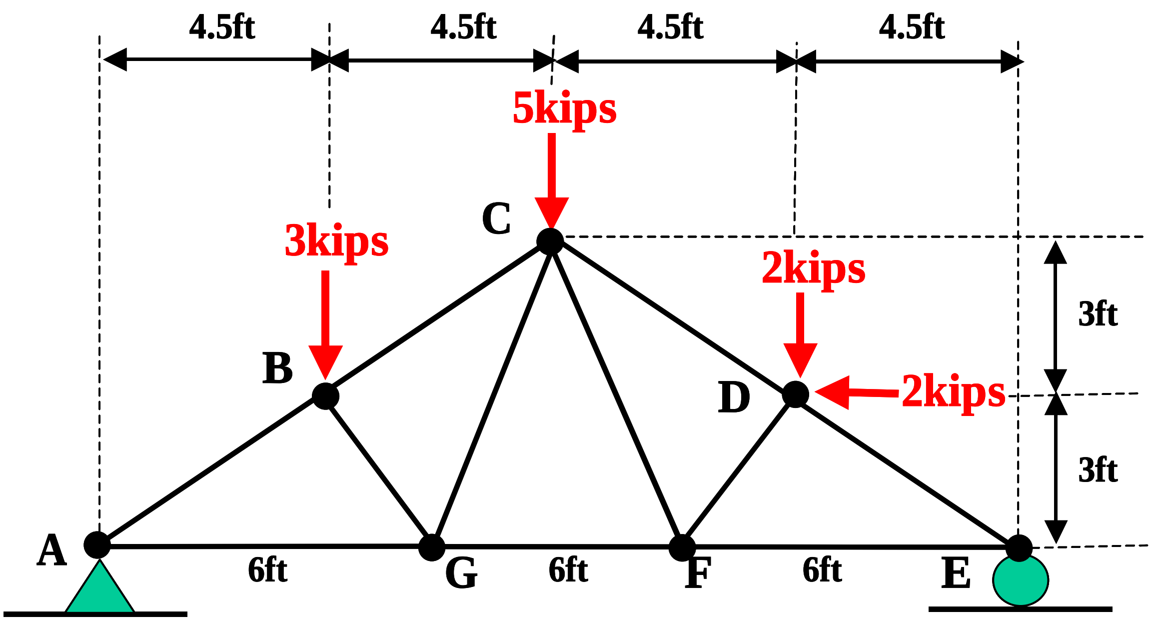
3.11 Mechanics of Materials F01
Exam #1 Solutions : Friday 10/05/01

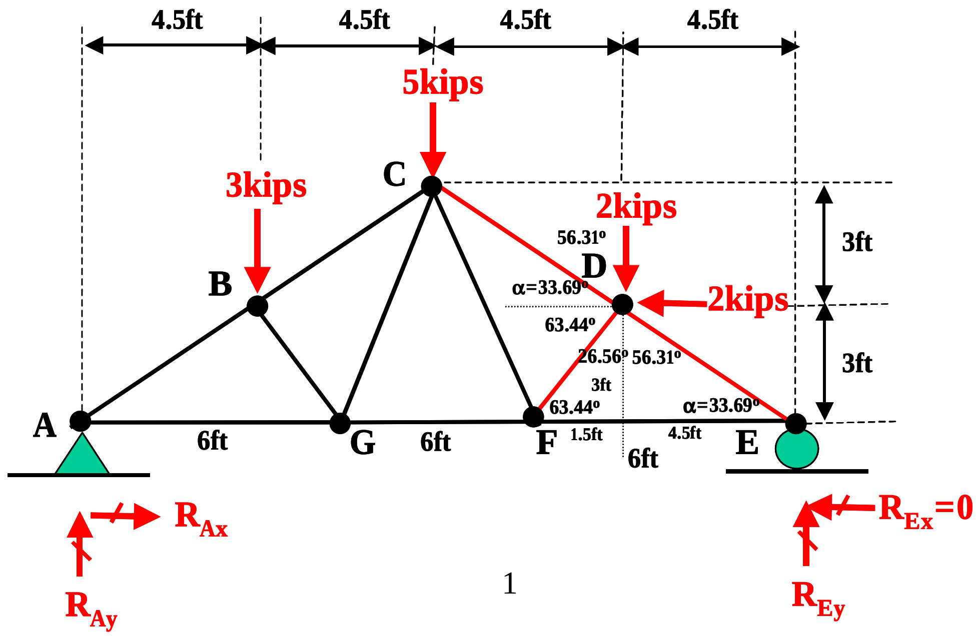
*(*show all of your work / calculations to get as much credit as possible)*

