3.11 Mechanics of Materials F02
Exam #2 SOLUTIONS

*(*show all of your work / calculations to get as much credit as possible)*

(Each question worth 10 PTS)

1. The attractive interatomic(ionic) potential between two oppositely charged ions of Na^+ and Cl^- in air, U(r), is given by :

$$U_{attractive}(r) = -[Q_1Q_2/4\pi\varepsilon_o]r^{-1}$$

where : $Q_1=Q_2=Q=1.602\bullet10^{-19}$C is the electric charge in Coulombs, ε_o is the dielectric permittivity of free space=$8.854*10^{-12}$ C^2 $J^{-1}m^{-1}$, and r is the interionic separation distance. The (steric) repulsive molecular interaction parameters using a soft, power law repulsion are : $B=10^{-135}$ Jm^{12}, m=12. Determine the binding energy, E_B, in units of k_BT. (*HINT : first *calculate* the equilibrium bond length).

ANS. Using a steric, soft, power law repulsion of :

$$U_{repulsive}(r) = \frac{+B}{r^m}$$

then the total interionic potential is :

$$U_{total}(r) = U_{attractive}(r) + U_{repulsive}(r) = \frac{-A}{r} + \frac{B}{r^{12}} \quad (1)$$

where :

$$A = \frac{Q_1Q_2}{4\pi\varepsilon_o} = \frac{\left(1.602\bullet10^{-19}C\right)^2}{4\pi\left(8.854\bullet10^{-12}C^2\ J^{-1}m^{-1}\right)} = 2.3\bullet10^{-28}Jm$$

To find the equilibrium bond length, r_e, take the negative derivative of equation *(1)* w/r/t to r, set=0, and solve for r_e :

$$F_{total}(r) = \frac{dU_{total}(r)}{dr} == \frac{-A}{r^2} + \frac{12B}{r^{13}}$$

$$F_{total}(r_e) = 0 = \frac{-A}{r_e^{\,2}} + \frac{12B}{r_e^{\,13}}$$

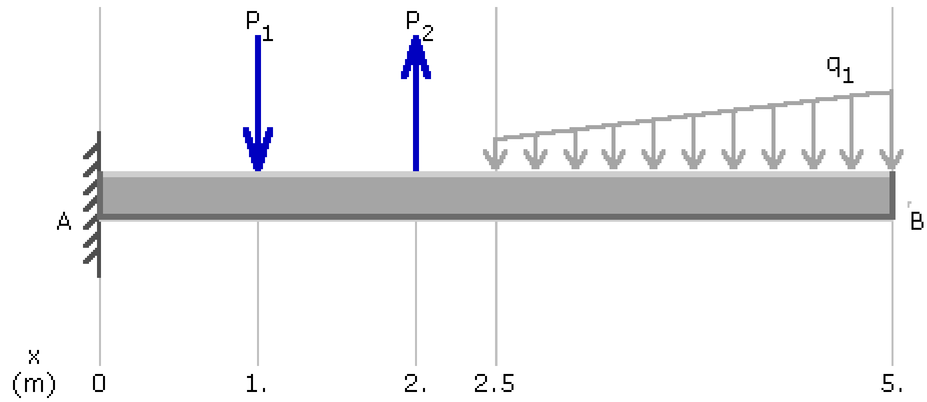
$$r_e = \sqrt[11]{\frac{12B}{A}} = \sqrt[11]{\frac{12\left(10^{-135}\ Jm^{12}\right)}{2.3\bullet10^{-28}\ Jm}} = 0.217\ nm$$

Substitute r_e into equation *(1)* :

$$E_B = U_{total}(r_e = 0.217\ nm) = \left[\frac{-2.3 \bullet 10^{-28}\,Jm}{0.217 \bullet 10^{-9}\,m} + \frac{10^{-135}\ Jm^{12}}{\left(0.217 \bullet 10^{-9}\,m\right)^{12}}\right]\left[\frac{1 k_B T}{4.1 \bullet 10^{-21}\,J}\right]$$

