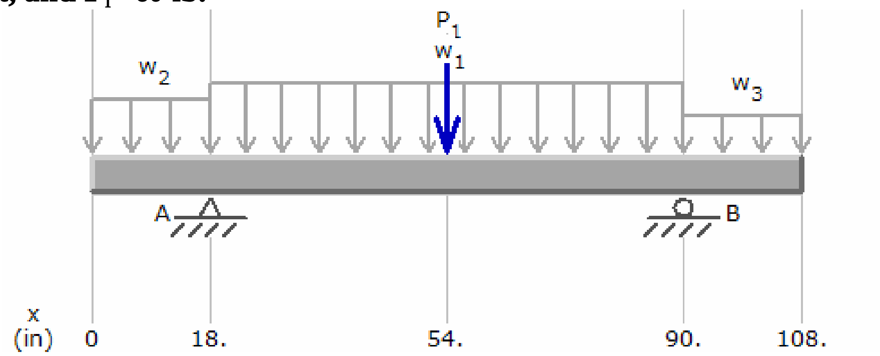
<u>3.11 Mechanics of Materials F03</u>

Exam #1 SOLUTIONS: Friday 10/23/03

THESE SOLUTIONS ARE WRITTEN IN EXCESSIVE DETAIL FOR CLARITY : YOU ARE NOT REQUIRED TO HAVE INCLUDED ALL THE DETAILS SHOWN HERE TO GET CREDIT.

TOTAL POINTS=39

1. In beam shown below, the length units are inches and w_1= 10 lb/ft, w_2=7 lb/ft, w_3=5 lb/ft, and P_1=60 lb.

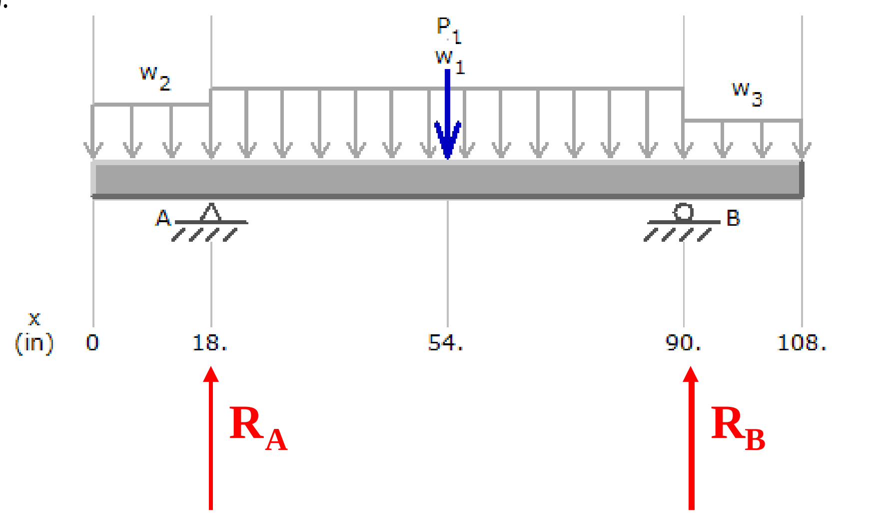


(a) Draw the shear force diagram, V(x), and label the location (along x-axis) and magnitude of the maximum shear force.

(b) Draw the bending moment diagram, M(x), and label the location (along x-axis) and magnitude of the maximum bending moment.

PLEASE LABEL NUMERICAL VALUES ON YOUR PLOTS AT x=18, 54, 90, 108 & USE APPROPRIATE SIGN CONVENTIONS

ANS. (a) Consider a free-body diagram of the entire beam and the equations of static equilibrium to determine the support reactions at A and B. Take CCW as (-) and CW as (+) ("=inches).



$$\sum M_A = 0 = -\left(\frac{7\text{ lb}}{12"}\bullet 18"\bullet 9"\right) + (60\text{ lb}\bullet 36") + \left(\frac{10\text{ lb}}{12"}\bullet 72"\bullet 36"\right) + \left(\frac{5\text{ lb}}{12"}\bullet 18"\bullet 81"\right) - R_B \bullet 72"$$

$$R_B = \frac{-94.5\text{ lb"}+2160\text{ lb"}+2160\text{ lb"}+607.5\text{ lb"}}{72"} = \mathbf{+67.125\ lb}$$

