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Review of Aerodynamic Shape Optimization using the Adjoint Method

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Introduction – Aerodynamic Shape Optimization

- **Objective:** Vary shape design variables subject to constraints to optimize aerodynamic performance
- **Research Developments:**
  - Airfoil optimization, Hicks et al. 1974
  - Aircraft wing design optimization, Hicks and Henne 1978
  - Transonic airfoil optimization using adjoint, Jameson 1988
  - Aircraft configuration using grid perturbation, Reuther 1999

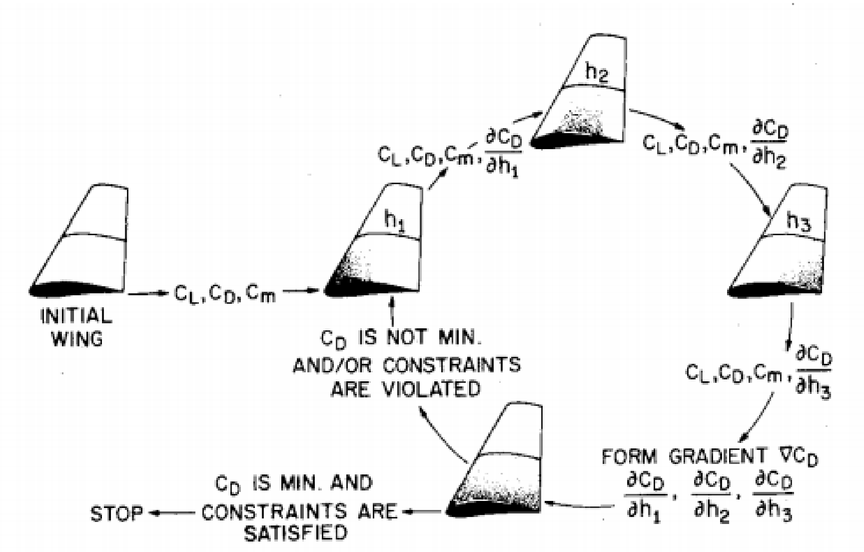

Figure 1. Optimization framework used in Hicks and Henne 1978

Gradient-based optimization

- Require values of objective and constraint functions, and gradients with respect to design variables

Methods for computing derivatives (Peter and Dwight 2010)

- Finite difference
- Complex step

- Analytic Methods
  - Direct Method
  - Adjoint Method

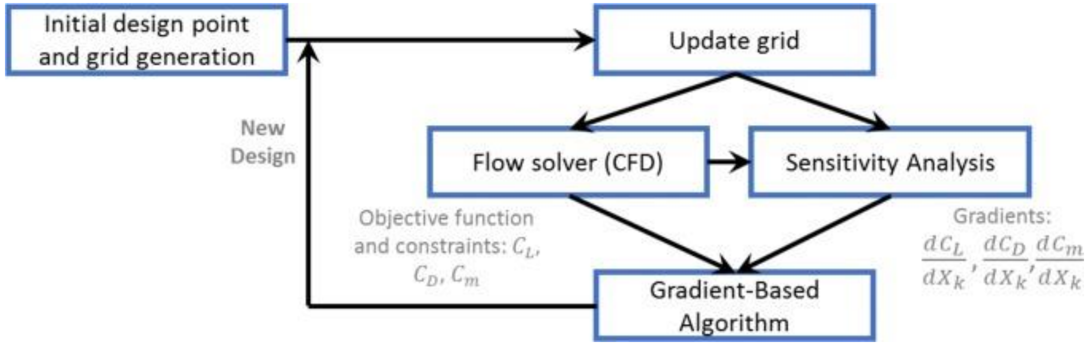

Figure 2. Schematic of gradient based aerodynamic optimization

Analytic Methods: Direct and Adjoint

- Quantity of interest f, independent variables x, state variables y
- Direct Method
  - Computational cost proportional to number of design variables
- Adjoint Method
  - Computational cost proportional to number of quantities of interest

$$f = F(x, y(x))$$

$$r = R(x, y(x)) = 0$$

$$\frac{df}{dx} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{dy}{dx}$$

$$\frac{dr}{dx} = \frac{\partial R}{\partial x} + \frac{\partial R}{\partial y}\frac{dy}{dx} = 0 \rightarrow \frac{\partial R}{\partial y}\frac{dy}{dx} = -\frac{\partial R}{\partial x}$$

$$\frac{df}{dx} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\left[\frac{\partial R}{\partial y}\right]^{-1}\frac{\partial R}{\partial x}$$