$$E_B = 220\ k_B T$$

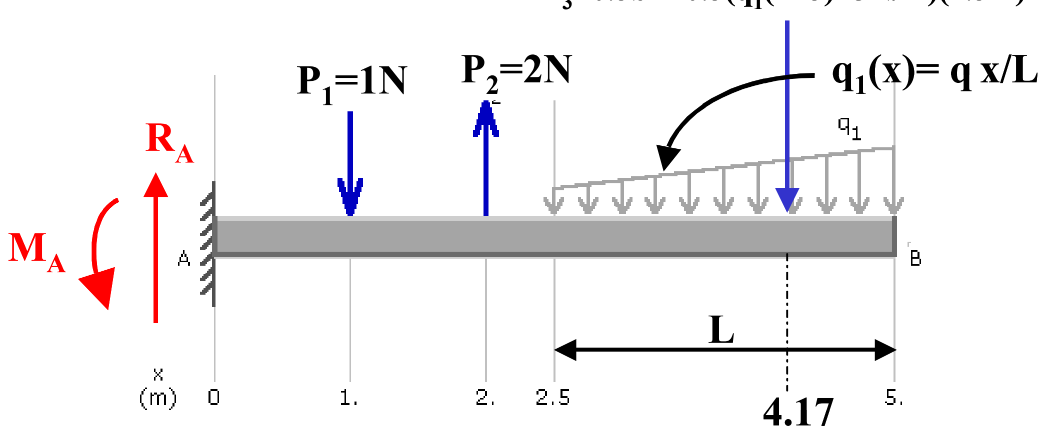
. A cantilever beam is used to support a linearly distributed load of intensity q_1 from 0 to 3 N/m and two concentrated loads $P_1 = 1$ N and $P_2 = 2$ N, as shown in the figure below.



(a) Draw the shear force diagram, V(x), and label the location (along x-axis) and magnitude of the maximum shear force.
(b) Draw the bending moment diagram, M(x), and label the location (along x-axis) and magnitude of the maximum bending moment.
****PLEASE LABEL NUMERICAL VALUES ON YOUR PLOTS AT x=0, 1, 2, 2.5, and 5. USE APPROPRIATE SIGN CONVENTIONS****

ANS. (a) Consider a free-body diagram of the entire beam and the equations of static equilibrium to determine the reaction shear force and bending moment at the fixed end **A**. Take upwards as (+) and downwards as (-).

P_3=0.5bh=0.5(q_1(x=5)=3N/m)(2.5m)=3.75N



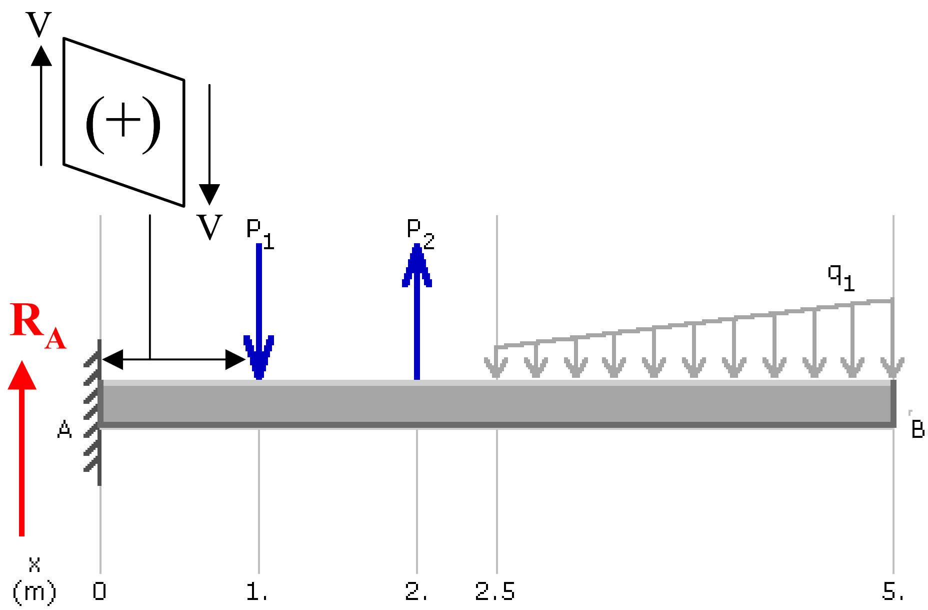
*(*acting through centroid=2b/3=0.83)*

$\Sigma F_Y=0= -P_1+P_2-P_3+R_A$
$R_A=P_1-P_2+P_3=(1-2+3.75)N$
$\mathbf{R_A=+2.75N}$
*(*positive answer means assumed direction is correct)*