(assumed CCW direction is correct)

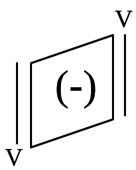
$$\sum F_Y = -10.5\text{ lb}+R_A\text{-60 lb-60 lb-7.5 lb+67.125 lb=0}$$

$$R_A = 10.5+60+60+7.5-67.125 = \mathbf{+70.875\ lb}$$

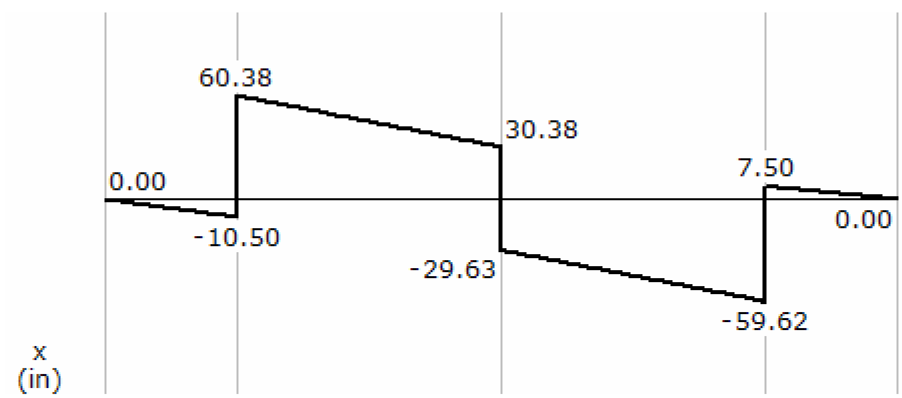
(assumed upward direction is correct)

(2 pts)

Starting from left hand side of beam, the distributed load, w_2, produces a negative shear



force from x=0 to 18"with a slope= $w_2 = -\left(\frac{7\text{ lb}}{12"}\right) = -0.5833\frac{\text{lb}}{\text{in}}$**,** yielding V=-10.50 lbs at x=18". At x=18, a discontinuous jump upwards in V(x) occurs due to R_A=70.875 lbs. Using the same rules, one can continue moving from right to left to obtain the rest of the V(x) diagram.



(4 pts)

(b) By definition : M(x)=$\int$V(x)dx . For example:

$$M(0<x<18) = \int -0.5833x\,dx = \frac{-0.5833x^2}{2} + C_1 = -0.2917x^2 + C_1$$

From the initial FBD, M(x=0)= 0=C_1

M(x)=$-0.2917x^2$

$$M(x=18) = -0.2917(18^2) = -94.51\text{ lbin}$$

(4 pts)

$\text{V(x=18)=60.38=}\left(\text{-0.833}\bullet\text{18}\right)\text{+b} \rightarrow \text{b=60.38} + \left(0.833 \bullet 18\right) = 75.37$

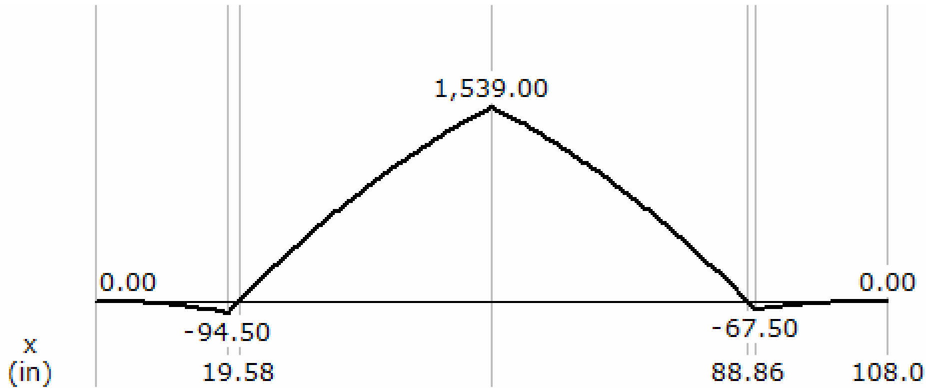
V(18<x<54)=-0.833x+75.37

$\text{M(18<x<54)} = \int\left(\text{-0.833x+75.37}\right)\ \text{dx} = \frac{-0.833x^2}{2} + 75.37x + C_1 = -0.4165x^2 + 75.37x + C_1$

