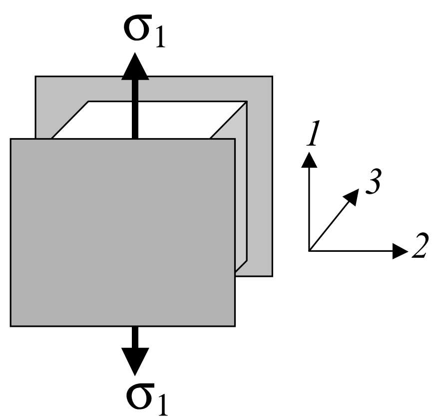
3.11 Mechanics of Materials F02

Exam #3 SOLUTIONS

(TOTAL 42 pts)

1. Q. A tensile stress is applied to a block of rubber in the 1-direction. Free expansion is allowed in the 2-direction and the sample is constrained in the 3-direction.



(a) Assuming Gaussian rubber elasticity theory and constant volume deformation calculate the stress versus extension ratio law in the 1-direction. (5pts)

A. (a) Starting with the constant volume constraint :

$$\lambda_1\lambda_2\lambda_3 = 1 \quad (1)$$

The 3-direction constraint leads to :

$$\lambda_3 = \frac{(L_f)_1}{(L_o)_1} = 1 \quad (2)$$

Substituting *(2)* into *(1)* :

$$\lambda_1\lambda_2 = 1 \quad (3)$$

Solve for λ_2 :

$$\lambda_2 = \frac{1}{\lambda_1}\ \text{(expansion)},\ \lambda_1 = \lambda_1\ \text{(expansion)},\ \lambda_3 = 1\ \text{(zero strain)} \quad (4)$$

The Helmholtz free energy of a Gaussian rubber netowrk is :

$$\Delta F = \frac{k_B T \nu_x}{2}\left[\lambda_1{}^2 + \lambda_2{}^2 + \lambda_3{}^2 - 3\right] \quad (5)$$

Substitute *(4)* into *(5)* :

$$\Delta F = \frac{k_B T \nu_x}{2}\left[\lambda_1{}^2 + \left[\frac{1}{\lambda_1}\right]^2 + 1^2 - 3\right] = \frac{k_B T \nu_x}{2}\left[\lambda_1{}^2 + \left[\frac{1}{\lambda_1}\right]^2 - 2\right]$$

Take the first derivative of *(6)* with respect to λ:

$$\sigma_1 = \frac{d\Delta F}{d\lambda_1} = \frac{d\left[\frac{k_B T \nu_x}{2}\left[\lambda_1{}^2 + \left[\frac{1}{\lambda_1}\right]^2 - 2\right]\right]}{d\lambda_1} = \left[\frac{k_B T \nu_x}{2}\right]\left[\frac{d\left[\lambda_1{}^2 + \left[\frac{1}{\lambda_1}\right]^2 - 2\right]}{d\lambda_1}\right]$$

$$\sigma_1 = \left[\frac{k_B T \nu_x}{2}\right]\left[2\lambda_1 - \frac{2}{\lambda_1{}^3}\right] = k_B T \nu_x \left[\lambda_1 - \frac{1}{\lambda_1{}^3}\right] \textit{(6)}\text{ANS}$$

(b) If σ_1= 3MPa produces ε_2 = 50% (NOTE: strain in 2-direction), what is ν_x? (5pts)
(there was a typo in this problem ε_2 =- 50%, you will get credit for either solution)
Solve for ν_x from eq.*(6)* :

$$\nu_x = \frac{\sigma_1}{k_B T\left[\lambda_1 - \frac{1}{\lambda_1{}^3}\right]} \textit{(7)}$$

From the definition of engineering strain :

$$\varepsilon_2 = \lambda_2 - 1$$
$$\lambda_2 = \varepsilon_2 + 1$$

For ε_2 =+ 50% :

$$\lambda_2 = 0.5 + 1 = 1.5$$
$$\lambda_1 = \frac{1}{\lambda_2} = \frac{1}{1.5} = 0.67$$

