22.02 Fall 2006 Problem Set #8 Solutions

1. Krane 3.9: $B = [Z\, m(^1H) + N m_n - m(^AX)]\, c^2$

$c^2 = 931.502$ MeV/amu, $m(^1H) = 1.007825$ amu, $m_n = 1.00866501$ amu

a) ^7Li: $Z=3, A=7, N=4$, $m(^7Li) = 7.016003$ amu $\Rightarrow$ $\boxed{B = 39.246 \text{ MeV}}$

$\boxed{B/A = 5.607 \text{ MeV}}$

b) ^{20}Ne: $Z=10$, $A=20$, $N=10$, $m(^{20}Ne) = 19.992436$ amu $\Rightarrow$ $\boxed{B = 160.651 \text{ MeV}}$

$\boxed{B/A = 8.0325 \text{ MeV}}$

c) ^{56}Fe: $Z=26, A=56, N=30$, $m(^{56}Fe) = 55.934939$ amu $\Rightarrow$ $\boxed{B = 492.26 \text{ MeV}}$

$\boxed{B/A = 8.79 \text{ MeV}}$

d) $^{235}U \rightarrow Z=92, A=235, N=143, m(^{235}U) = 235.043924$ amu $\Rightarrow$ $\boxed{B = 1783.89 \text{ MeV}}$

$\boxed{B/A = 7.591 \text{ MeV}}$

Krane 3.10: $B(A,Z) = 15.5A - 16.8A^{2/3} - 0.72\frac{Z(Z-1)}{A^{1/3}} - 23\frac{(A-2Z)^2}{A} + \delta$

$\delta(e\text{-}e, \text{odd } A, o\text{-}o) = (34A^{-3/4}, 0, -34A^{-3/4})$, $E_{coul} = -0.72\frac{Z(Z-1)}{A^{1/3}}$

a) ^{21}Ne: $Z=10$, $A=21$, odd A $\Rightarrow$ $\boxed{B(21,10) = 173.041 \text{ MeV}}$

$\boxed{E_{coul} = -23.487 \text{ MeV}}$ $\boxed{B/A = 8.24 \text{ MeV}}$

b) ^{57}Fe: $Z=26$, $A=57$, odd A $\Rightarrow$ $\boxed{B(57,26) = 502.98 \text{ MeV}}$

$\boxed{E_{coul} = -121.606 \text{ MeV}}$ $\boxed{B/A = 8.824 \text{ MeV}}$

c) ^{209}Bi: $Z=83$, $A=209$, o-o $\Rightarrow$ $\boxed{B(209,83) = 1618.0 \text{ MeV}}$

$\boxed{E_{coul} = -825.74 \text{ MeV}}$ $\boxed{B/A = 7.742 \text{ MeV}}$

d) ^{256}Fm: $Z=100$, $A=256$, e-e $\Rightarrow$ $\boxed{B(256,100) = 1886.86 \text{ MeV}}$

$\boxed{E_{coul} = -1122.59 \text{ MeV}}$ $\boxed{B/A = 7.37 \text{ MeV}}$

Krane 3.17: $I^\pi(^9Be) = \frac{3}{2}^-$, $I^\pi(^9B) = \frac{3}{2}^-$

Parity $= (-)\cdot(-) = +$

Spin $= (\frac{3}{2} - \frac{3}{2}) \rightarrow (\frac{3}{2} + \frac{3}{2}) = 0 \rightarrow 3 = 0, 1, 2, 3$

$\boxed{I^\pi(^{10}B) = 0^+, 1^+, 2^+, 3^+}$

2. $2G = \frac{2\pi Z_\alpha Z_D e^2}{\hbar v}$, $2G \approx \kappa r$

$\frac{\hbar^2 \kappa^2}{2\mu} \approx Q_\alpha \rightarrow \kappa \approx \frac{\sqrt{2\mu Q_\alpha}}{\hbar}$

$r \approx \frac{Z_\alpha Z_D e^2}{Q_\alpha} \Rightarrow 2G \approx \left(\frac{\sqrt{2\mu Q_\alpha}}{\hbar}\right)\left(\frac{Z_\alpha Z_D e^2}{Q_\alpha}\right) = \left(\frac{2\mu}{Q_\alpha}\right)^{1/2} Z_\alpha Z_D \frac{e^2}{\hbar}$

$v = \sqrt{\frac{2Q_\alpha}{\mu}} \Rightarrow \frac{1}{v} = \sqrt{\frac{\mu}{2Q_\alpha}} \Rightarrow \left(\frac{2\mu}{Q_\alpha}\right)^{1/2} = \frac{2}{v} \Rightarrow \boxed{2G \approx \frac{2 Z_\alpha Z_D e^2}{\hbar v}}$