Adjoint Method

$$\left[\frac{\partial R}{\partial y}\right]^{T}\psi = -\left[\frac{\partial F}{\partial y}\right]^{T}$$

$$\frac{df}{dx} = \frac{\partial F}{\partial x} + \psi^{T}\frac{\partial R}{\partial x}$$

Adjoint Method in Fluid Mechanics

- Define cost function I, flow-field variables w, and physical location of boundary F (Jameson 2003)
- Independent of number of design variables

Research Development:

- Pioneering paper, Pironneau 1974
- Discrete adjoint using automatic differentiation, Mader et al. 2008
- Automatic differentiation adjoint of RANS equations, Lyu et al. 2013

$$I = I(w, F)$$

$$R(w, F) = 0$$

$$\frac{dI}{dw} = \frac{\partial I^T}{\partial w} + \frac{\partial I^T}{\partial F}\frac{dF}{dw}$$

$$\frac{dR}{dw} = \frac{\partial R}{\partial w} + \frac{\partial R}{\partial F}\frac{dF}{dw} = 0$$

$$\frac{dI}{dw} = \left[\frac{\partial I^T}{\partial w} + \frac{\partial I^T}{\partial F}\frac{dF}{dw}\right] - \psi^T\left[\frac{\partial R}{\partial w} + \frac{\partial R}{\partial F}\frac{dF}{dw}\right]$$

$$\left[\frac{\partial R}{\partial w}\right]^T \psi = \left[\frac{\partial I}{\partial w}\right]$$

$$\boldsymbol{\frac{dI}{dF} = \frac{\partial I^T}{\partial F} - \psi^T\frac{\partial R}{\partial F}}$$

Numerical Example: Aircraft Wing Optimization (Lyu et al 2014)

Problem: Lift-constrained drag minimization of NASA Common Research Model Wing

Geometric Parameterization: Free-form deformation approach. Shape design variables are the displacement of FFD control points in z-direction.

Mesh Perturbation: Algebraic grid generation.

Flow Equations: Steady RANS with Spalart-Allmaras turbulence model.

CFD Solver: Cell centered finite-volume. Main flow solved using Runge-Kutta algorithm, along with geometric multigrid. Spalart-Allmaras turbulence equation iterated with ADI method.

Optimization Algorithm: Gradient based optimizer with adjoint gradient evaluations.

Figure 3. FFD volume and 720 geometric control points

Numerical Example: Wing Design Benchmark

- Baseline transonic wing geometry developed by NASA
- Under nominal flight condition, M = 0.85, Re = 5 x 10^6
- Mesh generation: Hyperbolic mesh-generator marched out from surface using O-grid topology to a far field 25 times the span. Using multilevel optimization acceleration method.
- Full optimization problem:

$$\boldsymbol{Minimize}\ C_D$$

$$\boldsymbol{w.r.t}\ \ \alpha, z$$

$$\begin{gathered}\boldsymbol{s.t.}\quad C_L = 0.5\\ C_{M_y} \geq -0.17\\ t \geq 0.25 t_{base}\\ V \geq V_{base}\\ \Delta z_{TE,upper} = \Delta z_{TE,lower}\\ \Delta\, z_{LE,upper,root} = \Delta z_{LE,lower,root}\end{gathered}$$

