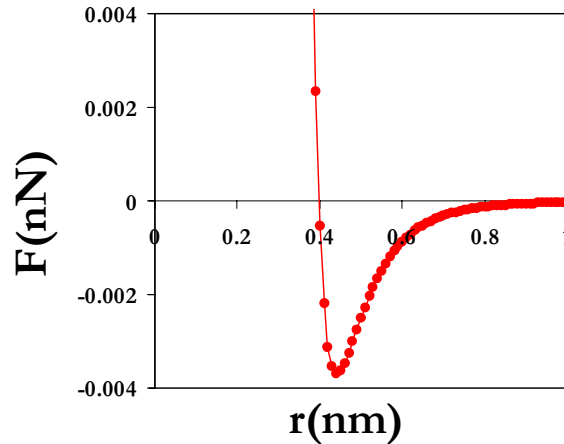


3.11 Mechanics of Materials F00

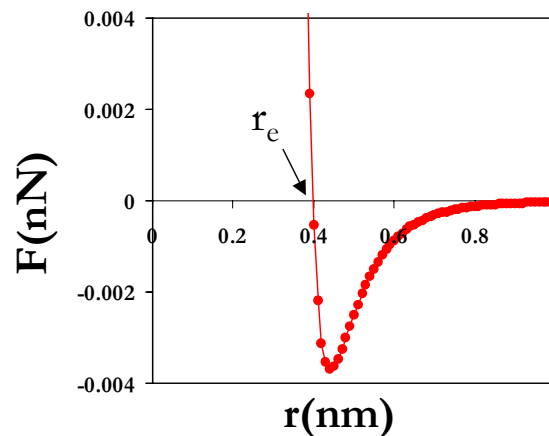
Exam #3 Sample Problems

1. Q. Two atoms interact at $T=0^\circ\text{K}$ via a van der Waals Lennard-Jones potential with $A=4.7\cdot 10^{-78}\text{ Jm}^6$. The interaction force versus separation distance plot is given in the following figure.



- (a) Calculate the binding energy ($k_B T$) and the bond stiffness (N/m).
(b) The two atoms are held at a particular separation distance r using an atomic force microscope so that the *attractive component* of the force is equal to -0.003 nN . At this distance are the atoms attracted to each other or repelled away from each other? Justify your answer with a numerical calculation.

A. (a) the equilibrium bond length, r_e , can be read directly off the plot as the separation distance where the $F(r_e)=0 \Rightarrow r_e=0.4\text{ nm}$.



$$r_c = [2B/A]^{1/6} \quad (1)$$

Solve equation (1) for : $B = Ar_c^6/2$ (2)

Substitute in equation (2) known values for r_c and A :

$$B = (4.7 \cdot 10^{-78} \text{Jm}^6)(0.4 \cdot 10^{-9} \text{m})^6/2 = 10^{-134} \text{Jm}^{12}$$

$$E_B = -[A^2/4B] = -[(4.7 \cdot 10^{-77} \text{Jm}^6)^2/4 \cdot 10^{-134} \text{Jm}^{12}] = -5.522 \cdot 10^{-22} \text{J} \cdot k_B T / 4.1 \cdot 10^{-21} \text{J}$$

$$E_B = -0.135 k_B T$$

$$k_{\text{bond}} = 42A/r_c^8 - 156B/r_c^{14} = 42 \cdot 4.7 \cdot 10^{-77} \text{Jm}^6 / (0.4 \cdot 10^{-9})^8 - 156 \cdot 10^{-134} \text{Jm}^{12} / (0.4 \cdot 10^{-9})^{14}$$

$$k_{\text{bond}} = 0.3 \text{ N/m}$$

$$(b) F = -6A/r^7 + 12B/r^{13}$$

$$F_{\text{attractive}} = -6A/r^7 \quad (3)$$

$$F_{\text{repulsive}} = 12B/r^{13} \quad (4)$$

Solve equation(3) for r \Rightarrow $r = [-6A/-F_{\text{attractive}}]^{1/7}$

$$r = [-6 \cdot 4.7 \cdot 10^{-77} \text{Jm}^6 / -0.003 \cdot 10^{-9} \text{N}]^{1/7} = 0.5 \text{ nm}$$

Substitute r and B into equation (4) \Rightarrow $F_{\text{repulsive}} = 12 \cdot 10^{-134} \text{Jm}^{12} / (0.5 \cdot 10^{-9} \text{m})^{13}$

$$F_{\text{repulsive}} = 0.0009 \text{ nN}$$

$$F_{\text{net}} = F_{\text{attractive}} + F_{\text{repulsive}} = -0.003 \text{ nN} + 0.0009 \text{ nN}$$

$$F_{\text{net}} = -0.00262$$

[NET FORCE IS ATTRACTIVE]

2. Q. (a) Calculate the true stress necessary to deform a rubber material of initial length, $L_o = 0.1 \text{ m}$ to $L_f = 0.2 \text{ m}$ in uniaxial tension assuming the Gaussian theory of rubber elasticity and a reasonable value for the Young's modulus.

(b) An elastomer that is put in a liquid solvent typically exhibits a macroscopic increase in volume, i.e. it "swells." Explain the molecular origin of this phenomenon in a few sentences.