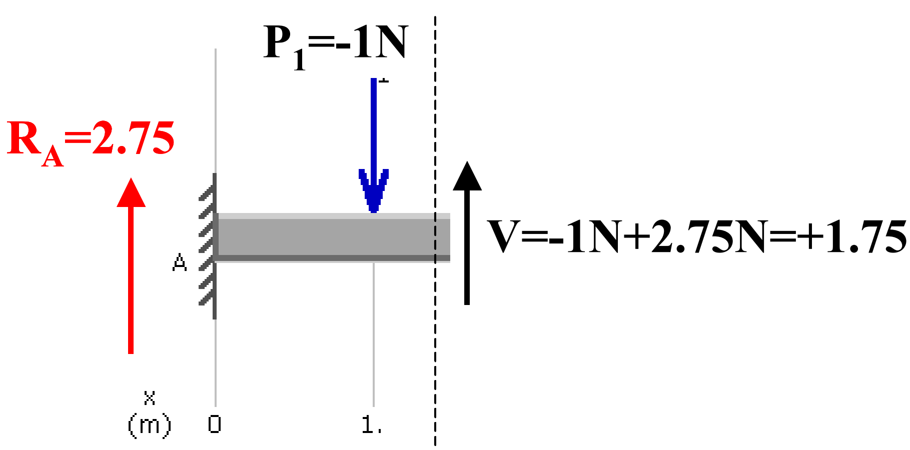
Taking (CCW+) and (CW-) :
$\Sigma M_A=0=M_A-P_1(1\ m)+P_2(2\ m)-P_3(4.17m)$
$\Sigma M_A=0=M_A-(1N)(1m)+(2N)(2m)-(3.75N)(4.17m)$
$\Sigma M_A=0=M_A-12.6\ Nm$
$\mathbf{M_A=+12.6Nm}$
*(*positive answer means assumed direction is correct)*

<u>Shear Force Diagram :</u>
You may start from either side of the beam.

0.0<x<1.0 Starting from the left hand side of the beam, the shear force is constant with x and equal in magnitude to R_A=2.75N. The sign is obtained by noting the shear directions as follows:



1.0<x<2.0 For the concentrated load, P_1(x=1)=1N, the shear will exhibit a discontinuous jump in the value of the V(x=1) equal in magnitude to P_1. We can tell that V(x) will jump down to V(x=1)=2.75-1=+1.75 N via a free body diagram :



2.0<x<2.5. For the concentrated load, $P_2(x=2)=2N$, the shear will exhibit a discontinuous jump in the value of the V(x=2) up to V(x=2.0)=1.75 N+2N=+3.75 N.

2.5<x<5.0 For linearly distributed loads, the shear force is parabolic with distance:

$$q_1(x)=\frac{qx}{L}$$

$$V(x)=-\int q_1(x)dx=-\int \frac{qx}{L}dx=\frac{-qx^2}{2L}+C_1$$

From above, $V(x=0)=+3.75 \rightarrow C_1=+3.75N$

$$V(x)=\frac{-qx^2}{2L}+3.75$$

$$V(x=L)=\frac{-qL}{2}+3.75=0$$

V_{max} is then equal to V(x=2.5) =+3.75 N.

Bending Moment Diagram :

By definition : $M(x)=\int V(x)dx$

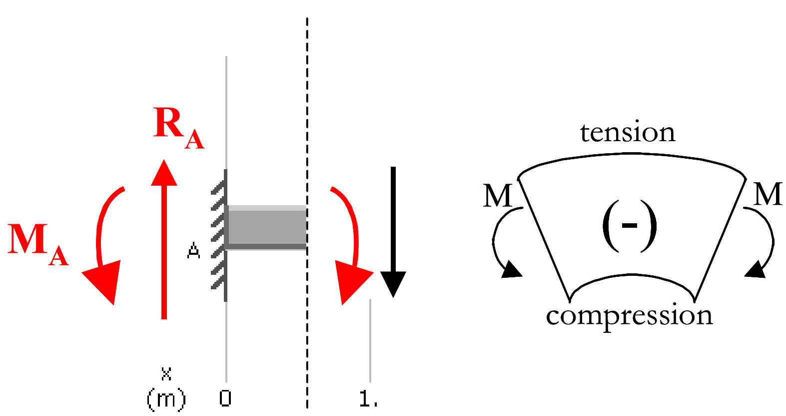
0<x<1.0

$$M(x)=\int V(x)dx=\int 2.75\,dx=2.75x+C_1$$

From the initial FBD, M(x=0)= -12.63 $=C_1$

(the sign is determined from the FBD below)

