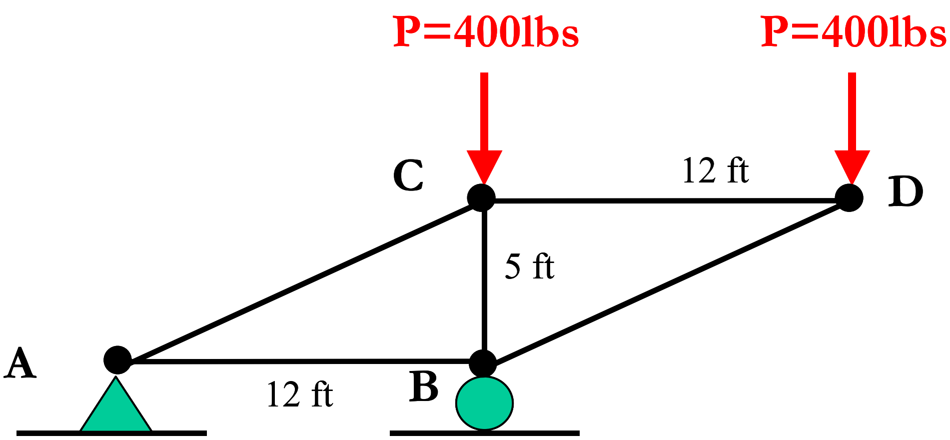
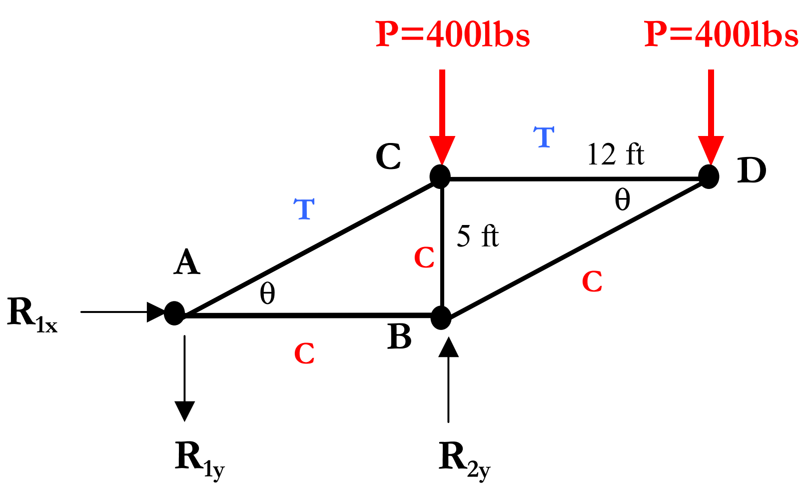
3.11 Mechanics of Materials F02
Exam #2 Solutions : Friday 10/04/01

1. Calculate the magnitude of the forces in all members of the truss shown below indicating whether each member is in tension or compression. (10 pts total)



ANSWER 1.



where : AC=13, $\tan\theta = 5/12=0.4166$, $\cos\theta = 12/13=0.923$, $\sin\theta= 5/13=0.3846$

2) Support reactions

$\Sigma F_x = 0 \quad => \quad R_{1x} = 0$
$\Sigma F_y = 0 \quad => \quad R_{2y} - R_{1y} - 2P = 0$

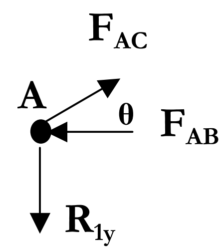
$\Sigma M_A = 0 \Rightarrow (R_{2y})(12)-(P)(12)-(P)(24) = 0$

$\Rightarrow R_{1x} = 0 \quad R_{1y} = P \quad R_{2y} = 3P$

3) Joint A

$\Sigma F_x = 0 \Rightarrow F_{AC} \cos\theta - F_{AB} = 0$
$\Sigma F_y = 0 \Rightarrow F_{AC} \sin\theta - R_{1y} = 0$

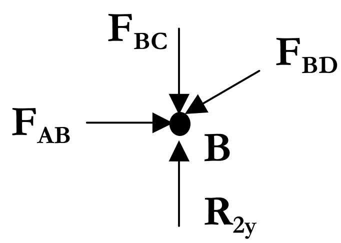
$\Rightarrow F_{AC} = \dfrac{1}{\sin\theta} P \qquad F_{AB} = \dfrac{1}{\tan\theta} P$