$\text{M(x=18)= -94.51=}\left(-0.4165 \bullet 18^2\right) + \left(75.37 \bullet 18\right) + C_1 \rightarrow C_1 = \text{-94.51+134.95-1356=-1315.56}$

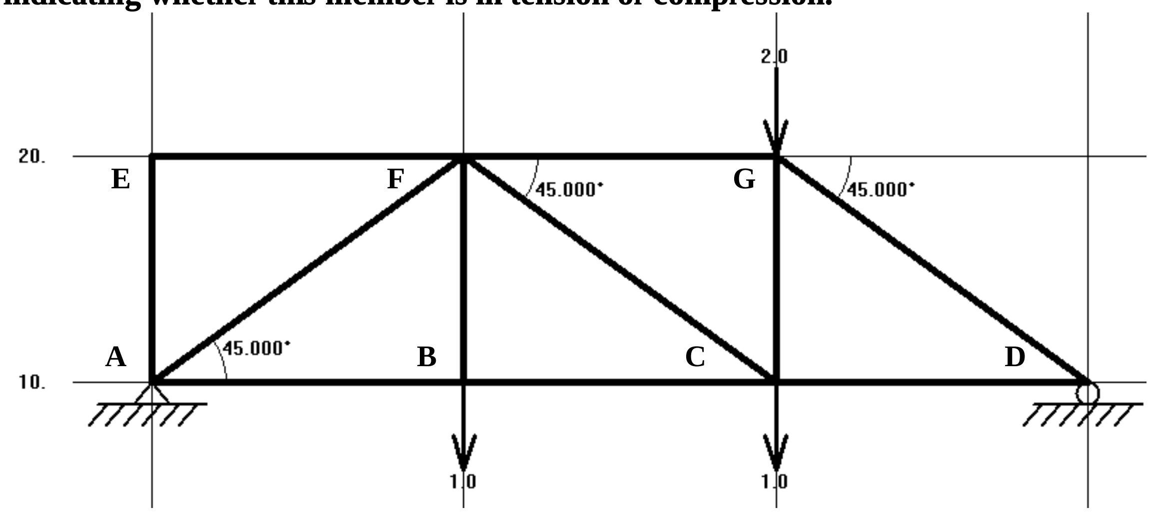
$\text{M(18<x<54)=} -0.4165x^2 + 75.37x\text{-1315.56}$

$M(x = 54) = \left(-0.4165 \bullet 54^2\right) + \left(75.37 \bullet 54\right)\text{-1315.56} = -1214.5 + 4069 - 1315 = 1540$



Continue along the rest of the beam similarly from left to right.

2. Calculate the magnitude of the forces member FC of the truss shown below indicating whether this member is in tension or compression.



a) Consider a free-body diagram of the entire truss and the equations of static equilibrium to determine the support reactions at A and D. Take CCW as (-) and CW as (+).

Find x-axis member length : $\tan(45) = \frac{10}{x} \rightarrow x = \frac{10}{\tan(45)} = 10$

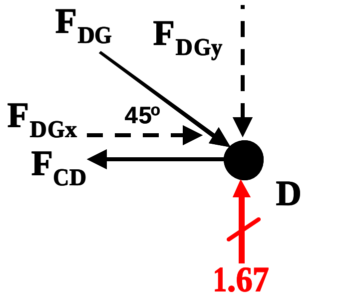
$\sum M_A = 0 = (1 \bullet 10) + (1 \bullet 20) + (2 \bullet 20) - (R_D \bullet 30)$

$R_D = \frac{70}{30} = 2.33$

$\sum F_Y = -2 - 1 - 1 + 2.33 + R_A \rightarrow R_A = 1.67$

(2 pts)

Use method of joints and start at joint D:



FBD of JOINT D (labeled joint G in solution below)

