFE-TRI1 scheme; CFL = 0.125.

Adapted from KT. Kim *et al.*, Computers and Structures, 2018

# References

Zhang L, Kim KT, Bathe KJ. The new paradigm of finite element solutions with overlapping elements in CAD–Computational efficiency of the procedure. Computers & Structures. 2018 Apr 1;199:1-7.

Kim KT, Zhang L, Bathe KJ. Transient implicit wave propagation dynamics with overlapping finite elements. Computers & Structures. 2018 Apr 1;199:18-33.

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