Substitute λ_1=1.5 into *(7)* :

$$\nu_x = \frac{3 \bullet 10^6 \frac{N}{m^2}}{4.1 \bullet 10^{-21}\,\text{Nm}\left[0.67 - \frac{1}{0.67^3}\right]} = 3.3 \bullet 10^{27}\,\frac{\text{strands}}{m^3}\ \text{ANS.}$$

For ε_2 =- 50% :

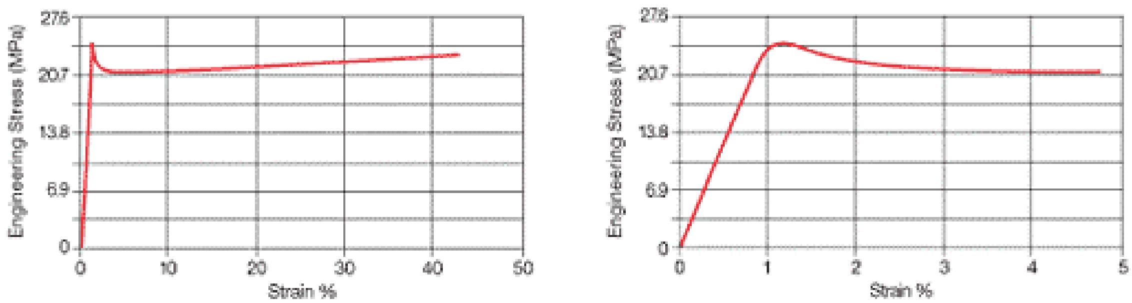
$$\lambda_2 = -0.5 + 1 = 0.5$$
$$\lambda_1 = \frac{1}{\lambda_2} = \frac{1}{0.5} = 2$$
$$\nu_x = \frac{3 \bullet 10^6 \frac{N}{m^2}}{4.1 \bullet 10^{-21}\,\text{Nm}\left[2 - \frac{1}{2^3}\right]} = 3.9 \bullet 10^{26}\,\frac{\text{strands}}{m^3}$$

(c) List 5 assumptions in classical rubber elasticity theory. (5pts)
1) chain end to end distance follows a Gaussian distribution
2) significant mobility of strands, low crosslink density, long chain molecules

3) entropy dominated, no internal energy or change in bond length
4) affine deformation, all network strands deform as if embedded in a continuum
5) molecular deformation of strands scales with macroscopic deformation
6) chain overlap allowed, no excluded volume

*NOTE: linear elasticity was not an assumption, it was a consequence of the derivation

2. Q. The following Figure is experimental data for the macroscopic uniaxial engineering stress versus strain curve. **(9pts total/ 1 pt each)**



(a) What kind of material is this? (extra credit if you can guess exactly what material it is)
(b) From the above plot, approximate the following numerically and briefly (one sentence) explain how you obtained each value :

(1) Young's modulus, E (GPa)
(2) 0.2% yield stress (MPa)
(3) % strain at failure
(4) failure strength (MPa)
(5) tensile ductility
(6) modulus of toughness (MPa)
(7) modulus of resilience (MPa)

A. (a) high impact polystyrene (HIPS) which is an amorphous glassy thermoplastic with embedded dispersed rubber particles

<u>If you didnt see the decimal point :</u>

(b) (1) Young's modulus, E (GPa) : ≈8.1 GPa
(2) 0.2% yield stress (MPa) ≈241 MPa
(3) % strain at failure=42%
(4) failure strength (MPa) ≈240 MPa
(5) tensile ductility (% strain) ≈42%
(6) modulus of toughness (MPa) ≈ 96.4 MPa
(7) modulus of resilience (MPa) ≈ 12 MPa

