Fall Term 2006

22.02 Introduction to APPLIED NUCLEAR PHYSICS

Problem Set #2

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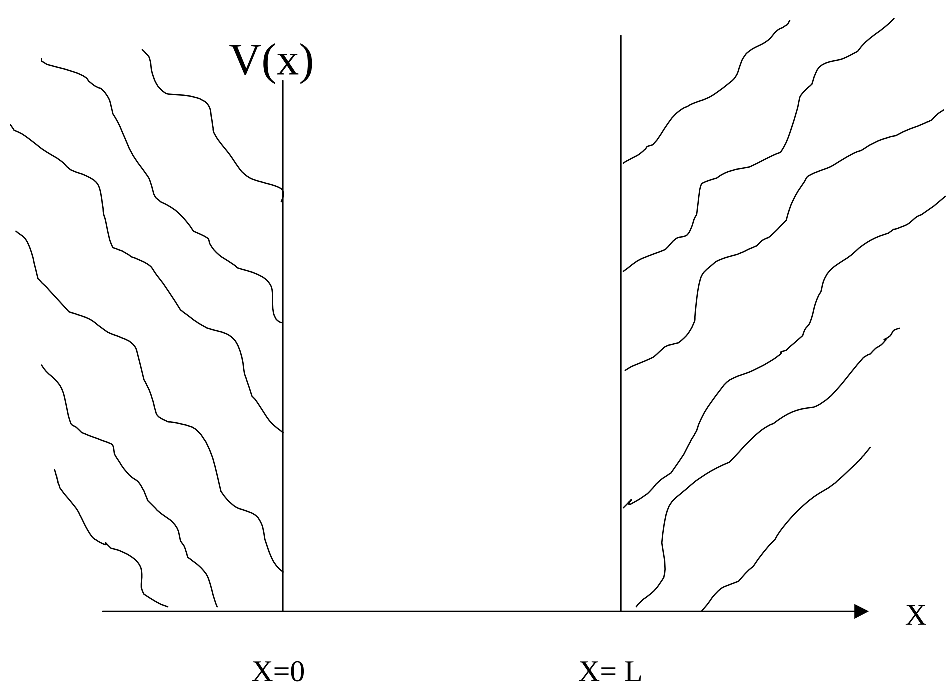
1. Liboff, 3.11 and 3.12. For 3.11 also calculate the mean momentum, $\langle p \rangle$. *Hint for 3.11: Use the following identity to rewrite, $\psi(x,t)$, before differentiating:*

$$\exp(x)\exp(y) = \exp(x+y)$$

or,

$$\exp(x_1)\exp(x_2)\exp(t) = \exp(x_1 + x_2 + t)$$

2. *Particle in 1D box:* Consider a free particle moving in box as shown in figure with infinite potential bounding the box at $x = 0$ and $x = L$.



- Solve the energy eigenvalue problem, $\hat{\mathcal{H}}\psi_n = E_n\psi_n$, and give a complete listing of both the normalized eigenfunctions, ψ_n, and the energy spectrum, E_n.

- Also for state $\psi = \psi_n$, as in b), write the probability that a position measurement finds the particle within the interval, Δx, about x? Be sure to get your answer dimensionally correct for a probablility.
- Now suppose the system is in a *superposition* quantum state:

$$\psi(x) = a_2\psi_2(x) + a_5\psi_5(x)$$

a. What are the possible outcomes of an energy measurement in this case?

b. Determine a_2 and a_5 if the probability of finding $E = E_2$ is $P_2 = 1/4$, and the probability of measuring, $E = E_5$ is $P_5 = 3/4$.

c. What is the probability of finding the particle at $x = \frac{1}{2}L$? Can you state more specifically what the measurement is?

3. *For this problem remember that:*

$$\hbar c = 197\ MeV - F$$

Calculate the de Broglie wavelength of a 1-KeV electron, and a 4-MeV alpha particle. Relate the numbers you obtain to characteristic physical dimensions, i.e. the size of an atom or nucleus, etc. Derive the expression relating the DeBroglie wavelength of a particle of mass, m, and velocity, v, from the fundamental relation, $p = \hbar k = mv$,

$$\lambda = 2\pi \frac{\hbar c}{mc^2} \frac{c}{v}$$

Now consider a piece of chalk as being approximately, $\sim\ 10^{24}$ Carbon atoms - a good classical object. If your good professor throws this to demonstrate Quantum effects, how slow must it go to possess a $\sim\ 1\ cm$, DeBroglie wavelength (that would allow it to exhibit certain "wave" effects?