Numerical Example: Wing Design Benchmark

- 8.5% reduction in drag
- Shock elimination in optimized wing

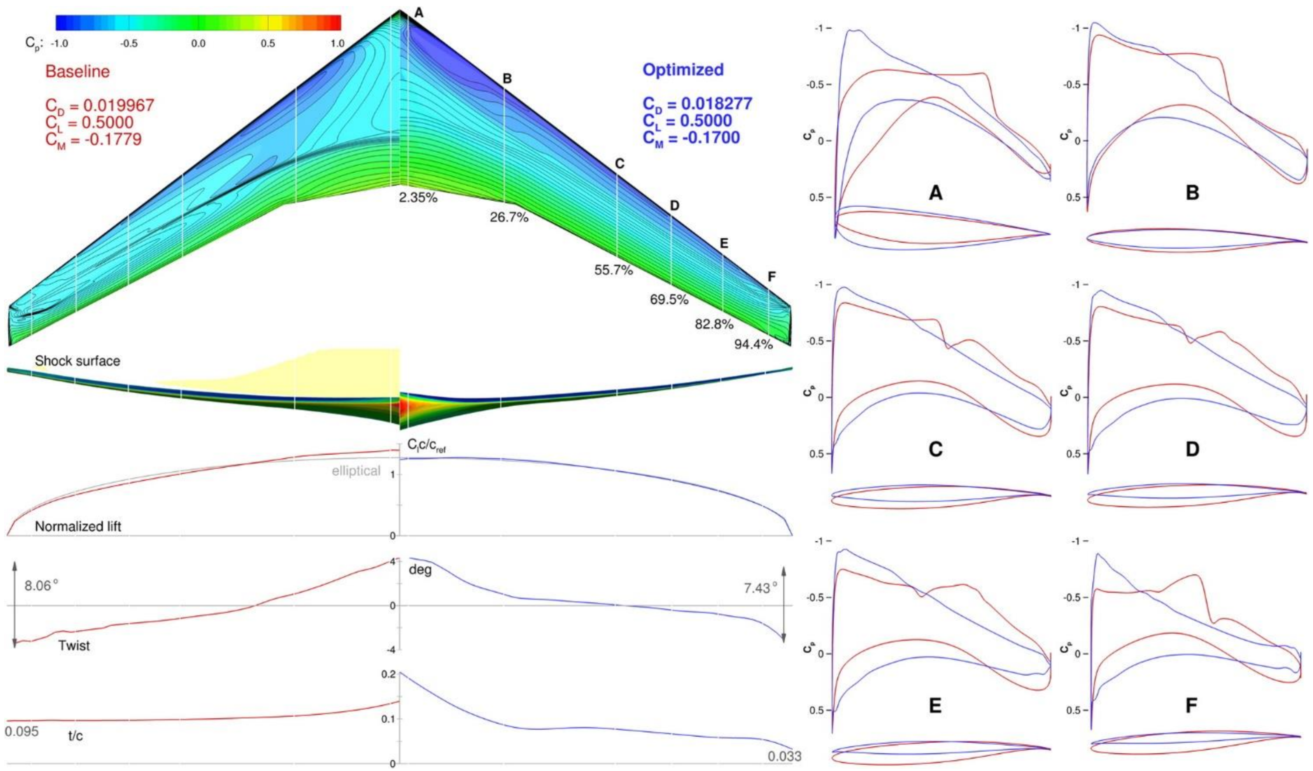

Figure 4. Baseline and optimized wing results. (Lyu et al 2014)

Numerical Example: Wing Design Benchmark

- Mesh convergence study to determine resolution accuracy of mesh
  - Three mesh sizes: L2 (451,000 cells), L1 (3.6 million cells), L0 (28.8 million cells)
  - Computed zero-grid spacing drag using Richardson's extrapolation

- Multilevel optimization: Perform optimization on coarser grid until optimality achieved, use optimal design variables on finer grid