<u>If you did see the decimal point :</u>

(b) (1) Young's modulus, E (GPa) : ≈0.81 GPa
(2) 0.2% yield stress (MPa) ≈24.1 MPa
(3) % strain at failure=42%
(4) failure strength (MPa) ≈24.0 MPa
(5) tensile ductility (% strain) ≈42%
(6) modulus of toughness (MPa) ≈ 9.64 MPa

(7) modulus of resilience (MPa) ≈ 0.12 MPa

3 Q. (a) A sheet of glass measuring 2 mm by 200 mm by 2 mm contains an edges slit parallel to the 200 mm side. The sheet is restrained at one end and loaded in tension with a mass of 500 kg. Assuming E = 60 GPa and surface energy is 0.5 J/m^2, what is the maximum allowable length of slit before fracture occurs? (5pts)

A. Linear elastic fracture mechanics tells us the fracture stress is :

$$\sigma_f = \sqrt{\frac{EG_c}{\pi a}},\ G_c = 2\gamma \quad (8)$$

Solve *(8)* for a :

$$a = \frac{2E\gamma}{\pi\sigma_f^2}$$

The externally applied stress is calculated from the applied mass to a load using the gravitational constant g and the area of the plate :

$$\sigma_f = \frac{500\ \text{Kg} \cdot 9.81\ \frac{\text{m}}{\text{s}^2}}{0.2\ \text{m} \cdot 0.002\ \text{m}}$$

$$1\text{N} = \frac{\text{Kg} \cdot \text{m}}{\text{s}^2}$$

$$\sigma_f = 12.26\ \text{MPa}$$

$$a = \frac{2\left(60 \bullet 10^9\ \frac{\text{N}}{\text{m}^2}\right)\left(0.5\frac{\text{N}}{\text{m}}\right)}{3.14\left(12.26 \bullet 10^6\ \frac{\text{N}}{\text{m}^2}\right)^2} = 1.27 \bullet 10^{-4}\ \text{m} = 0.127\text{mm}$$

(b) A thin sheet of maraging steel has a tensile strength of 1950 MPa. Calculate the percentage reduction in strength due to the presence of a edge crack in the sheet, which is 2 mm long and orientated perpendicular to the stressed direction. For this steel, E can be taken as 200 GPa, the energy of fracture surface as 2 J/m^2, and the work of plastic deformation of each crack tip is 2•10^4 J/m^2. (5pts)

$$\sigma_f = \sqrt{\frac{EG_c}{\pi a}},\ G_c = 2\gamma + W_p$$

$$\sigma_f = \sqrt{\frac{\left(200 \bullet 10^9\ \frac{\text{N}}{\text{m}^2}\right)\left(2 \bullet 2\frac{\text{N}}{\text{m}} + 2 \bullet 10^4\ \frac{\text{N}}{\text{m}}\right)}{3.14(0.002\ \text{m})}} = 798\ \text{MPa}$$

$$\text{reduction in strength} = \left[\frac{1950 - 798}{1950}\right] \bullet 100 = 59\%$$

4 Define the following terms in a few sentences (you may use formulas and schematics, but TEXT must explain it, zero credit will be given for just a formula or schematic with no explanation): (1pt each)

a. Instrumented impact tester

A. a machine which uses a stiff "impactor" to conduct high strain rate force versus time or displacement tests on a sample. Using a load cell and strain gauge the force and displacement are continuously monitored during the impact.

b. Theoretical cleavage stress

A. the maximum stress needed to separate 2 planes of covalently bonded atoms

c. Maxwell model

A. viscoelastic model of a Newtonian viscous dashpot and linear elastic Hookean spring in series

d. Creep

A. when a material exhibits a time-dependent strain in response to a constant externally applied stress

e. Edge Dislocation

A. an extra half plane of atoms in a crystalline material

f. Strain softening

A. in uniaxial tension, when the the stress drops with increasing strain due to localized plastic deformation or necking

g. Craze

A. a localized region of plastic deformation in a deformed glassy polymer which exhibits a fibrillar structure and hence, is load bearing

h. Ductility

A. the capability of a materials to undergo plastic deformation