1. Calculate the magnitude of the forces in members CD, DE, and DF, of the truss shown below indicating whether each member is in tension or compression (10 pts total)



1.A. <u>METHOD OF JOINTS :</u>

1. Draw a free-body diagram of the entire truss *(*shown below)*



2. Determine support reactions using the equations of static equilibrium
$\Sigma F_x=0$, $\Sigma F_y=0$, $\Sigma M_{xy}=0$ (CCW+, CW-) :

By definition : $\mathbf{R_{Ex}=0}$ **kips**

$\Sigma F_x=0= R_{Ax}+R_{Ex}-2$
$\mathbf{R_{Ax}=2}$ **kips**

$\Sigma M_A=0= -3(4.5)-5(4.5)(2)-2(4.5)(3)+2(3)+R_{Ey}(4.5)(4)$
$\Sigma M_A=0= -13.5-45-27+6+ R_{Ey}18$
$\Sigma M_A=0= -79.5+ R_{Ey}18$
$R_{Ey}=79.5/18$
$\mathbf{R_{Ey}=4.417}$ **kips**

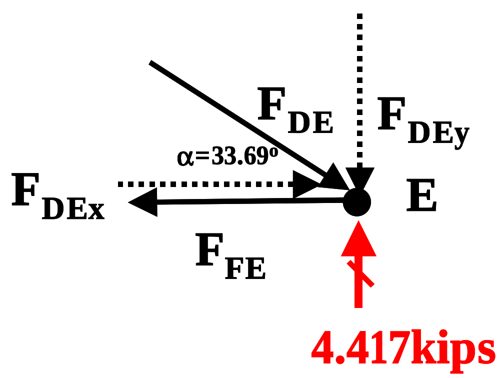
$\Sigma F_y=0=-3-5-2+R_{Ey}+R_{Ay}= -10+4.417+R_{Ay}$
$\mathbf{R_{Ay}=5.583}$ **kips**

(3 pts for determining reaction forces)

3. Identify a joint where you know the maximum amount of forces (e.g. a support with two members).

Start at Joint E.

4. Draw a free-body diagram of the joint and determine whether forces are compressive or tensile. We know that there has to be a force in the downwards y-direction counteracting the upwards reaction force at the roller. Hence, F_{DE} must be in compression, which also indictaes that F_{FE} must be in tension to counteract F_{DEx}.



5. Write and solve equations of static equilibrium* for diagram drawn in step 4.
From geometry : $F_{DEx}=F_{DE}\cos\alpha=0.832F_{DE}$
$F_{DEy}=F_{DE}\sin\alpha=0.5546F_{DE}$
where : $\alpha=33.69$

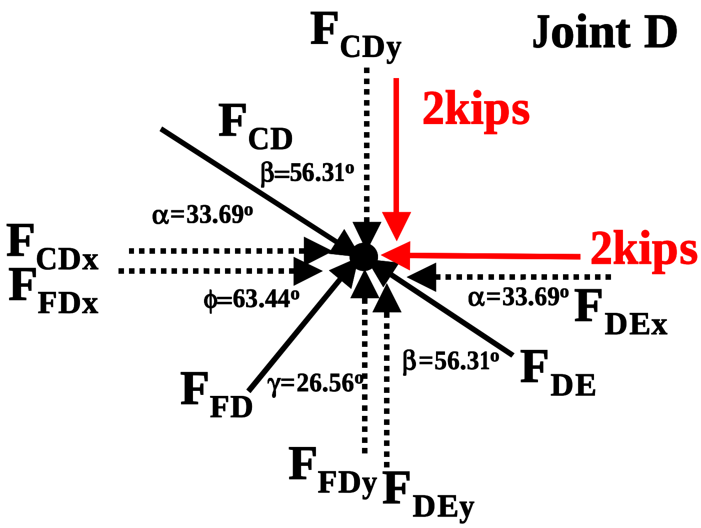
$\Sigma F_y=0=4.417-F_{DEy}=4.417-0.5546F_{DE}$

F_{DE}=7.963 kips (COMPRESSION) *Ans.*

(3 pts for determining F_{DE})

6. Move to an adjacent joint and repeat steps 4-5 until entire truss is solved

Move to Joint D and choose directions arbitrarily.