$M(x)=2.75x-12.63$



Hence, the M(x) diagram is linear with slope +2.75 and y-intercept -12.63.
M(x=1)=2.75(1)-12.63=-9.88Nm.

1.0<x<2.0 Following the same methodology abovethe M(x) diagram is linear with slope +1.75 and M(x=2)=-8.13 Nm.

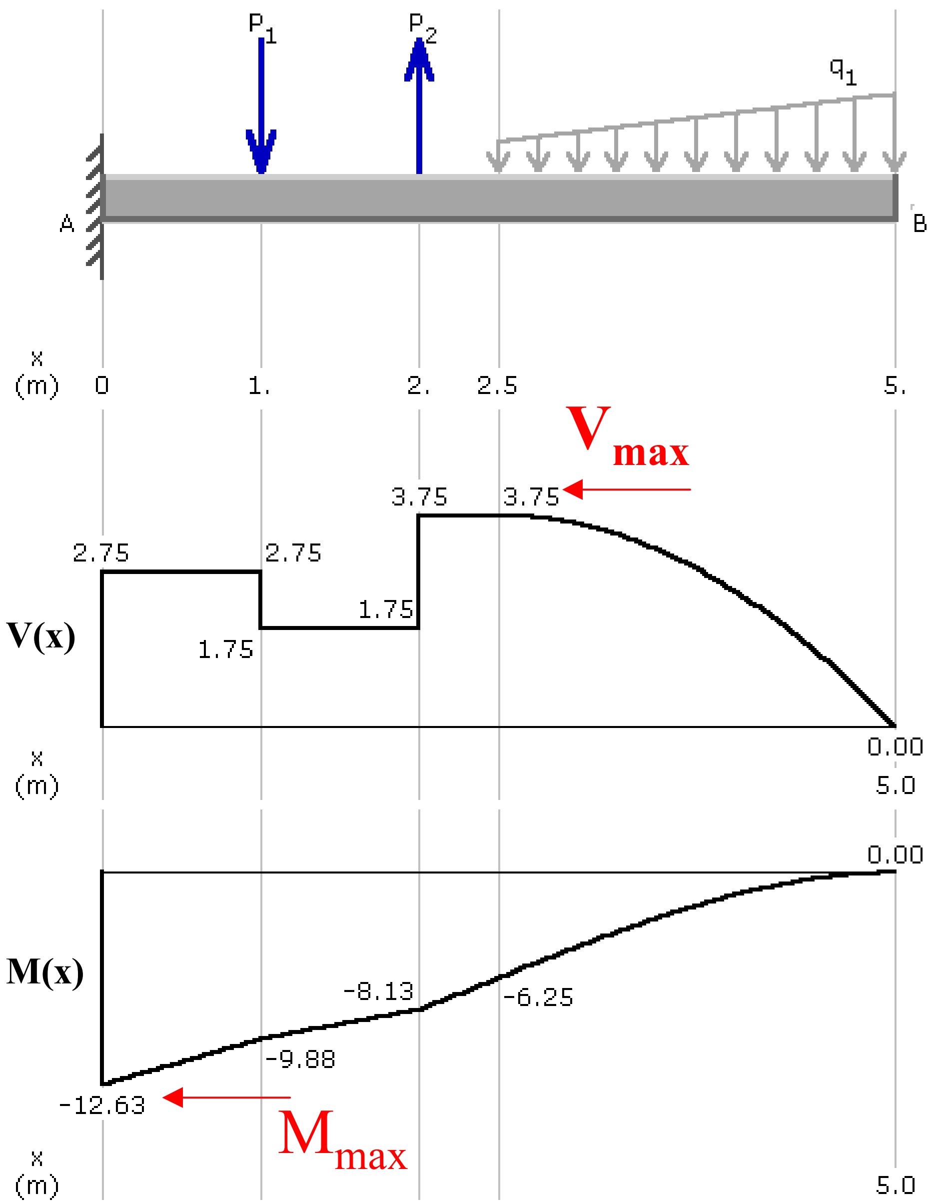
2.0<x<2.5 Following the same methodology above the M(x) diagram is linear with slope +3.75 and M(x=2.5)=-6.25 Nm.