$\cos(45) = \frac{DG_X}{DG} \rightarrow DG_X = DG\cos(45) = 0.707DG$

$\sin(45) = \frac{DG_Y}{DG} \rightarrow DG_Y = DG\sin(45) = 0.707DG$

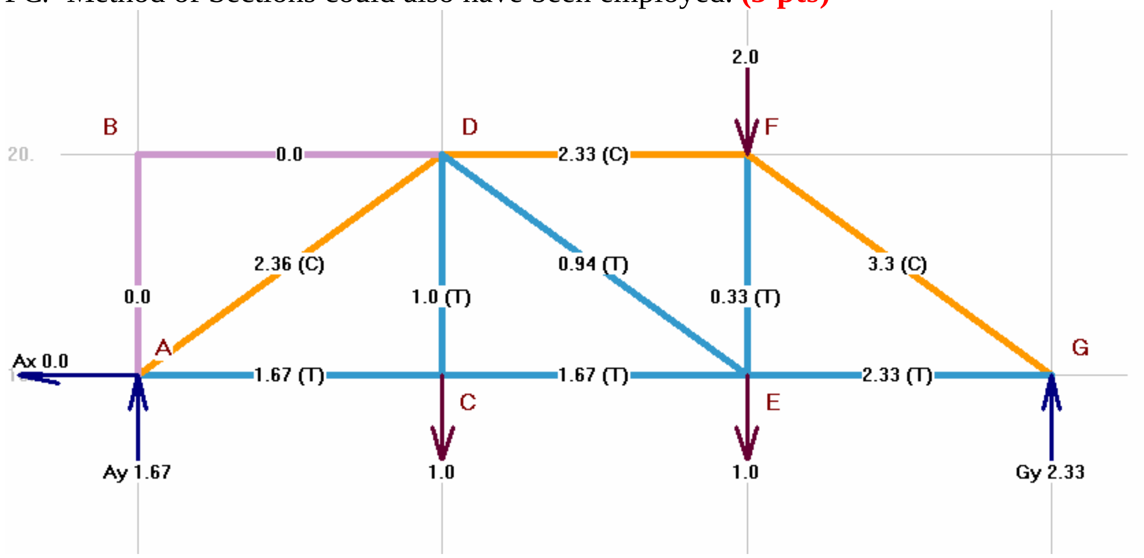
$\sum F_Y = 0 = -DG_Y + R_D = -0.707DG + 2.33$

$DG = \frac{2.33}{0.707} = 3.29\ C$

$\sum F_X = 0 = -DG_X + CD = -0.707DG + CD = (-0.707 \bullet 3.29) + CD$

$CD = 2.33T$

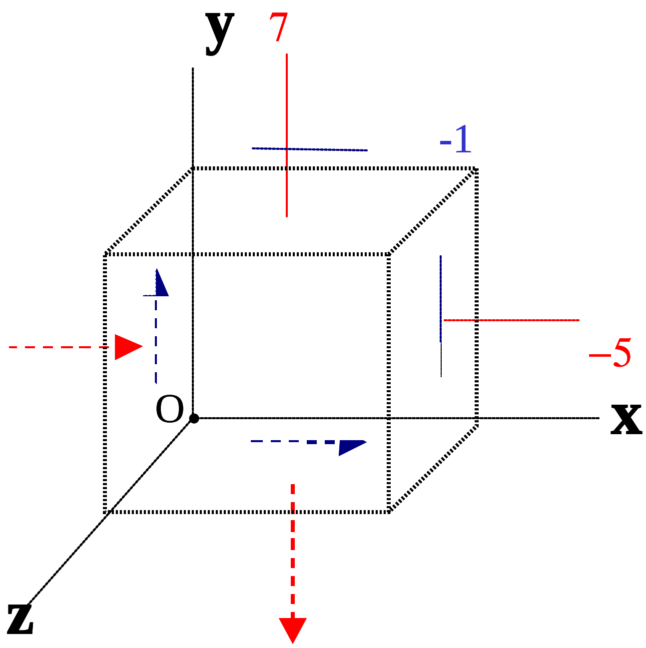
Then continue moving from neighboring joint to joint using same method until you reach FC. Method of Sections could also have been employed. **(5 pts)**



3) The stress pseudovector for a linear elastic isotropic material is given below (E=25 kPa and n**=0.35) :**

$$\begin{Bmatrix} s_x \\ s_y \\ s_z \\ t_{yz} \\ t_{xz} \\ t_{xy} \end{Bmatrix} = \begin{Bmatrix} -5 \\ 7 \\ 0 \\ 0 \\ 0 \\ -1 \end{Bmatrix}$$

a) Draw a 3-dimensional cubic stress element showing the magnitude and direction of each stress and the corresponding reaction stresses. (2 pts)



b) What is the magnitude and direction of the largest normal strain?