From geometry : $F_{DEx}=F_{DE}\cos\alpha=0.832F_{DE}=6.625$ kips
$F_{DEy}=F_{DE}\sin\alpha=0.5546F_{DE}=4.4162$ kips
$F_{FDx}=F_{FD}\cos\phi=0.4472F_{FD}$
$F_{FDy}=F_{FD}\sin\phi= 0.8943F_{FD}$
$F_{CDx}=F_{CD}\cos\alpha= 0.832F_{CD}$
$F_{CDy}=F_{CD}\sin\alpha=0.5546F_{CD}$

$\Sigma F_x=0=-2-F_{DEx}+F_{CDx}+F_{FDx}=-2-6.625+0.832F_{CD}+0.4472F_{FD}$

$\Sigma F_x=0=-8.625+0.832\mathbf{F_{CD}}+0.4472\mathbf{F_{FD}}$ *(1)*

$\Sigma F_y=0=-2-F_{CDy}+F_{FDy}+F_{DEy}=-2-0.5546F_{CD}+0.8943F_{FD}+4.4162$

$\Sigma F_y=0=2.4162-0.5546\mathbf{F_{CD}}+0.8943\mathbf{F_{FD}}$ *(2)*

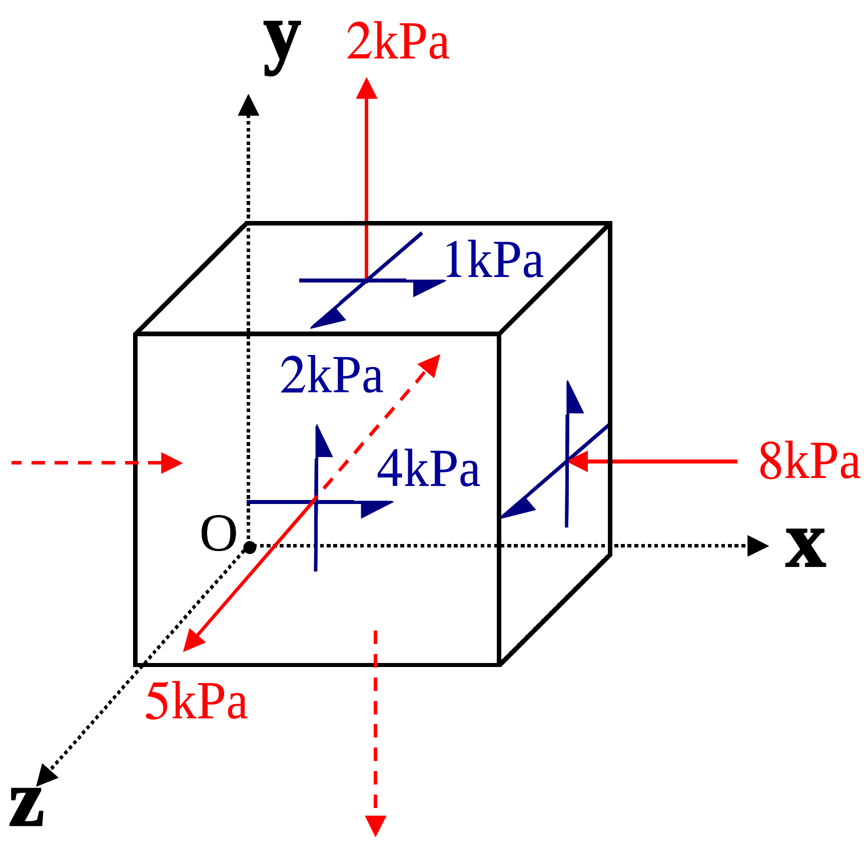
Hence, we have two equations and two unknowns :

Solve eq. *(1)* for $F_{CD}=8.625-0.4472F_{FD}/0.832=10.366-0.5375F_{FD}$
Plug into eq. *(1)* into eq. *(2)* and solve for F_{CD}:
$0=2.4162-0.5546[10.366-0.5375F_{FD}]+0.8943F_{FD}$
$0=2.4162-5.749+0.298F_{FD}+0.8943F_{FD}$
$0=-3.3328+1.1923F_{FD}$
F_{FD}=2.79 kips (COMPRESSION) *Ans.*

Plug into eq. *(1) :*
$0=-8.625+0.832F_{CD}+0.4472(2.79)$
F_{CD}=8.866 kips (COMPRESSION) *Ans.*

(4 pts for determining F_{FD} and F_{CD})

2. The stress state for a linear elastic isotropic material is shown on the following page (E=45kPa and ν=0.35).
a) What is the magnitude and direction of the largest normal strain?
b) What is the magnitude of the largest shear strain? What plane does it take place in?
(10 pts)



2.A. The stress pseudovector is : $\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} -8 \\ 2 \\ 5 \\ -2 \\ -4 \\ 1 \end{Bmatrix}$

Hooke's Law for a linear elastic, isotropic material under multiaxial loading in matrix form is :

