22.02 Fall 2006 Problem Set #7 Solutions

1. $E_N = \hbar\omega N - V_0$, $V_0 = 50$ MeV

$$\omega^2 = \frac{2V_0}{mR_0^2} \ , \quad \Delta E = \hbar\omega = \frac{\hbar c}{R_0}\sqrt{\frac{2V_0}{mc^2}} = 51.5\,A^{-1/3}\ \text{MeV} \ , \ A = 169 \Rightarrow \hbar\omega = 9.315\ \text{MeV}$$

$^{169}Os \Rightarrow Z = 76, N = 93$

N	# protons	# neutrons	# nucleons	E_N(MeV)	(# nucleons)* E_N (MeV)
0	2	2	4	−50	−200
1	6	6	12	−40.685	−488.22
2	12	12	24	−31.37	−752.88
3	20	20	40	−22.06	−882.4
4	30	30	60	−12.74	−764.4
5	6	23	29	−3.425	−99.325
6	0	0	0	5.89	0

Average binding energy / nucleon $= \sum_{N=0}^{5} -\left[(\#\text{nucleons}) * E_N\right] / 169 = \boxed{18.86\ \text{MeV/nucleon}}$

Protons fill up the N=5 level partially (6 protons) containing the 3p, 2f, and 1h levels.

Neutrons fill up the N=5 level partially (23 neutrons) containing the 3p, 2f, and 1h levels.

E_N goes above zero

2. Krane 5.1

Parity : $(-1)^{\ell}$ $\Rightarrow$

ℓ	0	1	2	3	4	5	6
	s	p	d	f	g	h	i
	+	−	+	−	+	−	+

a) $^{7}Li \Rightarrow A=7,\ Z=3,\ N=4 \Rightarrow$ 1 unpaired proton in $1p_{3/2}$ $\Rightarrow \boxed{\frac{3}{2}^-}$

b) $^{11}B \Rightarrow A=11,\ Z=5,\ N=6 \Rightarrow$ 1 unpaired proton in $1p_{3/2}$ $\Rightarrow \boxed{\frac{3}{2}^-}$

c) $^{15}C \Rightarrow A=15,\ Z=6,\ N=9 \Rightarrow$ 1 unpaired neutron in $1d_{5/2}$ $\Rightarrow \boxed{\frac{5}{2}^+}$

d) $^{17}F \Rightarrow A=17,\ Z=9,\ N=8 \Rightarrow$ 1 unpaired proton in $1d_{5/2}$ $\Rightarrow \boxed{\frac{5}{2}^+}$

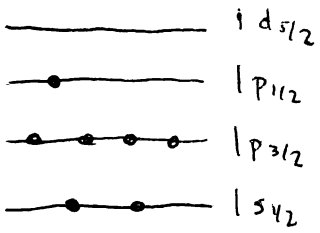
e) $^{31}P \Rightarrow A=31,\ Z=15,\ N=16 \Rightarrow$ 1 unpaired proton in $2s_{1/2}$ $\Rightarrow \boxed{\frac{1}{2}^+}$

f) $^{141}Pr \Rightarrow A=141,\ Z=59,\ N=82 \Rightarrow$ 1 unpaired proton in $2d_{5/2}$ $\Rightarrow \boxed{\frac{5}{2}^+}$

Krane 5.2

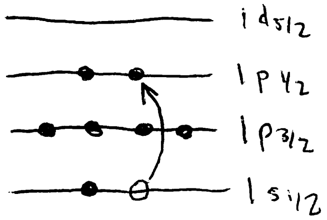
$^{13}C \Rightarrow A=13,\ Z=6,\ N=7$

Ground state :



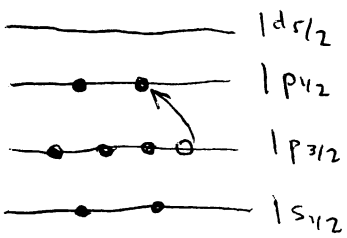
Unpaired neutron in $1p_{1/2}$ $\Rightarrow \boxed{\frac{1}{2}^-}$

1st Excited state :



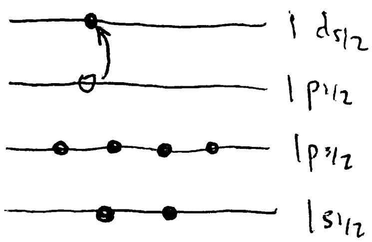
Unpaired neutron in $1s_{1/2}$ $\Rightarrow \boxed{\frac{1}{2}^+}$

2nd Excited state :



Unpaired neutron in $1p_{3/2}$ $\Rightarrow \boxed{\frac{3}{2}^-}$

3rd Excited state :

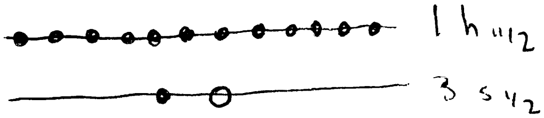


Unpaired neutron in $1d_{5/2}$ $\Rightarrow \boxed{\frac{5}{2}^+}$

Krane 5.3

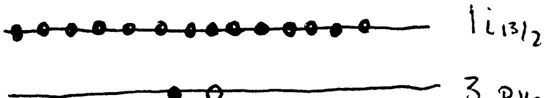
$^{203}Tl \Rightarrow A=207,\ Z=81 \Rightarrow$ 1 unpaired proton in $3s_{1/2} \Rightarrow \boxed{\frac{1}{2}+}$