4) Joint B

$\Sigma F_x = 0 \Rightarrow F_{AB} - F_{BD} \cos\theta = 0$
$\Sigma F_y = 0 \Rightarrow R_{2y} - F_{BC} - F_{BD} \sin\theta = 0$

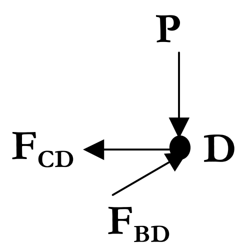
$\Rightarrow F_{BC} = 2P \quad F_{BD} = \dfrac{1}{\sin\theta} P$



5) Joint D

$\Sigma F_x = 0 \Rightarrow F_{BD} \cos\theta - F_{CD} = 0$

$\Rightarrow F_{CD} = \dfrac{1}{\tan\theta} P$



6) Therefore, we obtain the following result:

$F_{AC} = F_{BD} = \dfrac{1}{\sin\theta} P = 1040 \text{ lb}$

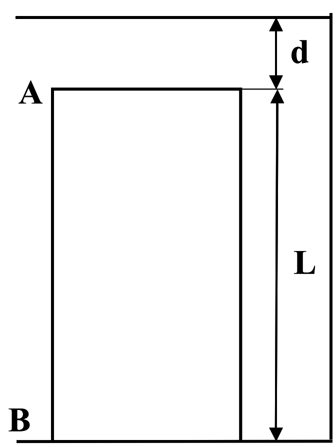
$F_{AB} = F_{CD} = \dfrac{1}{\tan\theta} P = 960 \text{ lb}$

$F_{BC} = 2P = 800 \text{ lb}$

The directions of these member forces are indicated in the free body diagram of the entire truss.

Member	Force (in terms of P)	Force (lbs)
AB	$\dfrac{-P}{\tan\theta}$	-960
AC	$\dfrac{P}{\sin\theta}$	1040
BC	-2P	-800
BD	$\dfrac{-P}{\sin\theta}$	-1040
CD	$\dfrac{P}{\tan\theta}$	960
R_{1y}	-P	-400
R_{2y}	3P	1200

2. A solid, cylindrical linear elastic bar AB having length, L, modulus, E, cross-sectional area, A_o, is fixed at end B (as shown in the Figure below). At the other end a small gap of dimensions, d, exists between the end of the bar (A) and a rigid surface. Calculate the axial compressive stress, σ_c, in the bar if the temperature increased by ΔT. **(10 pts total)**



$E=16 \cdot 10^6$ psi
$\alpha=9.8 \cdot 10^{-6}$ 1/°F
L=40 in
d=0.008 in
$\Delta T=90$°F

ANSWER 2.

free thermal strain of the bar : $\varepsilon_{\mathrm{T}} = \alpha\Delta T = \delta \mathrm{L}$

free thermal deformation of the bar : $\delta_{\mathrm{T}} = \alpha\Delta T\mathrm{L}$

strain that is prevented by the rigid support :

$$\varepsilon_{\mathrm{c}} = \frac{\delta_{\mathrm{c}}}{\mathrm{L}} = \frac{\delta_{\mathrm{T}} - \mathrm{d}}{\mathrm{L}} = \frac{\alpha\Delta T\mathrm{L} - \mathrm{d}}{\mathrm{L}}$$

stress in the bar : $\sigma_{\mathrm{c}} = \mathrm{E}\varepsilon_{\mathrm{c}} = \mathrm{E}\dfrac{\delta_{\mathrm{c}}}{\mathrm{L}} = \dfrac{\mathrm{E}}{\mathrm{L}}\left[\alpha\Delta T\mathrm{L} - \mathrm{d}\right]$

substitute in values : $\sigma_{\mathrm{c}} = \dfrac{16 \bullet 10^{6}\,\mathrm{psi}}{40\ \mathrm{in}}\left[\left(9.8 \bullet 10^{-6}\ {}^{\mathrm{o}}\mathrm{F}^{-1}\right)\left(90^{\mathrm{o}}\,\mathrm{F}\right)\left(40\ in\right) - 0.008\ \mathrm{in}\right]$

