Nuclear Physics: Lecture 2

Quantum Mechanics
and
Wave-Particle Duality
Prevailing View of Physics, circa ~ 1890

“Nature and nature’s laws lay hid in night:
God said, Let Newton be!, and all was light”
   -- Alexander Pope

“Give me the initial data on the particles,
and I’ll predict the future of the universe!”
   -- Marquis Pierre de Simon LaPlace (~1860)
Observations Implying Wave-Particle Duality

• “Wave” behavior from “particles”
  – Rutherford atom dilemma
  – Davisson Germer
  – deBroglie wave hypothesis
  – Bohr Atom

• “Particle” behavior from “waves”
  – Photoelectric effect
  – Blackbody radiation
  – Compton scattering
Fundamental Problem with Rutherford “Atom”

How To Avoid Electron Orbit Collapse

Accelerating electrons radiate electromagnetic energy => spiral into nucleus!

In the planetary model of atom, the electron should emit energy and spirally fall on the nucleus.

What Maintains Stable Atom Structure?

Answer: This will be a very long, convoluted answer involving Quantum Mechanics!
Wave Behavior from “Particles”

Light Emission from Excited Atoms:

WHAT IS EXPECTED FROM RUTHERFORD PICTURE?

Answer: Continuous frequency spectrum of light from classical EM radiation theory

BUT observed Spectral LINES are DISCRETE!
Wave Behavior from “Particles” . . . Cont’d

Davisson Germer Experiment:

Electrons scattered through crystal lattice

Scattered electrons form “Diffraction” pattern characteristic of “waves”??
DeBroglie Wave Hypothesis

Every particle has an associated “Wave”. “ψ” denotes the wave.

\[ \Psi \approx \cos(kx) = \cos\left(\frac{2\pi x}{\lambda}\right) \]

Wavelength found from Planck’s constant & Momentum:

\[ \lambda = \frac{\hbar}{mv} \]

The method for determining this wave and how it was used to actually determine particle behavior was left to others. And yet the Nobel prize in physics for the 55 page thesis!

\[ \hbar \equiv \frac{\hbar}{2\pi} \]
Waves have to “fit” in the “box”: 
=> Box must contain integer number of (half) wavelengths 

\[ L = \frac{n \lambda}{2} \]

Like natural frequencies of: 
- Organ Pipe
- Trumpet
- Drum Head
- Coke Bottle
- Violin
What are the “Waves”?  

Waves of Probability . . .

Particle has no definite position (we don’t know where in box/atom/nucleus the particle is actually located).

\[ P(x, t) = |\Psi(x, t)|^2 \]

Is probability of finding the particle at position, \( x \)

Probabilities for particle in box are “strange” (see blackboard . . .)
**Discrete Energy Levels**

- **If the wavelength determines the momentum . . .**

  \[ m \nu = \frac{h}{\lambda} \]

- **The Energies must be Quantized**

  \[ E_n = \frac{mv^2}{2} = n^2 \frac{h^2}{8mL^2} \]

  This is the origin of the term, "Quantum Mechanics"

**What is expected from Rutherford picture?**
Neils Bohr (1913) applied the quantum principles to electrons orbiting nucleus:

- Quantized Wavelengths:
  \[ 2\pi r = n\lambda \]

- deBroglie hypothesis:
  \[ \lambda = \frac{h}{mv} \]

- Force Balance (Centrifugal vs. Coulomb):
  \[ \frac{e^2}{r^2} = \frac{mv^2}{r} \]

=> Energy Levels of Hydrogen atom EXACTLY predicted!

\[ E_n = \frac{1}{2} mv^2 - \frac{e^2}{r} = -\frac{1}{n^2} m_e c^2 \frac{e^4}{2\hbar^2 c^2} \]

\[ E_n = -13.6 \text{ eV} \]
Atomic Electron Probabilities ("Orbitals")

- Probability clouds of electron in atom
- Note disconnected structure – For one electron!

\[ P(x, y, z, t) = |\Psi(x, y, z, t)|^2 \]
**Particle “Diffraction”**

Explain Davisson-Germer if electrons scatter from crystal lattice atoms as deBroglie waves:

Interference minima when path length from holes differs by half wavelength:

$$d \sin(\theta_{\text{min}}) = \lambda / 2$$

Electrons @ KeV energies “interfere” with Angstrom (~10^{-8} cm) scale atomic lattice structure
**Particle Behavior from “Waves”**

This is the final aspect of the “wave-particle duality” that is Quantum Mechanics

**Photoelectric Effect:**

Electrons ejected from metal by incoming light

**What is expected, classically, as function of light intensity and frequency?**

**Answer:** Higher intensity => higher electron energy & frequency of light irrelevant
Photoelectric Effect -- Observations

Experiment shows:

- Electron energy proportional to light frequency
- Intensity effects current but not electron energy
**Photons of Light**

**Photoelectric Effect** explained if Light “waves” exchange energy with matter via PHOTON quanta

\[ E_\gamma = hf \]

\[ p = E_\gamma / c = h / \lambda \]

\[ E_\gamma = \hbar \omega \]

\[ p = E_\gamma / c = \hbar k \]

- Higher frequency => higher photon energy
- Number of photons proportional to intensity
- Einstein (1905)

The frequency is denoted by the symbol \( f \).
Blackbody Radiation Spectrum

Continuous spectra (thermal spectra) for different temperatures. At higher temperatures, there is more energy at all wavelengths and the peak of the spectrum shifts to smaller wavelengths (toward the blue side). Wien's law: $\lambda_{\text{peak}} = \frac{2.9 \times 10^6 \text{nm}}{T} \text{ (in K)}$. Find star's or planet's $\lambda_{\text{peak}}$ to measure their temperature! The Sun's thermal spectrum (5840 K) is shown for comparison. Star temperatures range from 2600–40,000+ K.

$$E_\gamma = \frac{hc}{\lambda}$$
Compton Scattering
Formal Quantum Mechanics

- Schrödinger Equation
- Measurement Probabilities
- Wave Mechanics
- Eigenvalue Problem
- “Confused” picture of matter
- Fundamental philosophical dilemma:
  - => “God doesn’t play dice”

\[ i\hbar \frac{\partial \Psi}{\partial t} = \hat{H} \Psi \]