Ground state:

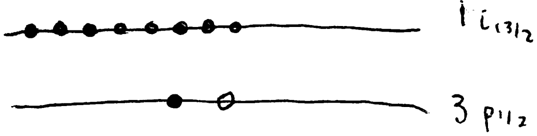


$^{207}Pb \Rightarrow A=207,\ N=125 \Rightarrow$ 1 unpaired neutron in $3p_{1/2} \Rightarrow \boxed{\frac{1}{2}-}$

Ground state:



$^{199}Hg \Rightarrow A=199,\ N=119 \Rightarrow$ 1 unpaired neutron in $3p_{1/2} \Rightarrow \boxed{\frac{1}{2}-}$



Krane 5.6

(a) $V_{nuc}(r) = V_0(r) + V_{so}(r)\dfrac{\hat{\ell}\cdot\hat{s}}{\hbar^2}$, $E_0 = V_0 \Rightarrow V_{nuc}(r) = E_0 + V_{so}(r)\dfrac{\hat{\ell}\cdot\hat{s}}{\hbar^2}$

$\hat{j}^2 = (\hat{\ell}+\hat{s})\cdot(\hat{\ell}+\hat{s}) = \hat{\ell}^2 + \hat{s}^2 + 2\hat{\ell}\cdot\hat{s} \Rightarrow \hat{\ell}\cdot\hat{s} = \dfrac{\hat{j}^2-\hat{\ell}^2-\hat{s}^2}{2}$

$\hat{\ell}\cdot\hat{s} = \dfrac{\hbar^2}{2}\left[j(j+1) - \ell(\ell+1) - s(s+1)\right]$

$$\left.\frac{\hat{\ell}\cdot\hat{s}}{\hbar^2}\right|_{j=\ell+\frac{1}{2}} = \frac{1}{2}\left[(\ell+\tfrac{1}{2})(\ell+\tfrac{1}{2}+1) - \ell(\ell+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)\right] = \frac{\ell}{2}$$

$$\left.\frac{\hat{\ell}\cdot\hat{s}}{\hbar^2}\right|_{j=\ell-\frac{1}{2}} = \frac{1}{2}\left[(\ell-\tfrac{1}{2})(\ell-\tfrac{1}{2}+1) - \ell(\ell+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)\right] = \frac{-(\ell+1)}{2}$$

$V_{nuc1}(r) = E_0 + V_{so}(r)\frac{\ell}{2} = E_0 + \frac{\ell}{2}V_{so}$

$V_{nuc2}(r) = E_0 + V_{so}(r)\left[-\frac{(\ell+1)}{2}\right] = E_0 - \frac{\ell+1}{2}V_{so}$

$\Rightarrow \boxed{V_{nuc1} = E_0 + \frac{\ell}{2}V_{so},\quad V_{nuc2} = E_0 - \frac{\ell+1}{2}V_{so}}$

(b) "center of gravity" : $\dfrac{V_{nuc1}D_1 + V_{nuc2}D_2}{D_1+D_2}$, D_1 & D_2 are degeneracies for each level

$D_1 = 2j_1+1$, $D_2 = 2j_2+1$

$$\frac{V_{nuc1}D_1 + V_{nuc2}D_2}{D_1+D_2} = \frac{(E_0+\frac{\ell}{2}V_{so})[2(\ell+\frac{1}{2})+1] + (E_0-\frac{\ell+1}{2}V_{so})[2(\ell-1)+1]}{[2(\ell+\frac{1}{2})+1]+[2(\ell-\frac{1}{2})+1]}$$

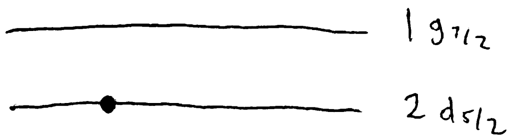
$\dfrac{V_{nuc1}D_1 + V_{nuc2}D_2}{D_1+D_2} = E_0 \Rightarrow \boxed{\text{"Center of gravity"} = E_0}$

3.

	Z	N	Expected	Observed
^{119}Sb	51	68	$\frac{7}{2}^{+}$	$\frac{5}{2}^{+}$
^{121}Sb	51	70	$\frac{7}{2}^{+}$	$\frac{5}{2}^{+}$
^{123}Sb	51	72	$\frac{7}{2}^{+}$	$\frac{7}{2}^{+}$
^{125}Sb	51	74	$\frac{7}{2}^{+}$	$\frac{7}{2}^{+}$
^{127}Sb	51	76	$\frac{7}{2}^{+}$	$\frac{7}{2}^{+}$

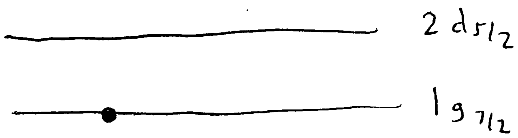
Pairing force depends on the number of neutrons.

Inversion for $N < 72$



$\Rightarrow \frac{5}{2}^{+}$

No inversion for $N \geq 72$



$\Rightarrow \frac{7}{2}^{+}$