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = 1/E \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ . & 1 & -\nu & 0 & 0 & 0 \\ . & . & 1 & 0 & 0 & 0 \\ . & . & . & 2(1+\nu) & 0 & 0 \\ . & . & . & . & 2(1+\nu) & 0 \\ . & . & . & . & . & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}$$

which corresponds to the following set of six equations :

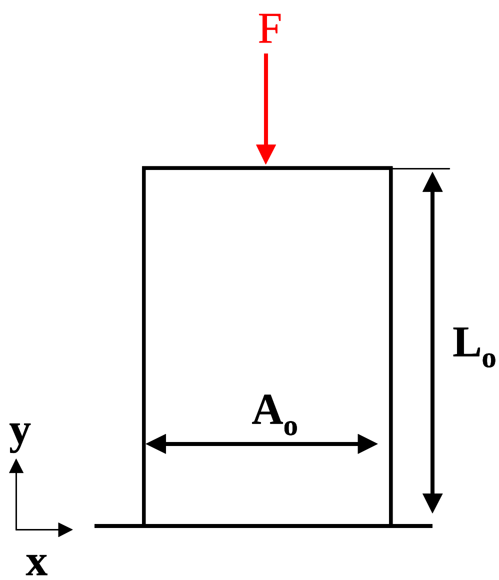
$$\varepsilon_x = \sigma_x / E - \nu\sigma_y / E - \nu\sigma_z / E$$
$$\varepsilon_y = -\nu\sigma_x / E + \sigma_y / E - \nu\sigma_z / E$$
$$\varepsilon_z = -\nu\sigma_x / E - \nu\sigma_y / E + \sigma_z / E$$
$$\gamma_{yz} = 2(1+\nu)\tau_{yz} / E$$
$$\gamma_{xz} = 2(1+\nu)\tau_{xz} / E$$
$$\gamma_{xy} = 2(1+\nu)\tau_{xy} / E$$

Substituting in the given numerical values yields :

$$\varepsilon_x = -8/45 - 0.35*2/45 - 0.35*5/45 = -0.177 - 0.0155 - 0.038 = -0.2305$$
$$\varepsilon_y = -0.35*-8/45 + 2/45 - 0.35*5/45 = -0.06 + 0.044 - 0.0388 = 0.0548$$
$$\varepsilon_z = -0.35*-8/45 - 0.35*2/45 + 5/45 = 0.0622 - 0.015 + 0.111 = 0.1583$$
$$\gamma_{yz} = 2(1+0.35)*-2/45 = -0.12$$
$$\gamma_{xz} = 2(1+0.35)*-4/45 = -0.24$$
$$\gamma_{xy} = 2(1+0.35)*1/45 = 0.06$$

a) The largest normal strain is ε_x=-0.23 (compressive). ***Ans.***
b) The largest shear strain is γ_{xz}=-0.24 ***Ans.***

3. Calculate the force needed to compress the steel bar shown below to $L_o/2$ if the bar is simultaneously heated up by 50°C (E=200GPa, $\alpha_L=12\bullet10^{-6}$ /°C, L_o=0.5m, $A_o=0.05m^2$). **(10 pts)**



3A. $\sigma=F/A_o=E\varepsilon_\sigma \Rightarrow \varepsilon_\sigma=F/EA_o$

$\varepsilon_{TOTAL}=\varepsilon_{THERMAL}+\varepsilon_\sigma=+\alpha_L\Delta T-F/EA_o$

$\varepsilon_{TOTAL}=\Delta L/L_o=-0.5L_o/L_o=-0.5$

$-0.5=+\alpha_L\Delta T-F/EA_o$

solve for : $F=[0.5+\alpha_L\Delta T]EA_o$

substitute in numerical values : $F=[0.5+(12\bullet10^{-6}/^{o}C*50\,^{o}C)]200GPa*0.05m^2$

F=5.006GN ***Ans.***

4. Define the following words in one to two sentences :
(2pts each=10 pts)

a) statically indeterminate : a structure which has more variables than number of equations of static equilibrium; hence, additional equations are needed to solve the entire structure (i.e. geometric compatibility, constitutive eqs., etc.)

b) membrane stresses : tensile stresses in the wall of a thin walled spherical pressure vessel tangential to the curved surface of the vessel, $\sigma_\phi=\sigma_\theta$

c) thermal shock : failure or fracture of a materials or structure due to stresses induced by thermal expansion

d) hydrostatic stress : a stress state with three normal, equal stresses and no shear stresses, $\sigma=\sigma_x=\sigma_y=\sigma_z,\ \tau_{xy}=\tau_{xz}=\tau_{yz}=0$

e) moment lever arm : the perpendicular distance from the point of interest to the line of action of the applied force