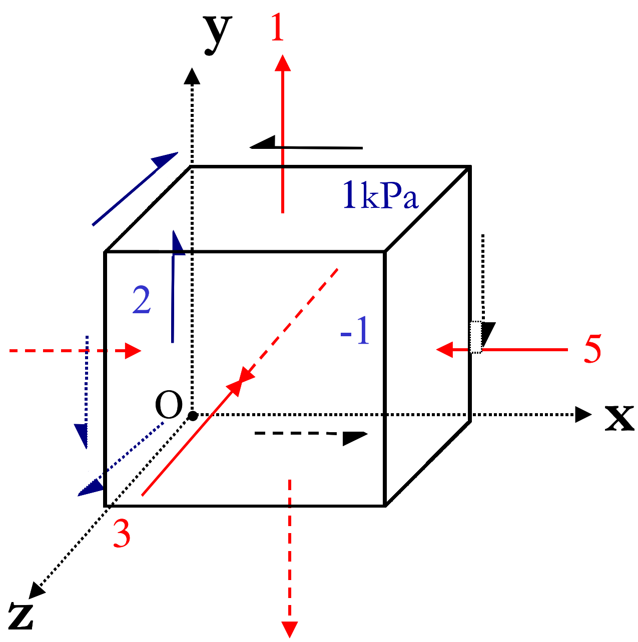
$\sigma_{\mathrm{c}} = -10,900\ \mathrm{psi}$

3. The stress pseudovector for a linear elastic isotropic material is given below (E=45kPa and ν=0.35) :

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} -5 \\ 1 \\ -3 \\ 2 \\ 0 \\ -1 \end{Bmatrix}$$

a) Draw a 3-dimensional cubic stress element showing each stress and the corresponding reaction stresses. (4 pts)
b) What is the magnitude and direction of the largest normal strain? (3 pts)
c) What is the magnitude of the largest shear strain? What plane does it take place in? (3 pts)

ANSWER 2.a.



2.b. Hooke's Law for a linear elastic, isotropic material under multiaxial loading in matrix form is :

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = 1/E \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ . & 1 & -\nu & 0 & 0 & 0 \\ . & . & 1 & 0 & 0 & 0 \\ . & . & . & 2(1+\nu) & 0 & 0 \\ . & . & . & . & 2(1+\nu) & 0 \\ . & . & . & . & . & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}$$

which corresponds to the following set of six equations :

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu\sigma_y - \nu\sigma_z]$$

$$\varepsilon_y = \frac{1}{E}[-\nu\sigma_x + \sigma_y - \nu\sigma_z]$$

$$\varepsilon_z = \frac{1}{E}[-\nu\sigma_x - \nu\sigma_y + \sigma_z]$$

$$\gamma_{yz} = 2(1+\nu)\tau_{yz}/E$$

$$\gamma_{xz} = 2(1+\nu)\tau_{xz}/E$$

$$\gamma_{xy} = 2(1+\nu)\tau_{xy}/E$$

Substituting in the given numerical values yields :

$$\varepsilon_x = \frac{1}{25}\left[-5-(0.35*1)-(0.35*-3)\right] = 0.04\left[-5-0.35+1.05\right] = -0.172$$

$$\varepsilon_y = \frac{1}{25}\left[-(0.35*-5)+1-(0.35*-3)\right] = 0.04\left[1.75+1+1.05\right] = +0.152$$

$$\varepsilon_z = \frac{1}{25}\left[-(0.35*-5)-(0.35*1)-3\right] = 0.04\left[1.75-0.35-3\right] = -0.064$$

$$\gamma_{yz} = \frac{2(1+0.35)*2}{25} = +0.216$$

$$\gamma_{xz} = \frac{2(1+0.35)*0}{25} = 0$$

$$\gamma_{xy} = \frac{2(1+0.35)*-1}{25} = -0.108$$

a) The largest normal strain is ε_x=-0.172 (compressive). ***Ans.***

b) The largest shear strain is γ_{yz}=+0.216 ***Ans.***

4. Define the following words in one to two sentences (<u>no formulas</u>): (2 pts each, 12 point total).

a) anisotropic :The mechanical properties of a materials are direction dependent.

b) compliance matrix : Multiplication of this matrix by the stress matrix yields the strain matrix (i.e. inverted Hooke's Law), the inverse of the stiffness matrix.

c) Poisson's ratio : The ratio of lateral normal strain to axial strain **in uniaxial deformation.**

d) failure strength : The **stress** at which a material fractures or fails such that the load is reduced to zero.

e) plane stress : A two dimensional stress state in which there are only three independent stress components,e.g. σ_x, σ_y, and τ_{xy} (all other stress components go to zero).

f) strain energy : The energy absorbed by a material when it is deformed which is equal to the external amount of work done on the material.