A. (a) The stress versus strain law for the Gaussian theory of rubber elasticity is :

$$\sigma = \nu_x k_B T (\lambda - 1/\lambda^2) \quad (1)$$

where : $\lambda = L_f/L_o = 2$ (2)

$$E = 3\nu_x k_B T \Rightarrow \nu_x k_B T = E/3 = 0.5 \text{ MPa}/3 = 0.166 \text{ MPa} \quad (3)$$

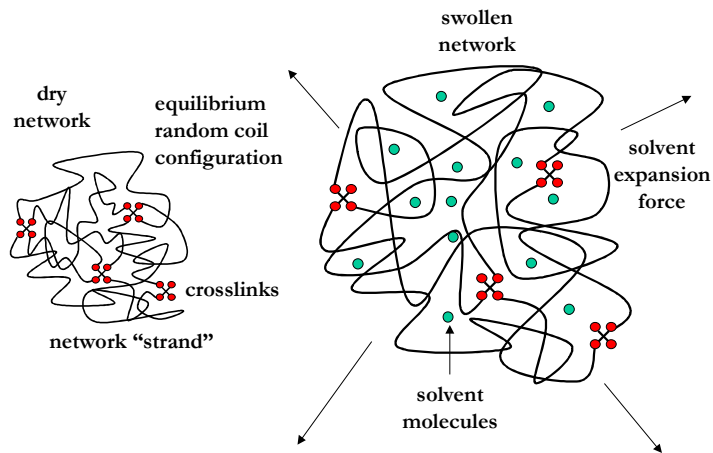
Substitute (2) and (3) into (1): $\sigma = 0.166 \text{ MPa} (2 - 1/2^2) = 0.2905 \text{ MPa}$

$$\sigma_f = \sigma(\epsilon + 1)$$

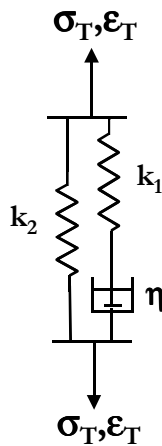
$$\lambda = 1 + \epsilon \Rightarrow \epsilon = \lambda - 1 = 2 - 1 = 1$$

$$\sigma_f = 0.2905 \text{ MPa} (1 + 1) = 0.581 \text{ MPa}$$

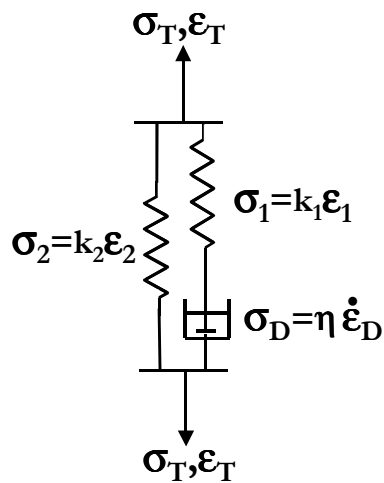
(b) An elastomer is a chemically crosslinked polymer structure where the chains between crosslinks are "random coils" due to entropic considerations. The solvent molecules enter the network producing an expansion force that expands the polymer chains isotropically in all directions, thus causing a macroscopic increase in volume.



3. Q. Derive the governing differential equation (σ_T as a function of ϵ_T) for the following viscoelastic model in terms of k_1 , k_2 , η .



A. The governing equations for the individual elements are shown below :



In series \Rightarrow the strains are additive, stresses equal
 In parallel \Rightarrow the strains are equal, stresses additive

$$\epsilon_T = \epsilon_2 = \epsilon_1 + \epsilon_D \quad (1)$$

$$\sigma_T = \sigma_2 + \sigma \quad (2)$$

where : $\sigma = \sigma_1 = \sigma_D$

$$\sigma_T = k_2 \epsilon_T + \sigma \Rightarrow \sigma = \sigma_T - k_2 \epsilon_T \quad (3)$$

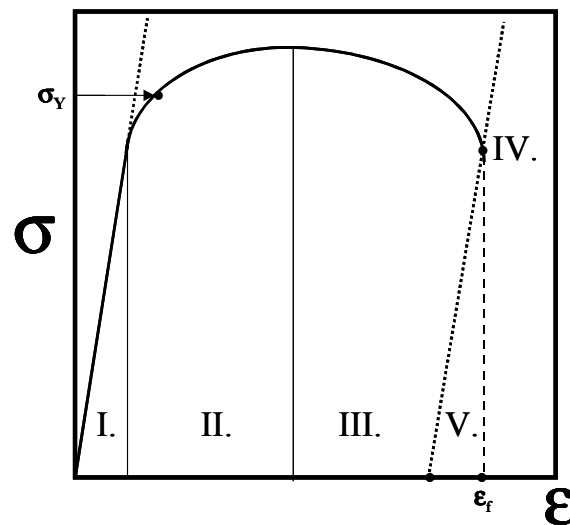
take time derivative of equation (1) : $d\epsilon_T/dt = d\epsilon_1/dt + d\epsilon_D/dt$

$$d\epsilon_T/dt = (d\sigma/dt)/k_1 + \sigma/\eta \quad (4)$$

substitute (3) into (4) : $d\epsilon_T/dt = (d\sigma_T/dt)/k_1 - k_2(d\epsilon_T/dt)/k_1 + \sigma_T/\eta - k_2\epsilon_T/\eta$

rearrange : $(d\sigma_T/dt)/k_1 + \sigma_T/\eta = d\epsilon_T/dt + k_2\epsilon_T/\eta + k_2(d\epsilon_T/dt)/k_1$

4. Q. The macroscopic uniaxial stress versus strain curve for a ductile polycrystalline metal is given in the Figure below. Explain what molecular or microscopic level phenomenon dominate the response in each of the labeled regions of the curve (*give one or two sentences for each region).



- A. **I. Linear Elasticity** : crystalline lattice strain dominates, i.e. bond stretching, bending, twisting.
II. Strain Hardening: dislocation movement, slip on crystallographic planes closest to 45°, dislocation pile-up within and at grain boundaries
III. Necking : unstable decrease in cross-sectional area with increasing strain due to plastic deformation described above
IV. Fracture : Separation of the material into new surfaces through bond breaking, the formation and propagation of a cracks
V. Elastic Strain Recovery : the leftover intact material recovers its elastic lattice strain