2.5<x<5.0

$$M(x)=\int\left(\frac{-qx^2}{2L}+3.75\right)dx=\frac{-qx^3}{6L}+3.75x+C_1$$

$$M(x=0)=-6.25 \rightarrow C_1=-6.25$$

$$M(x)=\frac{qx^3}{6L}-3.75x-6.25$$



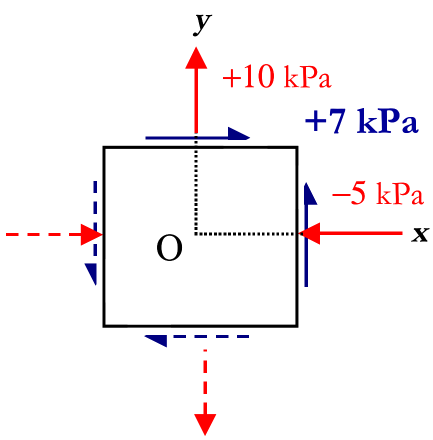
3. A state of plane stress consists of the following stresses : σ_x=-5 kPa, σ_y=+10 kPa, and τ_{xy}=+7kPa.

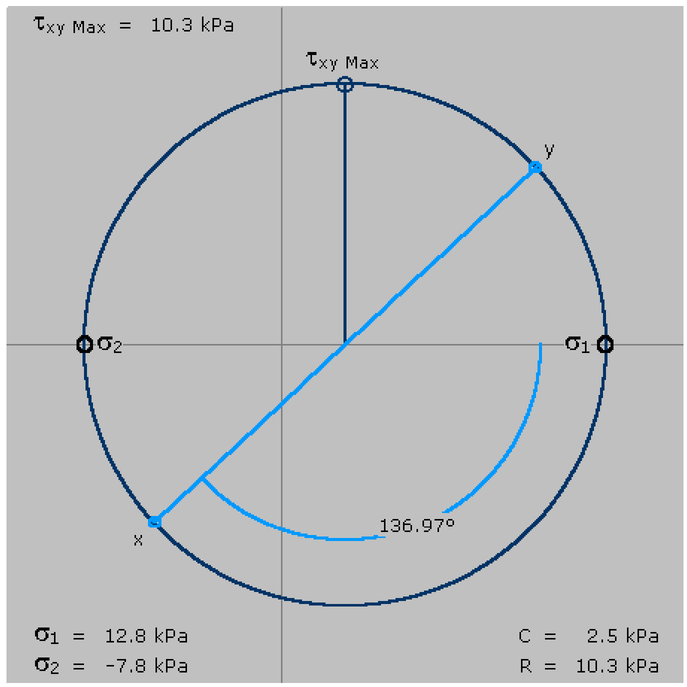
(a) **Draw the original unrotated element and the corresponding 2-D Mohr's circle construction showing the x-face and y-face coordinates.**

(b) **Calculate the principal stresses, σ_1 and σ_2 and their corresponding principal angles, θ_{p1},θ_{p2} and show all of these on your Mohr's circle construction.**

(c) **Calculate the maximum shear stresses, $\pm\tau_{MAX}$, and their corresponding angles of maximum shear stress, θ_{s1},θ_{s2} and show all of these on your Mohr's circle construction.**

(d) **Calculate the stresses on an element rotated by 75 degrees.**





$$\theta_{P1} = \frac{136.97^o}{2} = 68.4^o, \theta_{P2} = 158.4^o$$

$$\theta_{s1} = 113.4^o, \theta_{s2} = 203.4^o$$

(d) σx' = 12.5 kPa Tension (+)

σy' = 7.5 kPa Compression (-)

τx'y' = 2.3 kPa CW on x face (-)