Hooke's Law for a linear elastic, isotropic material under multiaxial loading in matrix form is :

$$\begin{Bmatrix} e_x \\ e_y \\ e_z \\ g_{yz} \\ g_{xz} \\ g_{xy} \end{Bmatrix} = 1/E \begin{bmatrix} 1 & -n & -n & 0 & 0 & 0 \\ . & 1 & -n & 0 & 0 & 0 \\ . & . & 1 & 0 & 0 & 0 \\ . & . & . & 2(1+n) & 0 & 0 \\ . & . & . & . & 2(1+n) & 0 \\ . & . & . & . & . & 2(1+n) \end{bmatrix} \begin{Bmatrix} s_x \\ s_y \\ s_z \\ t_{yz} \\ t_{xz} \\ t_{xy} \end{Bmatrix}$$

the 3 normal equations corresponding to the martix notation above are :

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\sigma_y - \nu\sigma_z\right]$$

$$\varepsilon_y = \frac{1}{E}\left[-\nu\sigma_x + \sigma_y - \nu\sigma_z\right]$$

$$\varepsilon_z = \frac{1}{E}\left[-\nu\sigma_x - \nu\sigma_y + \sigma_z\right]$$

Substituting in the given numerical values yields :

$$\varepsilon_x = \frac{1}{25}\left[-5 - (0.35 \bullet 7)\right] = 0.04\left[-5 - 2.45\right] = -0.298$$

$$\varepsilon_y = \frac{1}{25}\left[-(0.35 \bullet -5) + 7\right] = 0.04\left[1.75 + 7\right] = +0.35$$

$$\varepsilon_z = \frac{1}{25}\left[-(0.35 \bullet -5) - (0.35 \bullet 7)\right] = 0.04\left[1.75 - 2.45\right] = -0.028$$

The largest normal strain is ε_x=+0.35 (tensile). ***Ans.*** **(4 pts)**

c) Calculate the principal stresses and draw them on a corresponding properly oriented 2D stress element.

Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{-5+7}{2} \pm \sqrt{\left[\frac{-5-7}{2}\right]^2 + 1} = 1 \pm \sqrt{36+1} = 1 \pm 6.1$$

σ_1 = maximum normal stress = 7.1

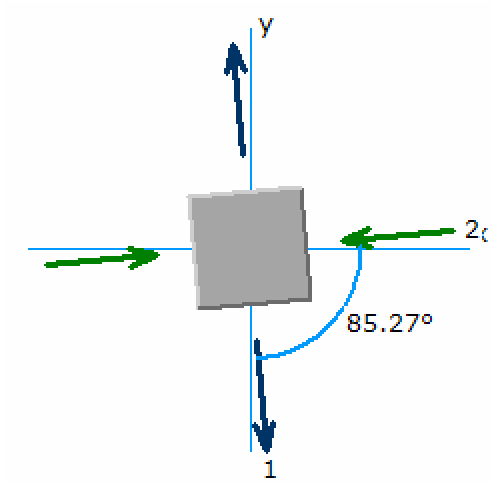
σ_2 = minimum normal stress = −5.1

(2 pts)

Principal Angles / Planes :

$$\tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{-2(1)}{-5-7} = 0.166$$ **(2 pts)**

$\theta_{p2} = 4.73,\ \theta_{p1} = 94.74$



4. Define the following words in one to two sentences using <u>no formulas</u>: (2 pts each, 12 pts total)

a) principal planes : the planes on which the maximum and minimum normal stresses exist in a multiaxial stress state

b) shear modulus : A constant which is equal to the ratio of shear stress to shear strain and hence, follows Hooke's law for linear elasticity

c) Hookean elasticity : linear elastic relation between stress and strain

d) continuum : a region of space filled with continuous matter (no gaps or spaces) with continuous properties that vary smoothly within the region.

e) internal moment : a measure of the tendency to produce rotation within the beam to oppose externally applied forces

f) equations of static equilibrium : describe force and moment requirements placed on a body to prevent the object from translating