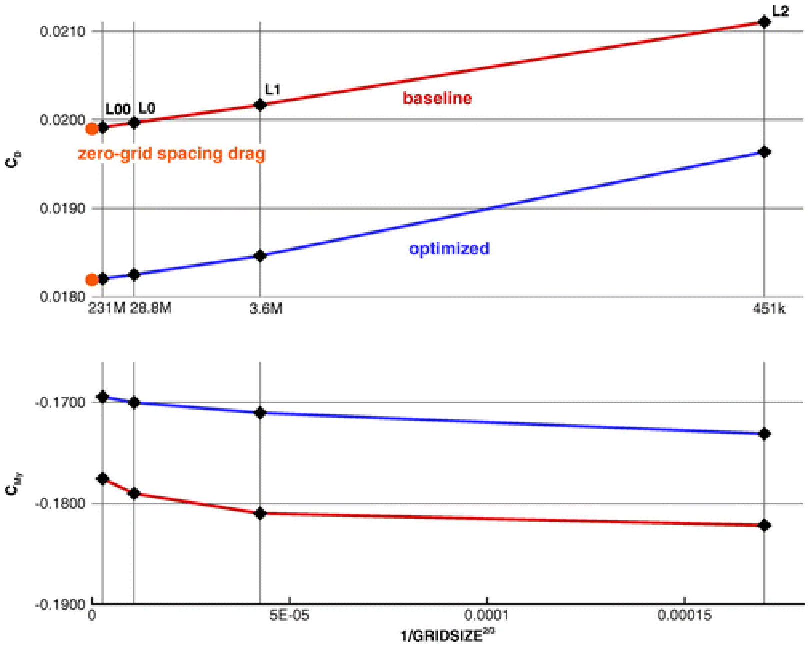

Figure 5. Mesh-convergence study

Grid Level	Iterations	Procs.	Time (hours)	Total proc-hr
L2	638	64	29.3	1,875.2
L1	89	256	20.2	5,171.2
L0	18	1,248	11.1	13,852.8

Table 1. Number of iterations and computational time spent on each grid level to reach optimal design.

Conclusions

- Gradient-based optimization using the adjoint method is efficient when the number of design variables exceeds number of quantities of interest

- Applications in airfoils, aircraft wings, aircraft configurations, wind turbine blades, hydrofoils

- Research Topics: Aerodynamic Design optimization via Machine Learning, Aerostructural optimization, Hybrid algorithms, Aeropropulsive design optimization of Boundary Layer Ingestion

References

A. Jameson, “Aerodynamic Shape Optimization Using the Adjoint Method,” in *Lectures at the Von Karman Institute, Brussels*, 06-Feb-2003.

G. K. W. Kenway, C. A. Mader, P. He, and J. R. R. A. Martins, “Effective adjoint approaches for computational fluid dynamics,” *Progress in Aerospace Sciences*, vol. 110, p. 100542, 2019.

J. E. V. Peter and R. P. Dwight, “Numerical sensitivity analysis for aerodynamic optimization: A survey of approaches,” *Computers & Fluids*, vol. 39, no. 3, pp. 373–391, 2010.

J. R. Martins and J. T. Hwang, “Review and Unification of Methods for Computing Derivatives of Multidisciplinary Computational Models,” *AIAA Journal*, vol. 51, no. 11, pp. 2582–2599, 2013.

S. N. Skinner and H. Zare-Behtash, “State-of-the-art in aerodynamic shape optimisation methods,” *Applied Soft Computing*, vol. 62, pp. 933–962, 2018.

Z. Lyu, G. K. Kenway, and J. R. Martins, “Aerodynamic Shape Optimization Investigations of the Common Research Model Wing Benchmark,” *AIAA Journal*, vol. 53, no. 4, pp. 968–985, 2015.

Z. Lyu, G. K. Kenway, C. Paige, and J. R. Martins, “Automatic Differentiation Adjoint of the Reynolds-Averaged Navier-Stokes Equations with a Turbulence Model,” *21st AIAA Computational Fluid Dynamics Conference*, 2013.