Exact answer includes a multiplying factor equal to π:

$$\boxed{2G = \frac{2\pi Z_\alpha Z_D e^2}{\hbar v}}$$

$2G = \frac{2\pi Z_\alpha Z_D e^2}{\hbar v} = \sqrt{\frac{E_G}{Q_\alpha}} \Rightarrow E_G = \frac{4\pi^2 Z_\alpha^2 Z_D^2 e^4 Q_\alpha}{\hbar^2 v^2}$

$v = \sqrt{\frac{2Q_\alpha}{\mu}} \Rightarrow \boxed{E_G = \frac{2\pi^2 Z_\alpha^2 Z_D^2 e^4 \mu}{\hbar^2}}$

3. Krane 8.3: $A \rightarrow B + C \Rightarrow Q = [m(A) - m(B) - m(C)]c^2$

a) ${}^{242}Pu \rightarrow {}^{238}U + \alpha$, $m({}^{242}Pu) = 242.058737$ amu, $m({}^{238}U) = 238.050785$ amu

$m({}^4He) = 4.002603$ amu $\Rightarrow \boxed{Q = 4.9826 \text{ MeV}}$

b) ${}^{208}Po \rightarrow {}^{204}Pb + \alpha$, $m({}^{208}Po) = 207.981222$ amu, $m({}^{204}Pb) = 203.973020$ amu

$\boxed{Q = 5.21548 \text{ MeV}}$

c) ${}^{208}Po \rightarrow {}^{196}Pt + {}^{12}C$, $m({}^{196}Pt) = 195.964926$ amu, $m({}^{12}C) = 12$ amu

$\boxed{Q = 15.1798 \text{ MeV}}$

d) ${}^{210}Bi \rightarrow {}^{208}Pb + {}^2H$, $m({}^{210}Bi) = 209.984095$ amu, $m({}^{208}Pb) = 207.976627$ amu

$m({}^2H) = 2.014102$ amu $\Rightarrow \boxed{Q = -6.1796 \text{ MeV}}$

Krane 8.4: $Q \pm \sigma = [m({}^{242}Cm) - m({}^{238}Pu) - m_\alpha]c^2 \Rightarrow m({}^{242}Cm) = \frac{Q}{c^2} + m({}^{238}Pu) + m_\alpha \pm \frac{\sigma}{c^2}$

$Q = 6.1129$ MeV, $\sigma = 10^{-4}$ MeV, $m({}^{238}Pu) = 238.049555$ amu, $m_\alpha = 4.002603$ amu

$$\boxed{m({}^{242}Cm) = 242.059 \pm 1.1 \times 10^{-7} \text{ amu}}$$

Krone 8.8:

^{203}Tl : $Q_\alpha = 0.91$ MeV

$$R_o = r_o\left((199)^{1/3} + 4^{1/3}\right) = 8.911 \text{ F}$$

$$R_c = \frac{e^2}{\hbar c}\left(\frac{\hbar c}{Q_\alpha}\right) Z_\alpha Z_D = \frac{197}{137}\left(\frac{2\cdot 79}{0.91}\right) = 249.667 \text{ F}$$

$$V_{in} = c\sqrt{\frac{2(Q_\alpha + V_0)}{m_\alpha c^2}} = c\sqrt{\frac{2\,(40 \text{ MeV})}{m_\alpha c^2}} = 4\times 10^{22} \text{ F/s}$$

$$E_G = \left(\frac{2\pi e^2 Z_\alpha Z_D}{\hbar c}\right)^2\left(\frac{\mu c^2}{2}\right) = \left(\frac{2\pi\cdot 2\cdot 79}{137}\right)^2 \cdot \frac{938}{2}\cdot\frac{4\cdot 199}{203} = 96566 \text{ MeV}$$

$$2G = \sqrt{\frac{E_G}{Q_\alpha}}\left[\frac{2}{\pi}\left(\cos^{-1}\sqrt{\frac{R_o}{R_c}} - \sqrt{\frac{R_o}{R_c}}\left(1-\frac{R_o}{R_c}\right)^{1/2}\right)\right] = 325.755(0.7609) = 247.866$$

$$\lambda_\alpha = \frac{V_{in}}{R_o} e^{-2G} = 1.0127\times 10^{-86} \text{ s}^{-1}$$

$$\tau_{1/2} = \ln 2/\lambda_\alpha = 6.8444\times 10^{85} \text{ s} = 2.2\times 10^{78} \text{ yr}$$

$$\boxed{\tau_{1/2} \text{ is very long} \Rightarrow \text{stable}}$$